Analysis for Steady Propagation of a Generic Ram Accelerator/Oblique Detonation Wave Engine Configuration

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Objective of Study

- Describe a methodology to determine a steady propagation speed of a projectile fired into a gaseous fuel and oxidizer mixture.

- Perform a simple theoretical and numerical analyses to illustrate the methodology.
Inert Oblique Shock

$M = 8.4$

$P = 16 \text{ bar}$

Reaction-inducing Oblique Shock (Oblique Detonation)

$M > 1$

$P \approx 600 \text{ bar}$

Projectile

$m = 70 \text{ g}$

$v = 2,475 \text{ m/s}$

$0.9\text{C}_2\text{H}_2 + 3\text{O}_2 + 5\text{CO}_2$

Accelerator Barrel

$166 \text{ mm}$

**Ram Accelerator, Hertzberg, et al., 1988, 1991**

Oblique Detonation Wave Engine, Dunlap, et al., 1958
Selected Past Work

1. Theoretical:

- Brackett and Bogdanoff, 1989, *(steady speeds)*
- Cambier, Adelman, and Menees, 1989, 1990,
- Pratt, Humphrey, and Glenn, 1991,
- Yungster and Bruckner, 1992, *(steady speed)*
- Powers and Gonthier, 1992a,b, *(steady speeds)*
- Grismer and Powers, 1992,
- Pepper and Brueckner, 1993

2. Experimental:

Modeling Difficulties

Multi-dimensional unsteady flow field

Diffusive processes:
- mass diffusion,
- momentum diffusion,
- energy diffusion.

Complex chemistry:
- multiple reactions,
- multiple species,
- complex chemical kinetics.

Complex wave interactions:
- compression waves,
- expansion waves,
- combustion waves.
Generic Configurations

upper cowl surface

incoming supersonic premixed flow

\( \tilde{y} \)

\( \tilde{x} \)

symmetric double wedge

\( \theta \)

\( \tilde{H} \)

\( \tilde{L} \)

lower cowl surface

Mach wave

oblique shock

rarefaction fan

flame sheet

axis of symmetry
Non-Dimensional Model Equations

Continuity:
\[
\frac{d\rho}{dt} + \rho \frac{\partial v_i}{\partial x_i} = 0 ,
\]

Momentum:
\[
\frac{dv_i}{dt} + \frac{1}{\rho} \frac{\partial P}{\partial x_i} = 0 ,
\]

Energy:
\[
\frac{dP}{dt} - \gamma \frac{P}{\rho} \frac{d\rho}{dt} = (\gamma-1) \rho \kappa q (1-\lambda) \exp\left(-\frac{\Theta}{T}\right) ,
\]

Species:
\[
\frac{d\lambda}{dt} = \kappa (1-\lambda) \exp\left(-\frac{\Theta}{T}\right) ,
\]

Caloric Equation of State:
\[
e = \frac{1}{\gamma-1} \frac{P}{\rho} - \lambda q ,
\]

Thermal Equation of State:
\[
P = \rho T .
\]
Non-Dimensional Variables

\[ \rho = \frac{\tilde{\rho}}{\rho_0}, \quad P = \frac{\tilde{P}}{\tilde{P}_0}, \quad T = \frac{\tilde{R}}{\tilde{P}_0/\rho_0} \tilde{T}, \]

\[ u = \frac{\tilde{u}}{\sqrt{\tilde{P}_0/\rho_0}}, \quad v = \frac{\tilde{v}}{\sqrt{\tilde{P}_0/\rho_0}}, \quad e = \frac{\tilde{e}}{\tilde{P}_0/\rho_0}, \]

\[ x = \frac{\tilde{x}}{\tilde{L}}, \quad y = \frac{\tilde{y}}{\tilde{L}}, \quad t = \frac{\sqrt{\tilde{P}_0/\rho_0}}{\tilde{L}} \tilde{t}. \]

Non-Dimensional Parameters

\[ q = \frac{\tilde{q}}{\tilde{P}_0/\rho_0}, \quad \Theta = \frac{\tilde{E}}{\tilde{P}_0/\rho_0}, \quad \gamma = 1 + \frac{\tilde{R}}{\tilde{C}_v}, \]

\[ \kappa = \frac{\tilde{L}}{\sqrt{\tilde{P}_0/\rho_0}}, \quad M_0 = \frac{\tilde{u}_0}{\sqrt{\gamma \tilde{P}_0/\rho_0}}. \]
Reaction Model

- Simple one-step, irreversible reaction:

\[ \lambda \]

\[ A \xrightarrow{\lambda} B \quad \text{(exothermic reaction)} \]

\[ \lambda = \text{reaction progress variable} \]

\[ Y_A = 1 - \lambda, \quad Y_B = \lambda. \]

- Arrhenius kinetics:

\[ \text{kinetic rate} \propto \exp(-E_a / RT) \]

