Analysis for Steady Propagation of a Generic Ram Accelerator/ Oblique Detonation Wave Engine Configuration

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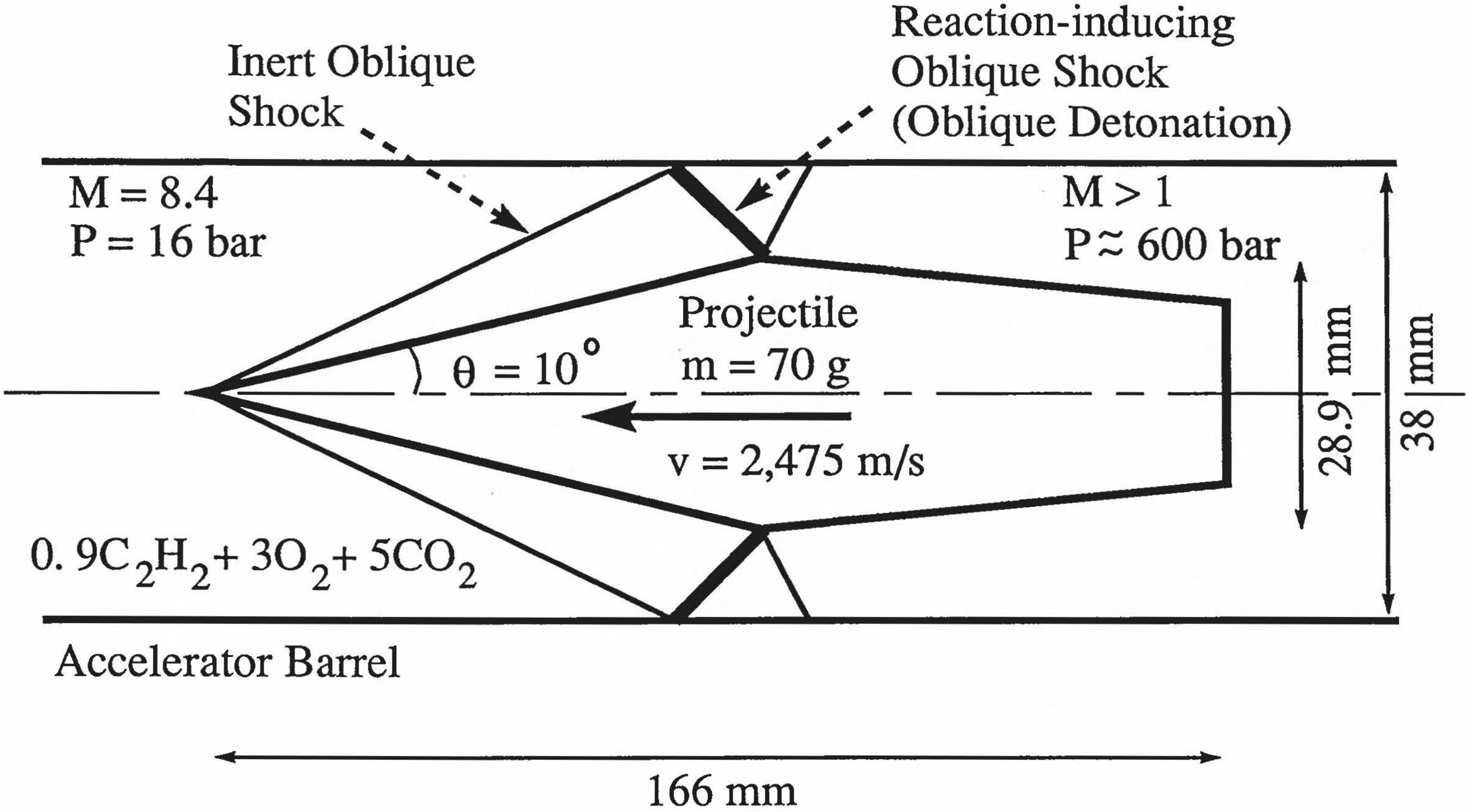
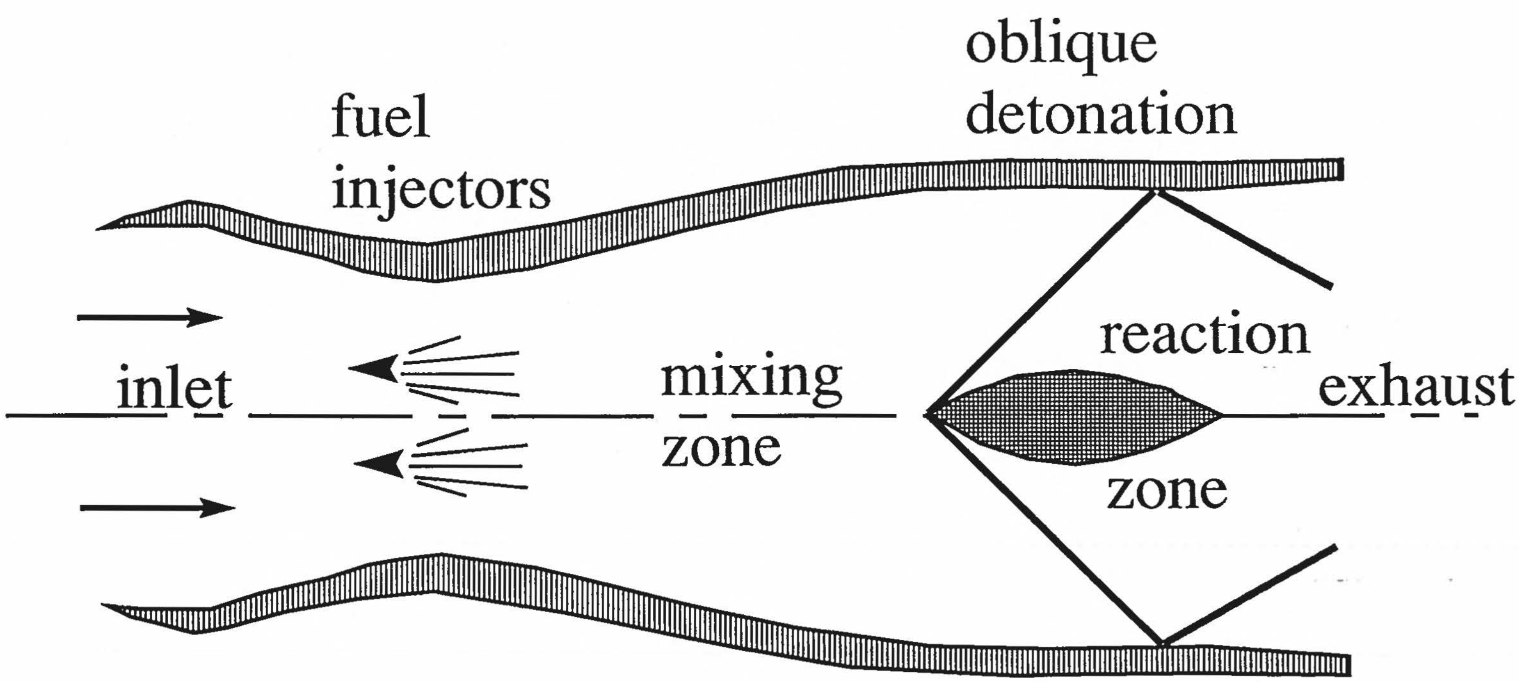
Objective of Study

- Describe a methodology to determine a steady propagation speed of a projectile fired into a gaseous fuel and oxidizer mixture.

- Perform a simple theoretical and numerical analyses to illustrate the methodology.



Ram Accelerator, Hertzberg, et al., 1988, 1991



Oblique Detonation Wave Engine, Dunlap, et al., 1958

Selected Past Work

1. Theoretical:

- *Brackett and Bogdanoff, 1989, (steady speeds)*
- Cambier, Adelman, and Menees, 1989, 1990,
- Pratt, Humphrey, and Glenn, 1991,
- *Yungster and Bruckner, 1992, (steady speed)*
- Powers and Stewart, 1992.
- *Powers and Gonthier, 1992a,*b, *(steady speeds)*
- Grismer and Powers, 1992,
- Pepper and Brueckner, 1993

2. Experimental:

- Hertzberg, Bruckner, and Knowlan, 1988, 1991.

Modeling Difficulties

Multi - dimensional unsteady flow field

Diffusive processes:

- mass diffusion,
- momentum diffusion,
- energy diffusion.

