Verification and Validation of Pseudospectral Shock Fitted Simulations of Supersonic Flow over a Blunt Body

Gregory P. Brooks
Air Force Research Laboratory, WPAFB, Ohio

Joseph M. Powers
University of Notre Dame, Notre Dame, Indiana

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Motivation

- Develop verified and validated high accuracy flow solver for Euler equations in space and time
  - *verification*: solving the equations “right”
  - *validation*: solving the right equations
- ultimate use for fundamental shock stability questions for inert and reactive flows, detonation shock dynamics, shape optimization

Review: Blunt Body Solutions

- Lin and Rubinov, *J. Math. Phys.*, 1948
- Van Dyke, *J. Aero/Space Sci.*, 1958
- Evans and Harlow, *J. Aero. Sci.*, 1958
- Moretti and Abbett, *AIAA J.*, 1966
- Kopriva, Zang, and Hussaini, *AIAA J.*, 1991
- Kopriva, *CMAME*, 1999
- Brooks and Powers, *J. Comp. Phys.*, 2004 (to appear)

Model: Euler Equations

- two-dimensional
- axisymmetric
- inviscid
- calorically perfect ideal gas

Model: Euler Equations

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r}\right) = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z} + \gamma p\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r}\right) = 0$$

Model: Secondary Equations

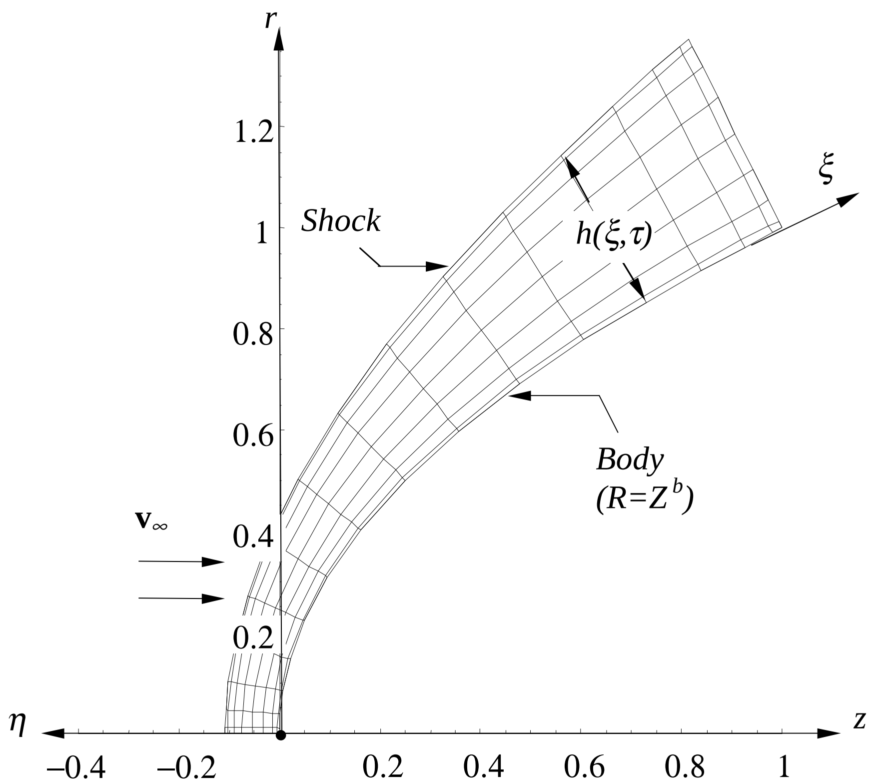
$$\omega_\theta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}$$

$$\frac{d\omega_\theta}{dt} = \frac{\omega_\theta}{\rho}\frac{d\rho}{dt} + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial r} - \frac{\partial \rho}{\partial r}\frac{\partial p}{\partial z}\right) + \omega_\theta \frac{u}{r}$$

$$T = \frac{1}{\gamma - 1}\frac{p}{\rho}, \quad s = \ln\left(\frac{p}{\rho^\gamma}\right), \qquad \frac{ds}{dt} = 0$$