- High activation energy limit.
Thermal Explosion Theory

Reduced Equations (assumed $v_i = 0$):

\[
\frac{dP}{dt} = (\gamma-1) \rho \kappa q (1-\lambda) \exp \left( -\frac{\Theta \rho}{P} \right),
\]

\[
\frac{d\lambda}{dt} = \kappa (1-\lambda) \exp \left( -\frac{\Theta \rho}{P} \right),
\]

\[P(0) = P_1, \quad \lambda(0) = 0.\]

Linearize the equations:

\[P = P_1 + P', \quad \lambda = \lambda'.\]

where

\[P' \ll 1, \quad \lambda' \ll 1.\]

Solve for the pressure perturbation $P'$. 
Solution:

\[ P' = - \frac{P_1^2}{\Theta \rho_1} + \ln \left[ 1 - \frac{\Theta \rho_1^2}{P_1^2} (\gamma - 1) q \kappa \left( \exp \frac{-\Theta \rho_1}{P_1} \right) t \right]. \]

Solve for the thermal explosion time:
(corresponds to the induction time)

\[ t_{ind} = \frac{P_1^2}{(\gamma - 1) \rho_1^2 \Theta \kappa q} \exp \left[ \frac{\Theta \rho_1}{P_1} \right]. \]

Induction distance:

\[ L_{ind} = t_{ind} \sqrt{u_1^2 + v_1^2}. \]
Calculation of Surface Forces

1. Wave Drag:

\[ F_D = \frac{1}{2} (P_1 - P_3) \tan \theta . \]

2. Net Thrust Force:

\[ F_{net} = P_3 \left( 2 L_{ind} \cos \theta - 1 \right) \tan \theta \\
+ \left[ P_4 \left( 2 - 2 L_{ind} \cos \theta \right) - P_1 \right] \tan \theta . \]

3. Combustion Induced Thrust:

\[ F_c = F_{net} + F_D . \]
Jump Relations Across Lead Shock

\[
\rho_1 = \left(\frac{1 + \gamma M_0^2 \sin^2\beta \pm \sqrt{A}}{(\gamma + 1) M_0^2 \sin^2\beta}\right)^{-1}
\]

\[
u_1 = \sqrt{\gamma} M_0 \left(\frac{1}{\rho_1} \sin^2\beta + \cos^2\beta\right)
\]

\[
v_1 = \sqrt{\gamma} M_0 \cos \beta \sin \beta \left(1 - \frac{1}{\rho_1}\right)
\]

\[
P_1 = 1 + \gamma M_0^2 \sin^2\beta \left(1 - \frac{1}{\rho_1}\right)
\]

where

\[
A = \left(1 + \gamma M_0^2 \sin^2\beta\right)^2 - (\gamma + 1) \gamma M_0^2 \sin^2\beta \\
\times \left(2 + (\gamma - 1) M_0^2 \sin^2\beta\right)
\]
Flow Expansion Region

Prandtl - Meyer Function:

\[ v(M_3) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M_3^2 - 1) - \tan^{-1} \sqrt{M_3^2 - 1}} \]

Isentropic Relations:

\[ \frac{\rho_3}{\rho_1} = \left( \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_3^2} \right)^{\frac{1}{\gamma-1}} \]

\[ \frac{P_3}{P_1} = \left( \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_3^2} \right)^{\frac{\gamma}{\gamma-1}} \]

Velocity Components:

\[ u_3 = M_3 \sqrt{\frac{\gamma P_3}{\rho_3}} \cos \theta \quad , \quad v_3 = -M_3 \sqrt{\frac{\gamma P_3}{\rho_3}} \sin \theta \]
Jump Relations Across Flame Sheet

\[
\frac{\rho_4}{\rho_3} = \left( \frac{1 + \gamma M_3^2 \pm \sqrt{B}}{(\gamma + 1) M_3^2} \right)^{-1}
\]

\[
\frac{P_4}{P_3} = 1 + \gamma M_3^2 \left( 1 - \frac{\rho_3}{\rho_4} \right)
\]

\[
u_4 = \left( \frac{\rho_3}{\rho_4} \right) u_3 \cos \theta, \quad v_4 = -\left( \frac{\rho_3}{\rho_4} \right) u_3 \sin \theta
\]

where

\[
B = (1 + \gamma M_3^2)^2 - (\gamma - 1) M_3^2
\]

\[
\times \left( 2 + (\gamma - 1) M_3^2 + 2 (\gamma - 1) \frac{\rho_3}{P_3} Q \right)
\]
Tail End Compression Region

\[
\rho_5 = \frac{\left(1 + \gamma M_4^2 \sin^2(\alpha + \theta) \pm \sqrt{C} \right)^{-1}}{(\gamma + 1) M_4^2 \sin^2(\alpha + \theta)}
\]

\[u_5 = \sqrt{\gamma} M_4 \left(\frac{\rho_4}{\rho_5} \sin^2(\alpha + \theta) + \cos^2(\alpha + \theta)\right)\]

\[v_5 = \sqrt{\gamma} M_4 \cos(\alpha + \theta) \sin(\alpha + \theta) \left(1 - \frac{\rho_4}{\rho_5}\right)\]

\[\frac{P_5}{P_4} = 1 + \gamma M_4^2 \sin^2(\alpha + \theta) \left(1 - \frac{\rho_4}{\rho_5}\right)\]

where

\[C = \left(1 + \gamma M_4^2 \sin^2(\alpha + \theta)\right)^2 - (\gamma + 1) \gamma M_4^2 \sin^2(\alpha + \theta) \times \left(2 + (\gamma - 1) M_4^2 \sin^2(\alpha + \theta)\right)\]
Numerical Analysis

**RPLUS Code:**

- developed at NASA Lewis,

- based on LU-SSOR numerical scheme.