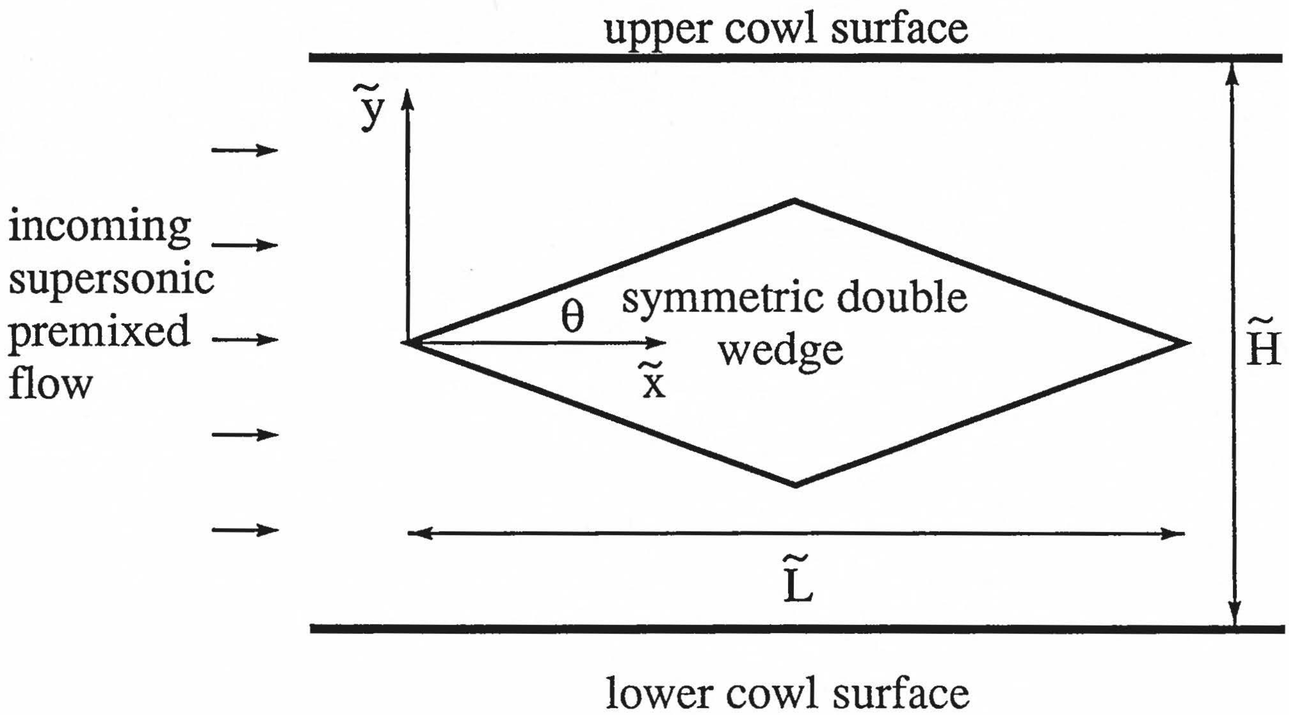
Complex chemistry:

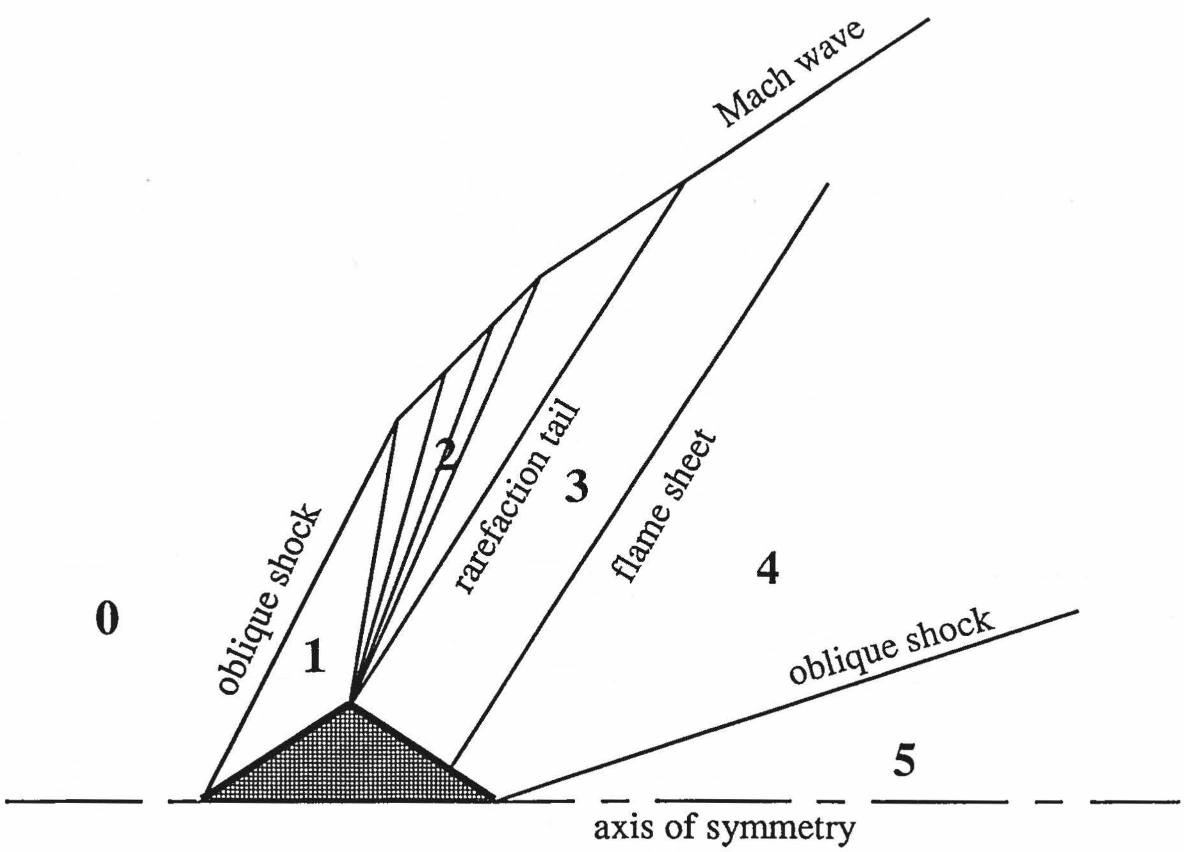
- multiple reactions,
- multiple species,
- complex chemical kinetics.

Complex wave interactions:

- compression waves,
- expansion waves,
- combustion waves.

Generic Configurations





Non-Dimensional Model Equations

Continuity:

$$\frac{d\rho}{dt} + \rho \frac{\partial v_i}{\partial x_i} = 0 ,$$

Momentum:

$$\frac{dv_i}{dt} + \frac{1}{\rho} \frac{\partial P}{\partial x_i} = 0 ,$$

Energy:

$$\frac{dP}{dt} - \gamma \frac{P}{\rho} \frac{d\rho}{dt} = (\gamma\text{-}1)\, \rho\, \kappa\, q\, (1\text{-}\lambda) \exp\left(\frac{-\Theta}{T}\right) ,$$

Species:

$$\frac{d\lambda}{dt} = \kappa\, (1\text{-}\lambda) \exp\left(\frac{-\Theta}{T}\right) ,$$

Caloric Equation of State:

$$e = \frac{1}{\gamma\text{-}1} \frac{P}{\rho} - \lambda\, q ,$$

Thermal Equation of State:

$$P = \rho\, T .$$

Non-Dimensional Variables

$$\rho = \frac{\tilde{\rho}}{\tilde{\rho}_0} \quad , \quad P = \frac{\tilde{P}}{\tilde{P}_0} \quad , \quad T = \frac{\tilde{R}}{\tilde{P}_0/\tilde{\rho}_0}\tilde{T} \quad ,$$

$$u = \frac{\tilde{u}}{\sqrt{\tilde{P}_0 / \tilde{\rho}_0}} \quad , \quad v = \frac{\tilde{v}}{\sqrt{\tilde{P}_0 / \tilde{\rho}_0}} \quad , \quad e = \frac{\tilde{e}}{\tilde{P}_0/\tilde{\rho}_0} \quad ,$$

$$x = \frac{\tilde{x}}{\tilde{L}} \quad , \quad y = \frac{\tilde{y}}{\tilde{L}} \quad , \quad t = \frac{\sqrt{\tilde{P}_0/\tilde{\rho}_0}}{\tilde{L}}\tilde{t} \quad .$$

Non-Dimensional Parameters

$$q = \frac{\tilde{q}}{\tilde{P}_0/\tilde{\rho}_0} \quad , \quad \Theta = \frac{\tilde{E}}{\tilde{P}_0/\tilde{\rho}_0} \quad , \quad \gamma = 1 + \frac{\tilde{R}}{\tilde{c}_v} \quad ,$$

$$\kappa = \frac{\tilde{L}}{\sqrt{\tilde{P}_0/\tilde{\rho}_0}} \quad , \quad M_0 = \frac{\tilde{u}_0}{\sqrt{\gamma\,\tilde{P}_0/\tilde{\rho}_0}} \quad .$$

Reaction Model

- Simple one-step, irreversible reaction:

$$A \xrightarrow{\lambda} B \qquad \text{(exothermic reaction)}$$

λ = reaction progress variable

$$Y_A = 1 - \lambda \ , \ \ Y_B = \lambda \ .$$

- Arrhenius kinetics:

$$\text{kinetic rate} \propto \exp(-E_a / RT)$$

- high activation energy limit.