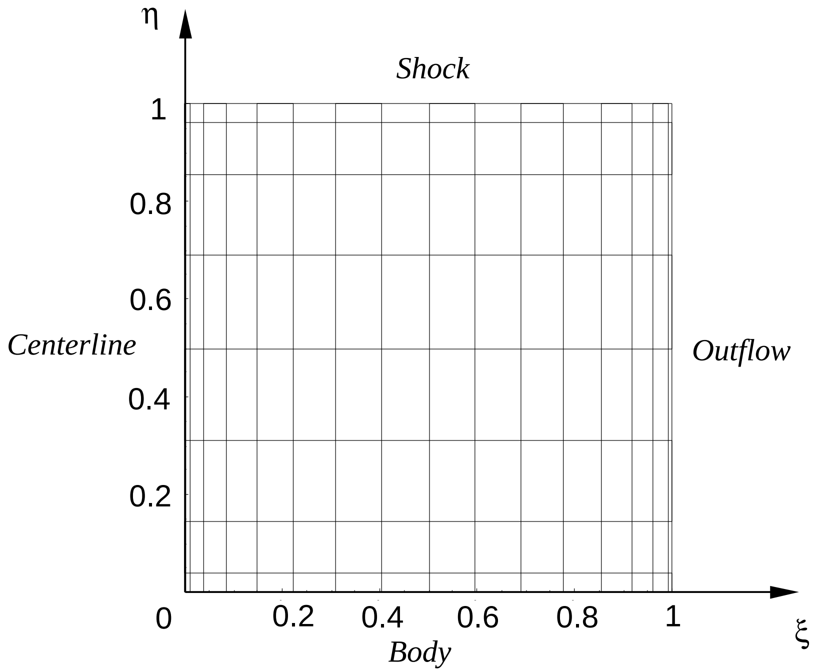
$$H_o = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} + \frac{1}{2}\left(u^2 + w^2\right) = constant$$

Flow Geometry and Boundary Conditions



- body: zero mass flux
- shock: RH jump
- center: homeoentropic
- outflow: supersonic

Flow Geometry in Transformed Space

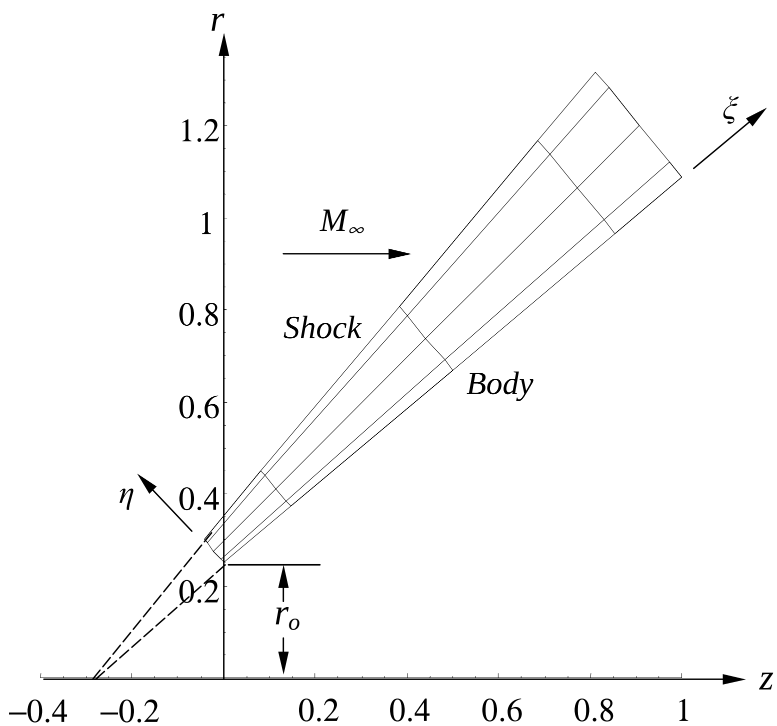


- $(r, z, t) \rightarrow (\xi, \eta, \tau)$
- unsteady
- *shock-fitted* to avoid low first order accuracy of shock capturing

Outline: Pseudospectral Solution Procedure

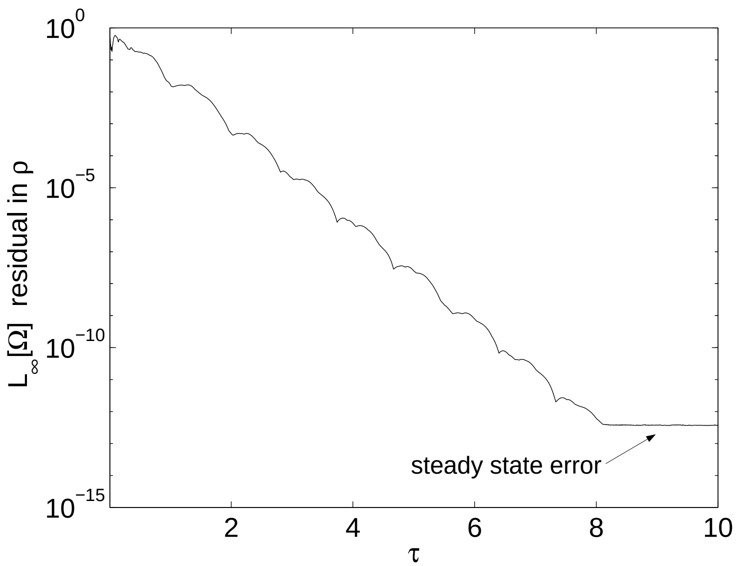
- Define collocation points in computational space.
- Approximate all continuous functions and their spatial derivatives with Lagrange interpolating polynomials, which have *global support for high spatial accuracy*.
- PDEs $\xrightarrow{spatial\ discretization}$ DAEs $\xrightarrow{algebra}$ ODEs.
- Cast ODEs as $\frac{d\mathbf{x}}{dt} = \mathbf{q}(\mathbf{x})$.
- Solve ODEs using high accuracy solver LSODA.