**Computational Grid:**

- 199 x 99 fixed grid.

**Convergence:**

- 500 iterations,

- residual unsteady terms had scaled values $\leq 1.0 \times 10^{-8}$.

**Computations:**

- run on IBM RS/6000 POWERstation 350

- run time about one hour.
Parameters

Geometric:

\[ \theta = 5^\circ, \quad \bar{L} = 0.10 \text{ m} . \]

Kinetic:

\[ \bar{k} = 1.0 \times 10^7 \text{ /sec}, \quad \bar{E} = 1.019 \times 10^6 \text{ J/kg}, \]

\[ 1.295 \times 10^6 \text{ J/kg} \leq \bar{q} \leq 1.704 \times 10^7 \text{ J/kg}. \]

Atmospheric free-stream conditions:

\[ \bar{P}_0 = 1.01325 \times 10^5 \text{ Pa}, \quad \bar{\rho}_0 = 1.225 \text{ kg/m}^3. \]

Thermodynamic constants:

\[ \gamma = \frac{7}{5}, \quad \bar{R} = 287 \text{ J/(kg K)}, \quad \bar{c}_v = 717.5 \text{ J/(kg/K)}. \]
Pressure on Wedge Surface

- rise on forebody due to shock and reaction
- drop on aftbody due to rarefaction
- lack of crisp shock indicates more resolution necessary
- propagation speed sensitive to local pressure
- trends plausible
Pressure traces on wedge surface.

$k = 1 \times 10^7, \quad Ea = 3550$

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<table>
<thead>
<tr>
<th>Mach 13.4</th>
<th>Inert</th>
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\[ \tilde{q}, \text{ Heat of Reaction (J/kg)} \]

\[ 1.30 \times 10^6 \, \text{ to } 1.33 \times 10^6 \]

\[ \begin{array}{cccc}
15.60 & 15.70 & 15.80 & 15.90 \\
\end{array} \]

\[ \begin{array}{cccc}
16.00 & 16.10 & 16.20 & 16.30 \\
\end{array} \]

\[ M_0, \text{ Mach Number} \]

\[ \begin{array}{cccc}
6 & 7 & 8 & 9 \\
\end{array} \]

\[ \begin{array}{cccc}
10 & 11 & 12 & 13 \\
\end{array} \]

\[ \tilde{q}, \text{ Heat of Reaction (J/kg)} \]

\[ 1.40 \times 10^6 \, \text{ to } 1.70 \times 10^6 \]

\[ \begin{array}{cccc}
16 & 17 & 18 & 19 \\
\end{array} \]

\[ \begin{array}{cccc}
20 & 21 & 22 & 23 \\
\end{array} \]

\[ M_0, \text{ Mach Number} \]

\[ \begin{array}{cccc}
10 & 12 & 14 & 16 \\
\end{array} \]

\[ \begin{array}{cccc}
18 & 20 & 22 & 24 \\
\end{array} \]
Quasi-Stable Configuration

\( M_0 = 17.53, \ \tilde{u}_0 = 5,965 \text{ m/s} \)

\( q = 18.13, \ \tilde{q} = 1.5 \times 10^6 \text{ J/kg} \)

Reactant mass fraction

(1-\( \lambda \)) contours

Unstable Configuration

\( M_0 = 13.4, \ \tilde{u}_0 = 4,560 \text{ m/s} \)

\( q = 18.13, \ \tilde{q} = 1.5 \times 10^6 \text{ J/kg} \)

Reactant Mass Fraction

(1-\( \lambda \)) contours
Quasi-Stable Configuration

\[ M_0 = 17.53, \quad \tilde{u}_0 = 5965 \text{ m/s} \]
\[ q = 18.13, \quad \tilde{q} = 1.5 \times 10^6 \text{ J/kg} \]

Unstable Configuration

\[ M_0 = 13.4, \quad \tilde{u}_0 = 4560 \text{ m/s} \]
\[ q = 18.13, \quad \tilde{q} = 1.5 \times 10^6 \text{ J/kg} \]
Conclusions

1. The interaction of kinetic length scales with geometric length scales are important for determining steady propagation speeds.

2. Bifurcation phenomena observed for equilibrium Mach numbers:
   - high Mach number solutions stable to quasi-static perturbations.
   - low Mach number solutions, unstable.

3. Near the bifurcation point, increasing heat release increases steady propagation speed

4. Qualitative agreement exists between theoretical and numerical results.

5. Higher resolution required for quantitative accuracy.