Thermal Explosion Theory

Reduced Equations (assumed $v_i = 0$):

$$\frac{dP}{dt} = (\gamma\text{-}1)\,\rho\,\kappa\,q\,(1\text{-}\lambda)\exp\left(\frac{-\,\Theta\,\rho}{P}\right),$$

$$\frac{d\lambda}{dt} = \kappa\,(1\text{-}\lambda)\exp\left(\frac{-\Theta\,\rho}{P}\right),$$

$$P(0) = P_1 \quad , \quad \lambda(0) = 0 \ .$$

Linearize the equations:

$$P = P_1 + P' \ , \quad \lambda = \lambda' \ .$$

where

$$P' << 1 \quad , \quad \lambda' << 1 \ .$$

Solve for the pressure perturbation P'.

Solution:

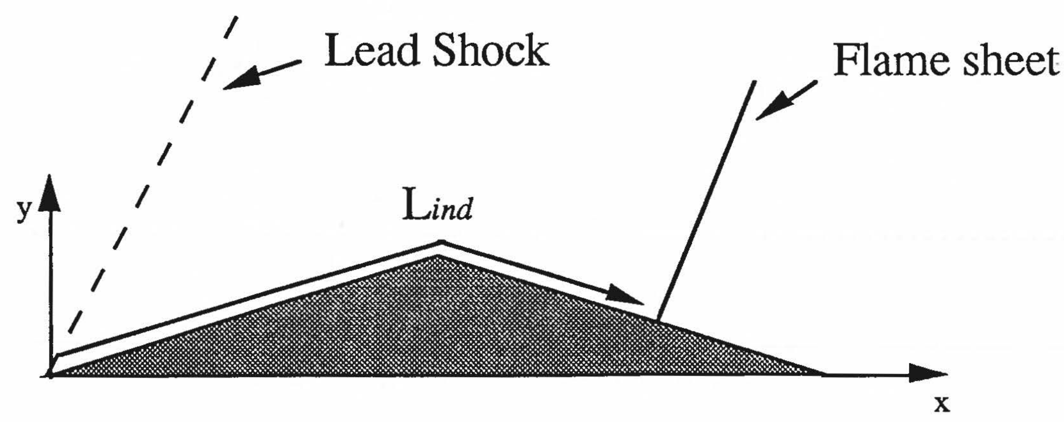
$$P' = -\frac{P_1^2}{\Theta\,\rho_1} + \ln\left[1 - \frac{\Theta\,\rho_1^2}{P_1^2}(\gamma-1)\,q\,\kappa\left(\exp\frac{-\Theta\,\rho_1}{P_1}\right)t\right] .$$

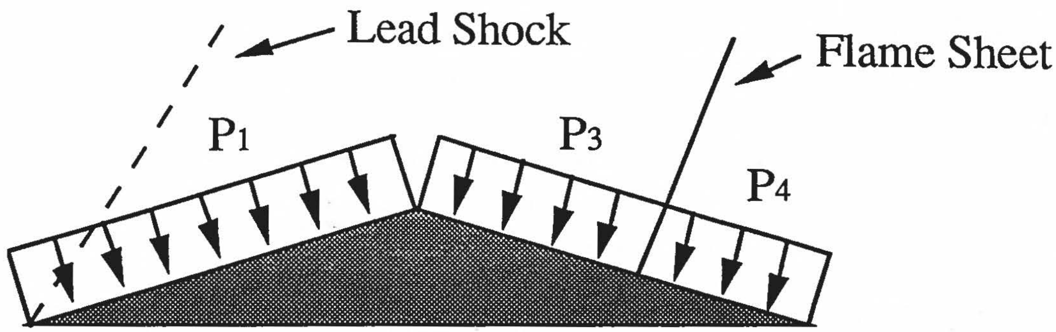
Solve for the thermal explosion time:
(corresponds to the induction time)

$$t_{ind} = \frac{P_1^2}{(\gamma-1)\,\rho_1^2\,\Theta\,\kappa\,q}\exp\left[\frac{\Theta\,\rho_1}{P_1}\right] .$$

Induction distance:

$$L_{ind} = t_{ind}\,\sqrt{u_1^2 + v_1^2} .$$





Calculation of Surface Forces

1. Wave Drag:

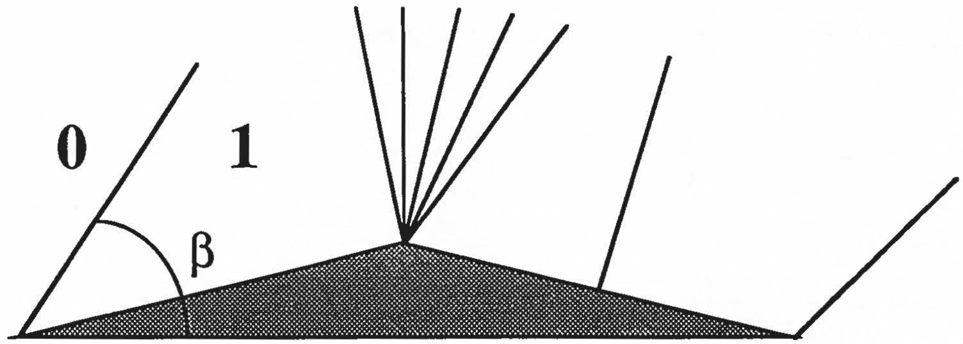
$$F_D = \frac{1}{2}\left(P_1 - P_3\right)\tan\theta \ .$$

2. Net Thrust Force:

$$F_{net} = P_3\left(2\,L_{ind}\cos\theta - 1\right)\tan\theta$$
$$+\left[P_4\left(2 - 2\,L_{ind}\cos\theta\right) - P_1\right]\tan\theta \ .$$

3. Combustion Induced Thrust:

$$F_c = F_{net} + F_D \ .$$



Jump Relations Across Lead Shock

$$\rho_1 = \left(\frac{1 + \gamma M_0^2 \sin^2\beta \pm \sqrt{A}}{(\gamma + 1) M_0^2 \sin^2\beta} \right)^{-1}$$

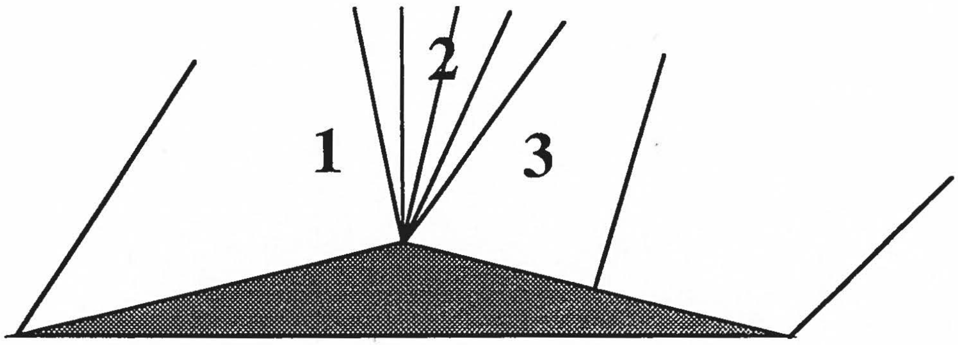
$$u_1 = \sqrt{\gamma}\, M_0 \left(\frac{1}{\rho_1} \sin^2\beta + \cos^2\beta \right)$$

$$v_1 = \sqrt{\gamma}\, M_0 \cos\beta \sin\beta \left(1 - \frac{1}{\rho_1} \right)$$

$$P_1 = 1 + \gamma M_0^2 \sin^2\beta \left(1 - \frac{1}{\rho_1} \right)$$

where

$$A = \left(1 + \gamma M_0^2 \sin^2\beta\right)^2 - (\gamma + 1)\gamma M_0^2 \sin^2\beta$$
$$\times \left(2 + (\gamma - 1) M_0^2 \sin^2\beta\right)$$



Flow Expansion Region

Prandtl - Meyer Function:

$$\nu(M_3) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_3^2 - 1\right)} - \tan^{-1}\sqrt{M_3^2 - 1}$$

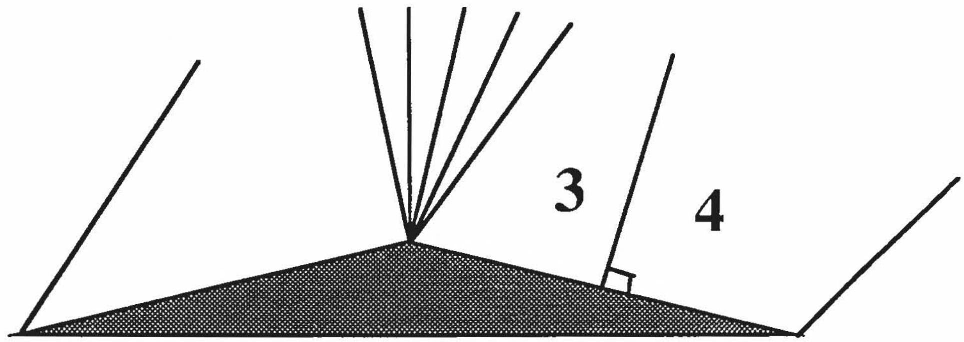
Isentropic Relations:

$$\frac{\rho_3}{\rho_1} = \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_3^2}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{P_3}{P_1} = \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_3^2}\right)^{\frac{\gamma}{\gamma-1}}$$

Velocity Components:

$$u_3 = M_3 \sqrt{\gamma \frac{P_3}{\rho_3}} \cos\theta \quad , \quad v_3 = - M_3 \sqrt{\gamma \frac{P_3}{\rho_3}} \sin\theta$$



Jump Relations Across Flame Sheet

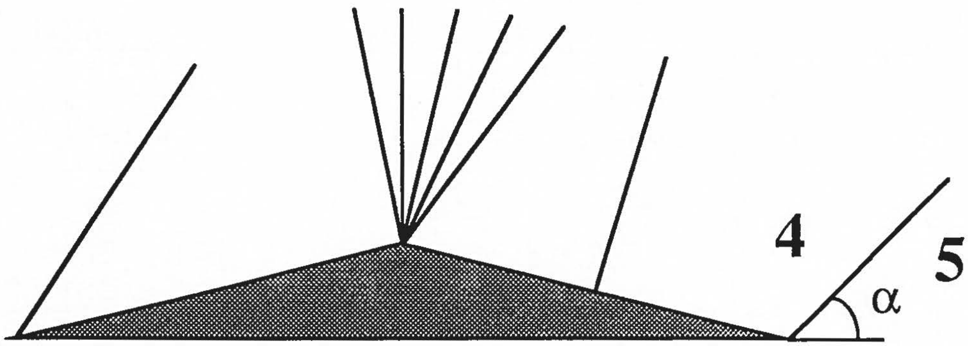
$$\frac{\rho_4}{\rho_3} = \left(\frac{1 + \gamma M_3^2 \pm \sqrt{B}}{(\gamma + 1) M_3^2} \right)^{-1}$$

$$\frac{P_4}{P_3} = 1 + \gamma M_3^2 \left(1 - \frac{\rho_3}{\rho_4} \right)$$

$$u_4 = \left(\frac{\rho_3}{\rho_4} \right) u_3 \cos \theta \,, \quad v_4 = - \left(\frac{\rho_3}{\rho_4} \right) u_3 \sin \theta$$

where

$$B = \left(1 + \gamma M_3^2\right)^2 - (\gamma - 1) M_3^2$$
$$\times \left(2 + (\gamma\text{-}1) M_3^2 + 2 (\gamma\text{-}1) \frac{\rho_3}{P_3} Q \right)$$



Tail End Compression Region

$$\frac{\rho_5}{\rho_4} = \left(\frac{1 + \gamma M_4^2 \sin^2(\alpha+\theta) \pm \sqrt{C}}{(\gamma + 1) M_4^2 \sin^2(\alpha+\theta)} \right)^{-1}$$

$$u_5 = \sqrt{\gamma}\, M_4 \left(\frac{\rho_4}{\rho_5} \sin^2(\alpha+\theta) + \cos^2(\alpha+\theta) \right)$$

$$v_5 = \sqrt{\gamma}\, M_4 \cos(\alpha+\theta)\ \sin(\alpha+\theta) \left(1 - \frac{\rho_4}{\rho_5} \right)$$

$$\frac{P_5}{P_4} = 1 + \gamma M_4^2 \sin^2(\alpha+\theta) \left(1 - \frac{\rho_4}{\rho_5} \right)$$

where

$$C = \left(1 + \gamma M_4^2 \sin^2(\alpha+\theta)\right)^2 - (\gamma + 1)\, \gamma M_4^2 \sin^2(\alpha+\theta) \times \left(2 + (\gamma - 1) M_4^2 \sin^2(\alpha+\theta)\right)$$

Numerical Analysis

RPLUS Code:

- developed at NASA Lewis,
- based on LU-SSOR numerical scheme.

Computational Grid:

- 199 x 99 fixed grid.

Convergence:

- 500 iterations,
- residual unsteady terms had scaled values $\leq 1.0 \times 10^{-8}$.

Computations:

- run on IBM RS/6000 POWERstation 350
- run time about one hour.

Parameters

Geometric:

$$\theta = 5^{\circ}\ ,\ \ \tilde{L} = 0.10\ \text{m}\ .$$

Kinetic:

$$\tilde{k} = 1.0 \text{ x } 10^{7}\ /\text{sec}\ ,\quad \tilde{E} = 1.019 \text{ x } 10^{6}\ \text{J/kg}\ ,$$

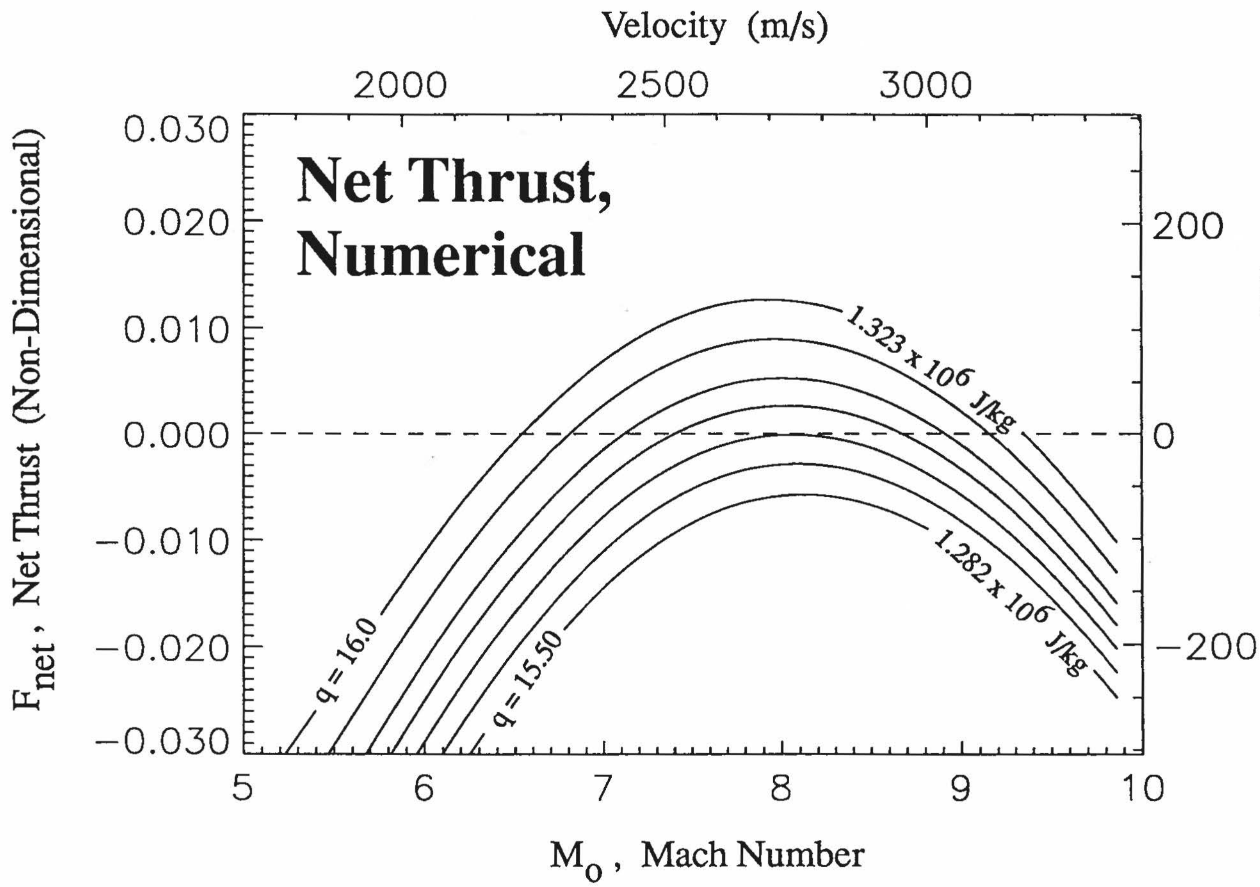
$$1.295 \text{ x } 10^{6}\ \text{J/kg} \ \leq\ \tilde{q}\ \leq\ 1.704 \text{ x } 10^{7}\ \text{J/kg}\ .$$

Atmospheric free-stream conditions:

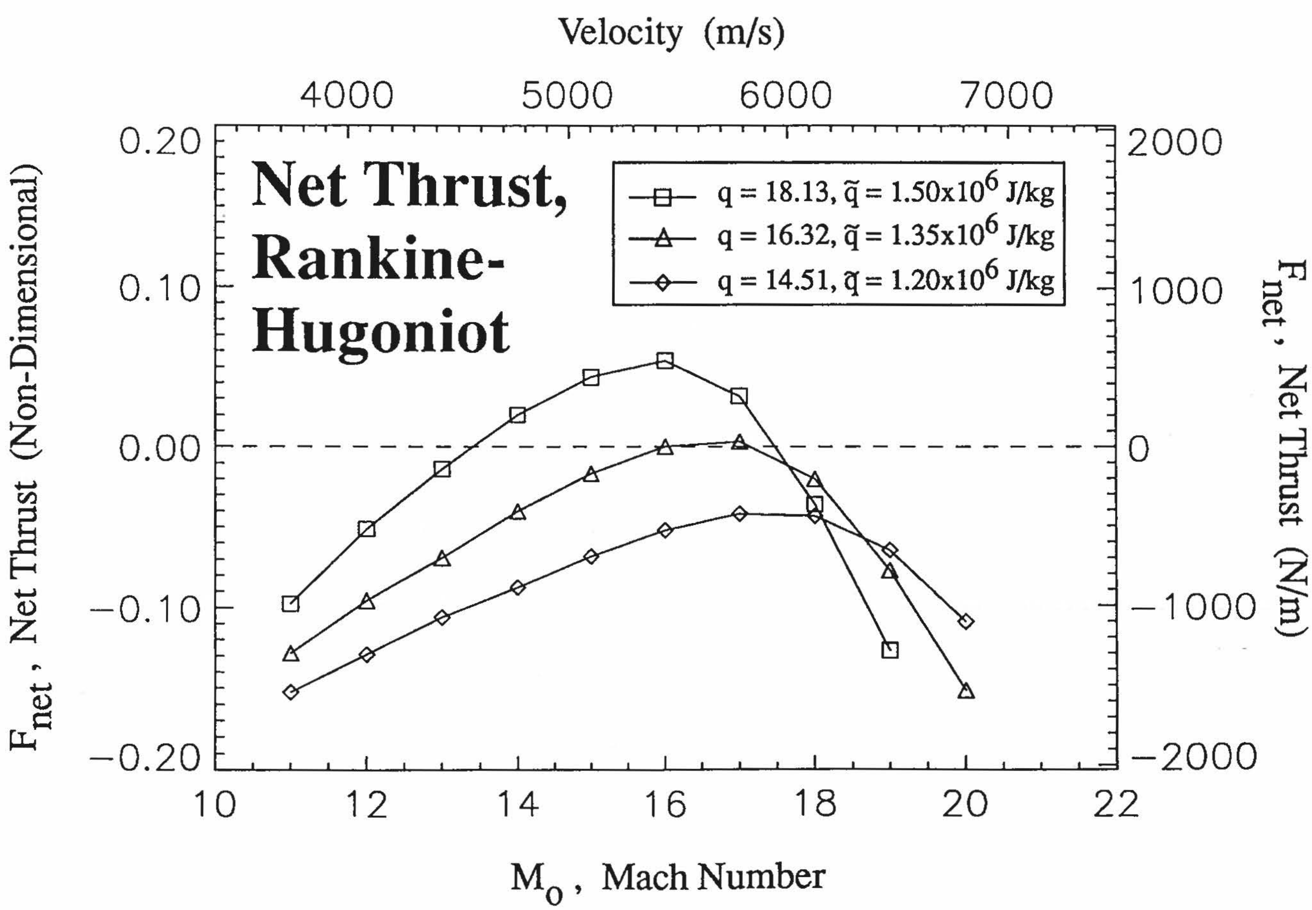
$$\tilde{P}_0 = 1.01325 \text{ x } 10^{5}\ \text{Pa}\ \ ,\quad \tilde{\rho}_0 = 1.225\ \text{kg/m}^3\ .$$

Thermodynamic constants:

$$\gamma = \frac{7}{5}\ \ ,\quad \tilde{R} = 287\ \text{J/(kg K)}\ \ ,\quad \tilde{c}_v = 717.5\ \text{J/(kg/K)}\ .$$

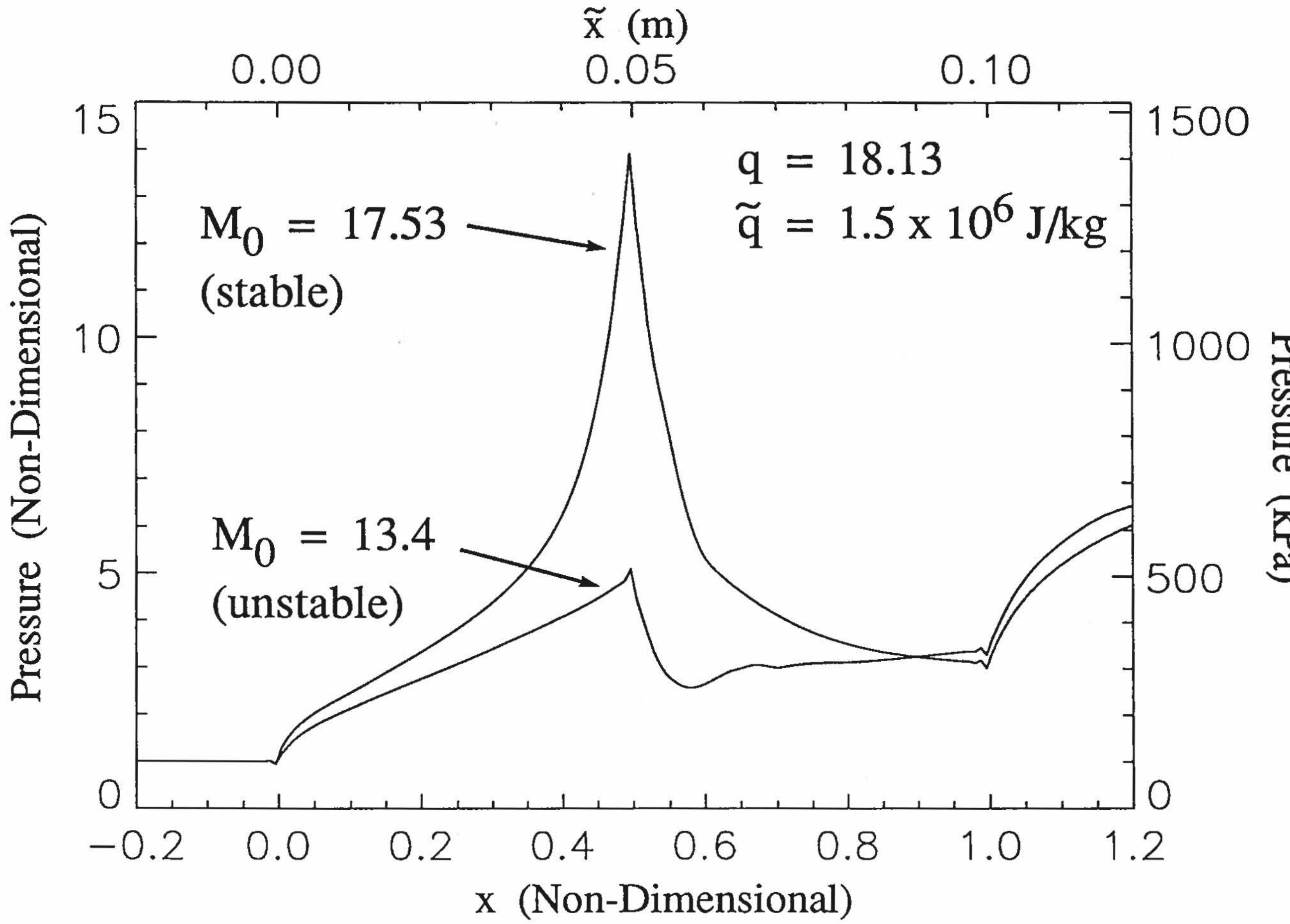


Velocity (m/s)
2000
2500
3000
Net Thrust,
Numerical
1.323 x 10^6 J/kg
1.282 x 10^6 J/kg
q = 16.0
q = 15.50
0.030
0.020
0.010
0.000
−0.010
−0.020
−0.030
200
0
−200
5
6
7
8
9
10
F_{net} , Net Thrust (Non-Dimensional)
F_{net} , Net Thrust (N/m)
M_o , Mach Number



Velocity (m/s)
4000
5000
6000
7000
Net Thrust,
Rankine-
Hugoniot
q = 18.13, $\tilde{q}$ = 1.50x10^6 J/kg
q = 16.32, $\tilde{q}$ = 1.35x10^6 J/kg
q = 14.51, $\tilde{q}$ = 1.20x10^6 J/kg
0.20
0.10
0.00
−0.10
−0.20
2000
1000
0
−1000
−2000
10
12
14
16
18
20
22
F_{net} , Net Thrust (Non-Dimensional)
F_{net} , Net Thrust (N/m)
M_o , Mach Number

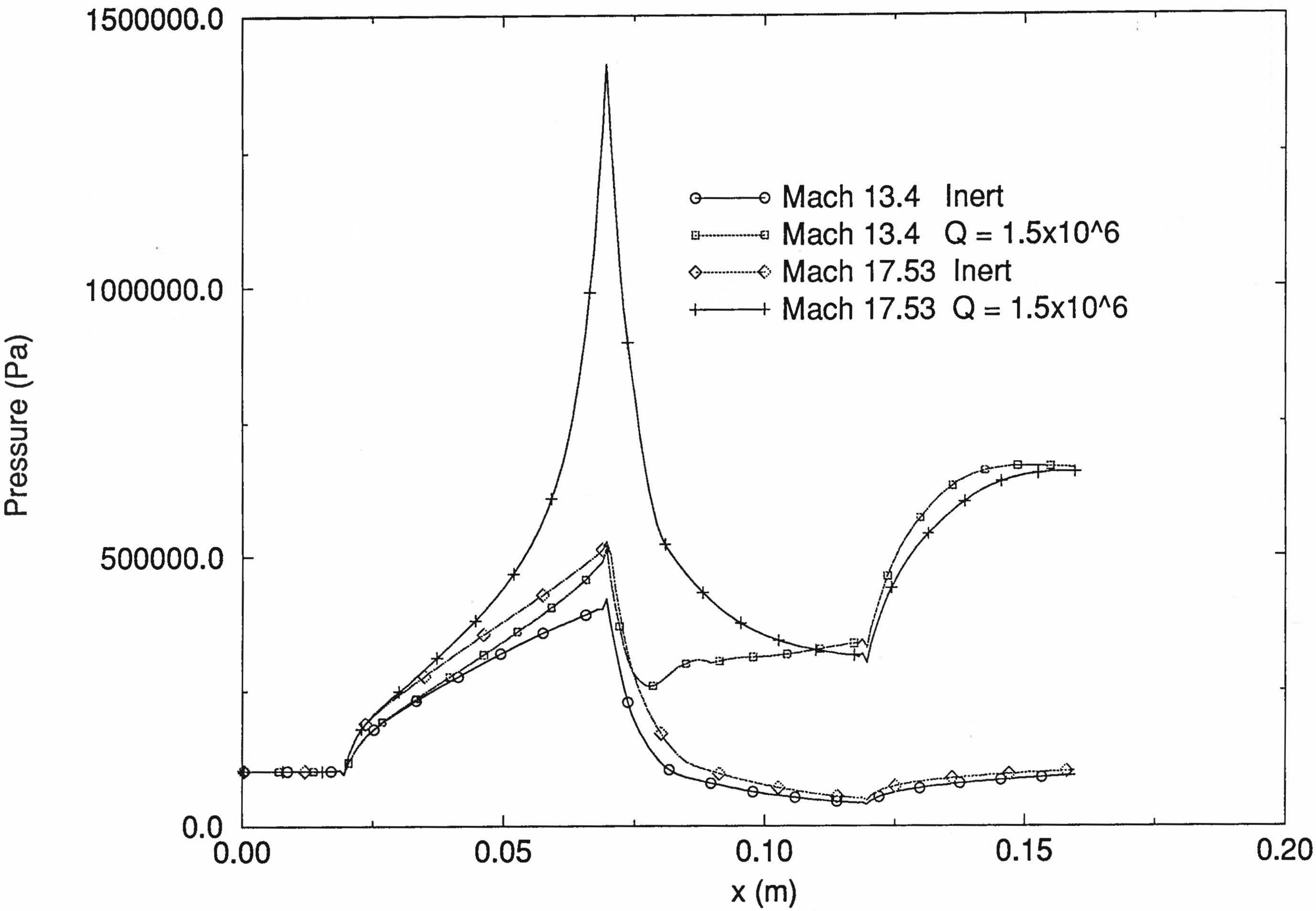
Pressure on Wedge Surface

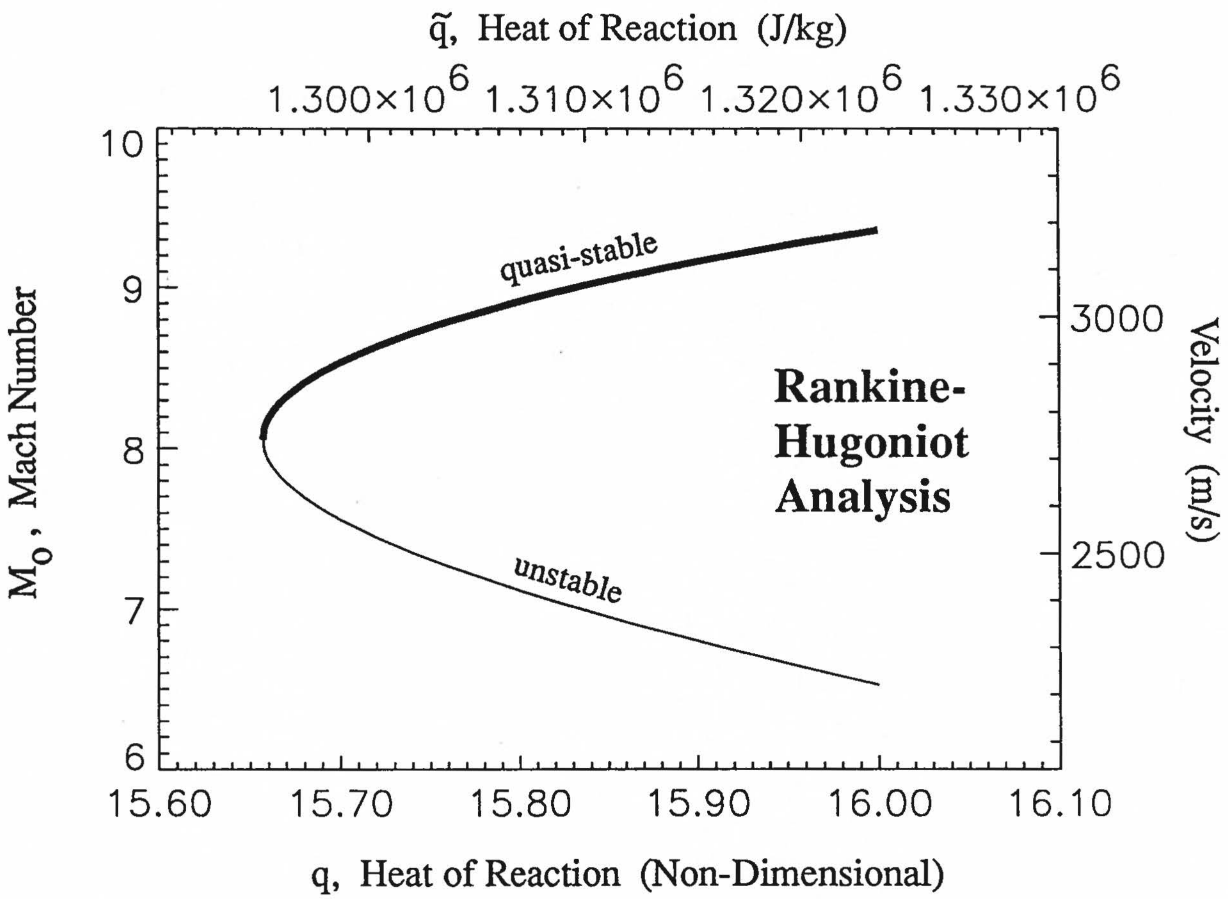


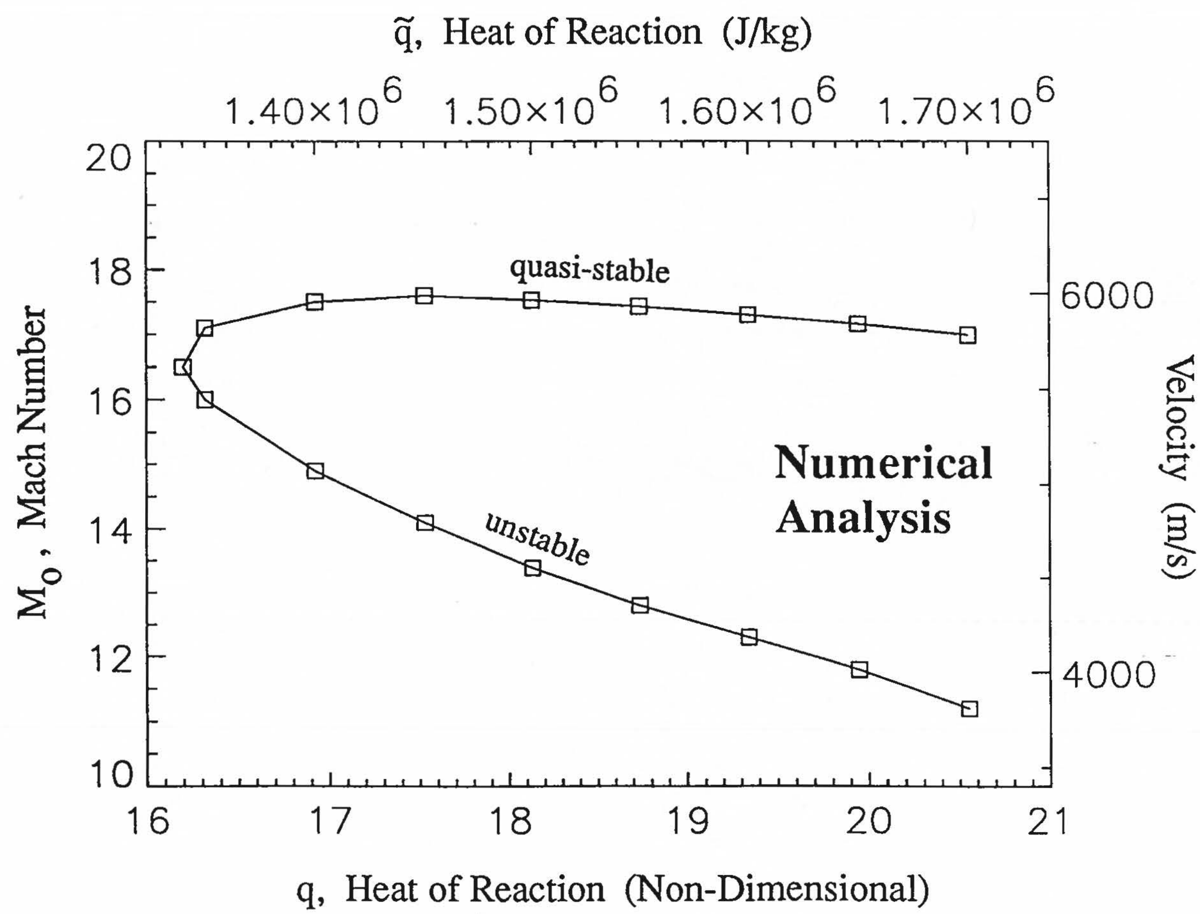
- **rise on forebody due to shock and reaction**
- **drop on aftbody due to rarefaction**
- **lack of crisp shock indicates more resolution necessary**
- **propagation speed sensitive to local pressure**
- **trends plausible**

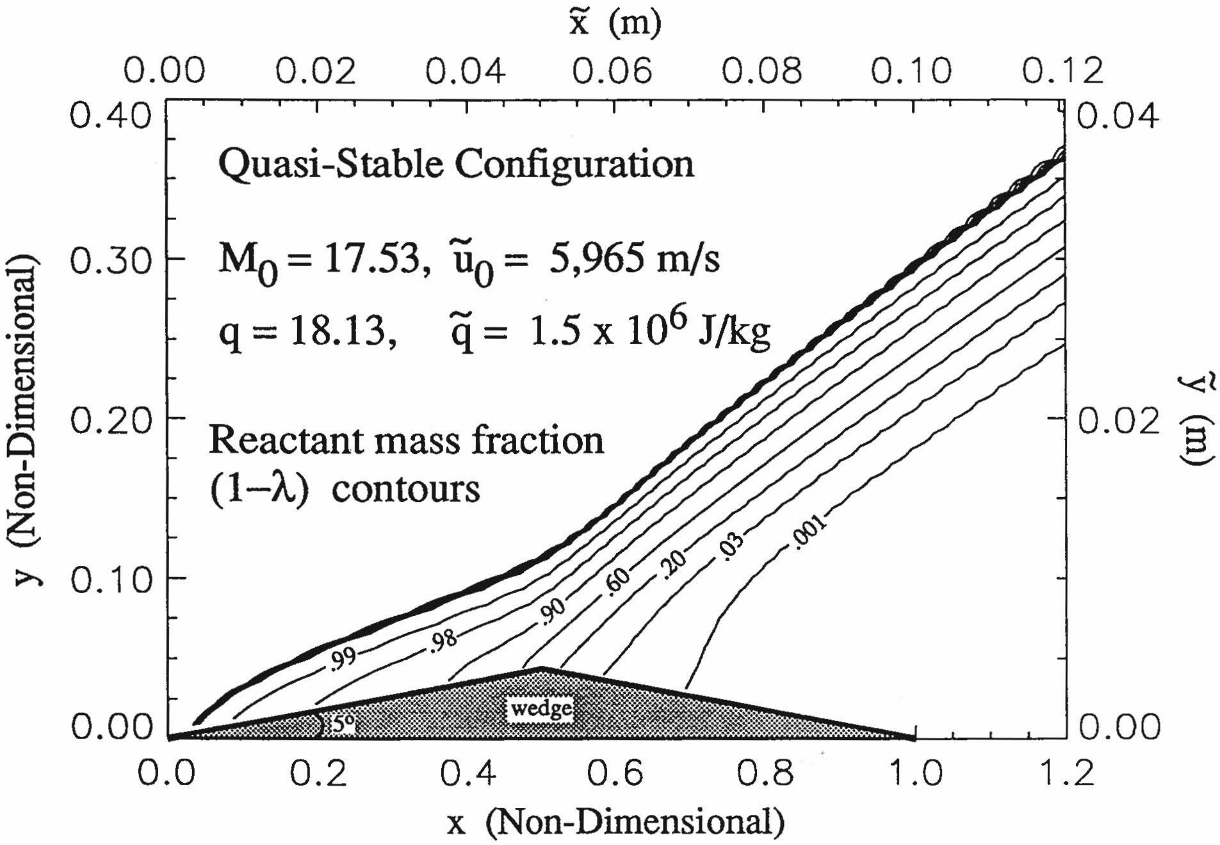
Pressure traces on wedge surface.

k=1x10^7   Ea=3550

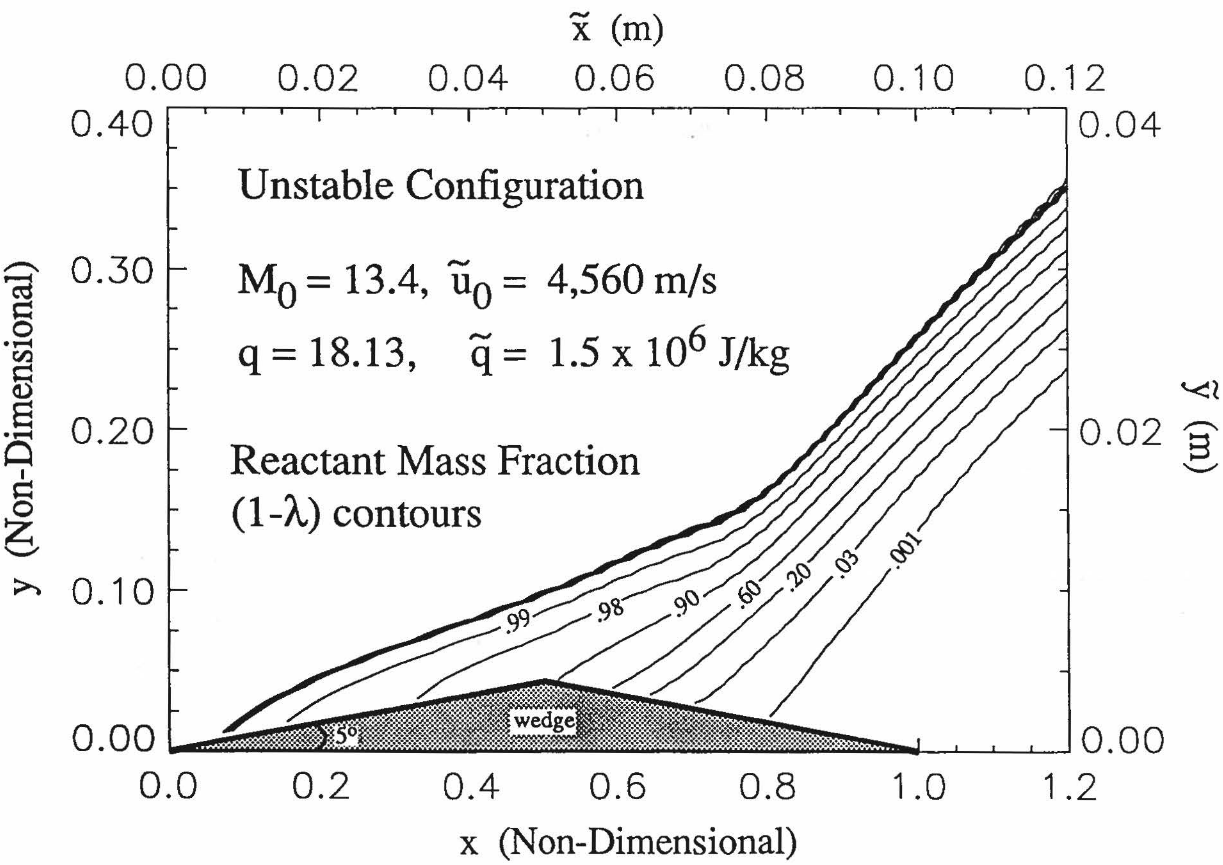



q̃, Heat of Reaction (J/kg)
1.300×10^6 1.310×10^6 1.320×10^6 1.330×10^6
M0 , Mach Number
10
9
8
7
6
Velocity (m/s)
3000
2500
quasi-stable
unstable
Rankine-Hugoniot Analysis
15.60 15.70 15.80 15.90 16.00 16.10
q, Heat of Reaction (Non-Dimensional)

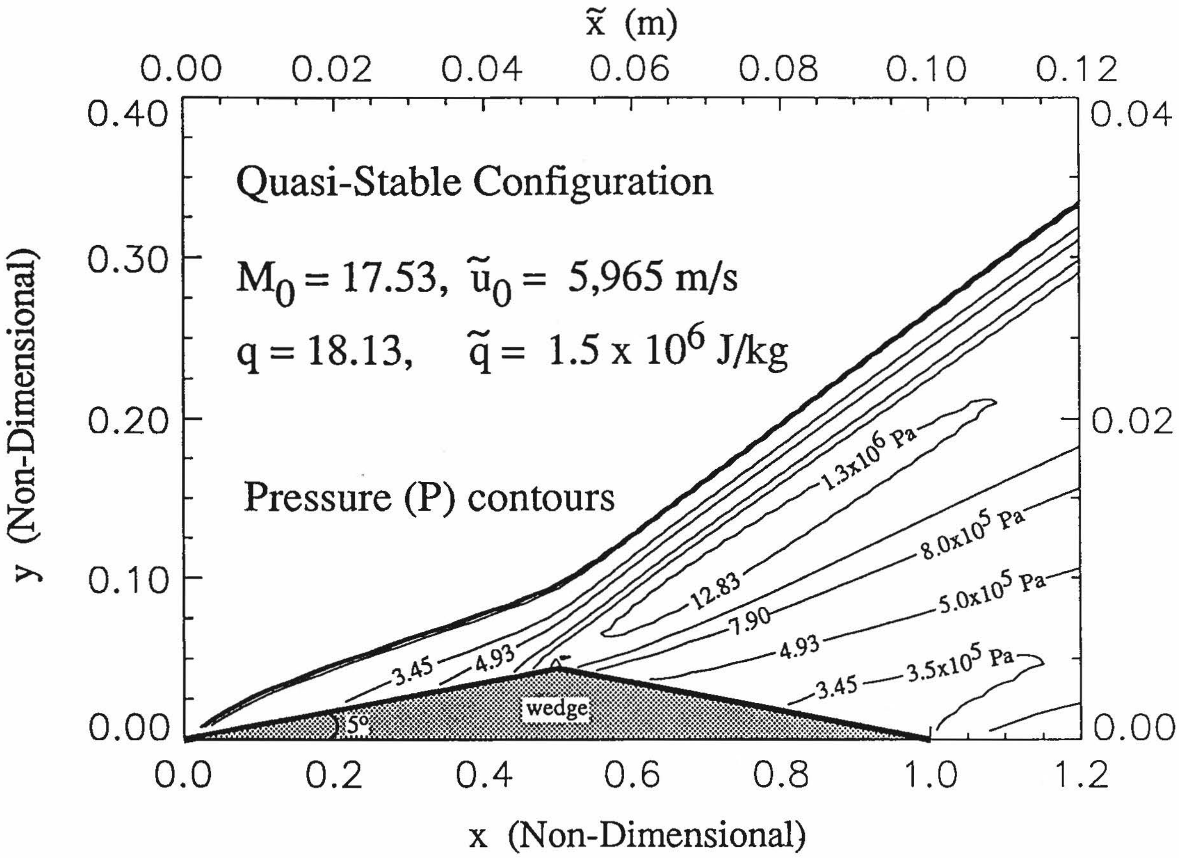

q̃, Heat of Reaction (J/kg)
1.40×10^6 1.50×10^6 1.60×10^6 1.70×10^6
M0 , Mach Number
20
18
16
14
12
10
Velocity (m/s)
6000
4000
quasi-stable
unstable
Numerical Analysis
16 17 18 19 20 21
q, Heat of Reaction (Non-Dimensional)



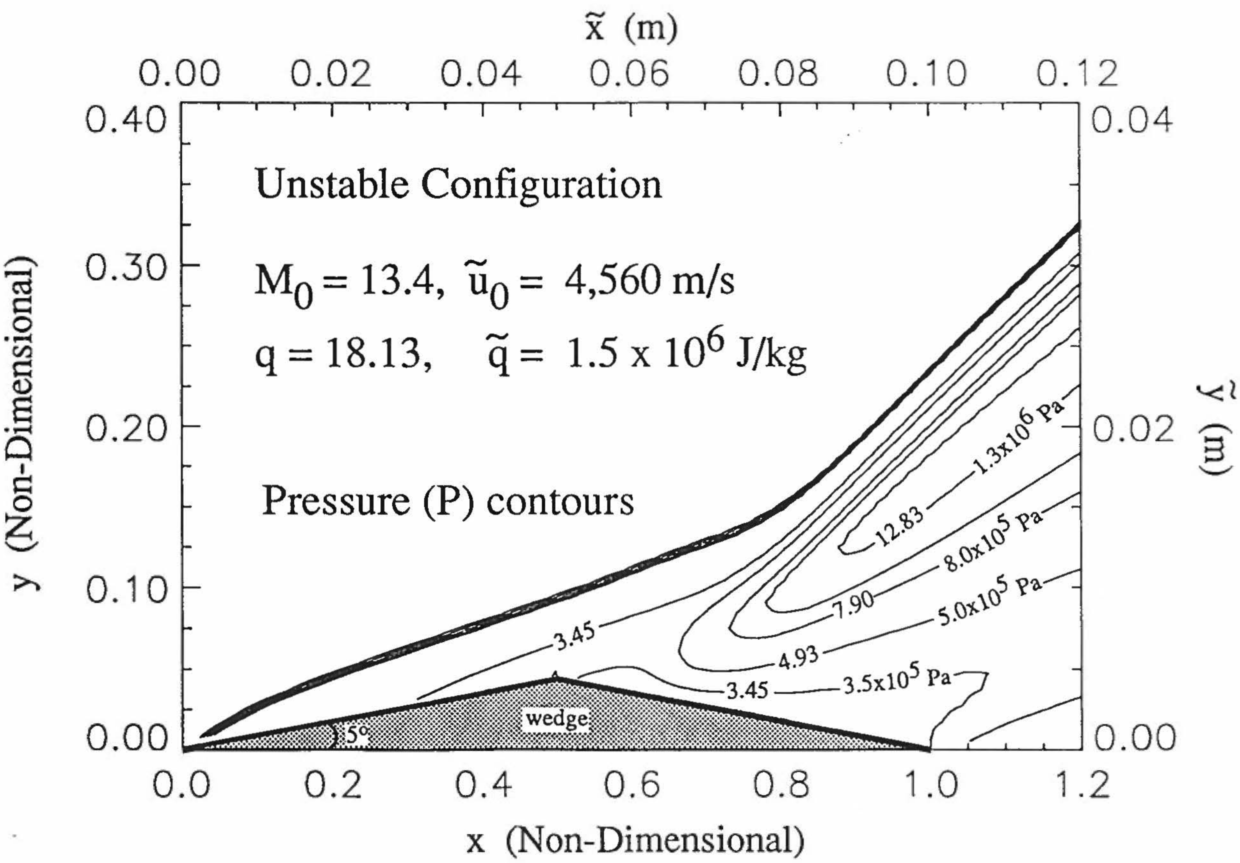
x̃ (m)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.40
0.30
0.20
0.10
0.00
0.04
0.02
0.00
y (Non-Dimensional)
ỹ (m)
Quasi-Stable Configuration
$M_0 = 17.53$, $\tilde{u}_0 =$ 5,965 m/s
$q = 18.13$, $\tilde{q} = 1.5 \times 10^6$ J/kg
Reactant mass fraction
(1−λ) contours
.99
.98
.90
.60
.20
.03
.001
wedge
5°
0.0
0.2
0.4
0.6
0.8
1.0
1.2
x (Non-Dimensional)



x̃ (m)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.40
0.30
0.20
0.10
0.00
0.04
0.02
0.00
y (Non-Dimensional)
ỹ (m)
Unstable Configuration
$M_0 = 13.4$, $\tilde{u}_0 =$ 4,560 m/s
$q = 18.13$, $\tilde{q} = 1.5 \times 10^6$ J/kg
Reactant Mass Fraction
(1-λ) contours
.99
.98
.90
.60
.20
.03
.001
wedge
5°
0.0
0.2
0.4
0.6
0.8
1.0
1.2
x (Non-Dimensional)



x̃ (m)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.40
0.30
0.20
0.10
0.00
0.04
0.02
0.00
Quasi-Stable Configuration
$M_0 = 17.53$, $\tilde{u}_0 =$ 5,965 m/s
$q = 18.13$, $\tilde{q} = 1.5 \times 10^6$ J/kg
Pressure (P) contours
1.3×10^6 Pa
8.0×10^5 Pa
5.0×10^5 Pa
3.5×10^5 Pa
12.83
7.90
4.93
3.45
wedge
5°
y (Non-Dimensional)
ỹ (m)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
x (Non-Dimensional)



x̃ (m)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.40
0.30
0.20
0.10
0.00
0.04
0.02
0.00
Unstable Configuration
$M_0 = 13.4$, $\tilde{u}_0 =$ 4,560 m/s
$q = 18.13$, $\tilde{q} = 1.5 \times 10^6$ J/kg
Pressure (P) contours
1.3×10^6 Pa
8.0×10^5 Pa
5.0×10^5 Pa
3.5×10^5 Pa
12.83
7.90
4.93
3.45
wedge
5°
y (Non-Dimensional)
ỹ (m)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
x (Non-Dimensional)

Conclusions

1. The interaction of kinetic length scales with geometric length scales are important for determining steady propagation speeds.

2. Bifurcation phenomena observed for equilibrium Mach numbers:

   - high Mach number solutions stable to quasi-static perturbations.

   - low Mach number solutions, unstable.

3. Near the bifurcation point, increasing heat release increases steady propagation speed

4. Qualitative agreement exists between theoretical and numerical results.

5. Higher resolution required for quantitative accuracy.