Taylor-Maccoll: Flow over a Sharp-Nose Cone



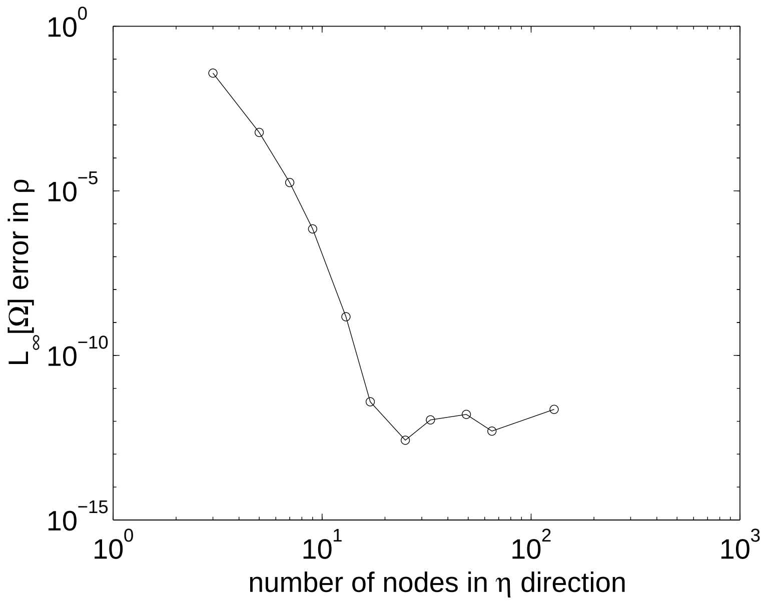
- Similarity solution available for flow over a sharp cone
- Non-trivial post-shock flow field
- Ideal verification benchmark

Verification: Taylor-Maccoll Time-Relaxation



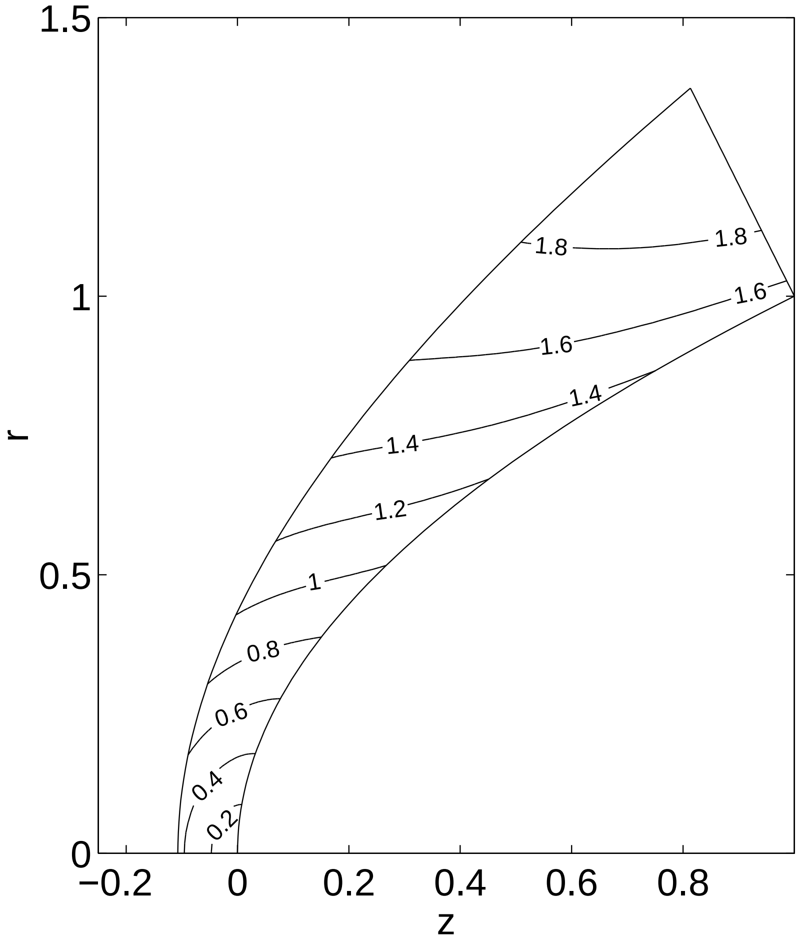
- $M_\infty = 3.5$
- 5×17 grid
- $t \to \infty$, error $\to 10^{-12}$

Verification: Taylor-Maccoll Spatial Resolution



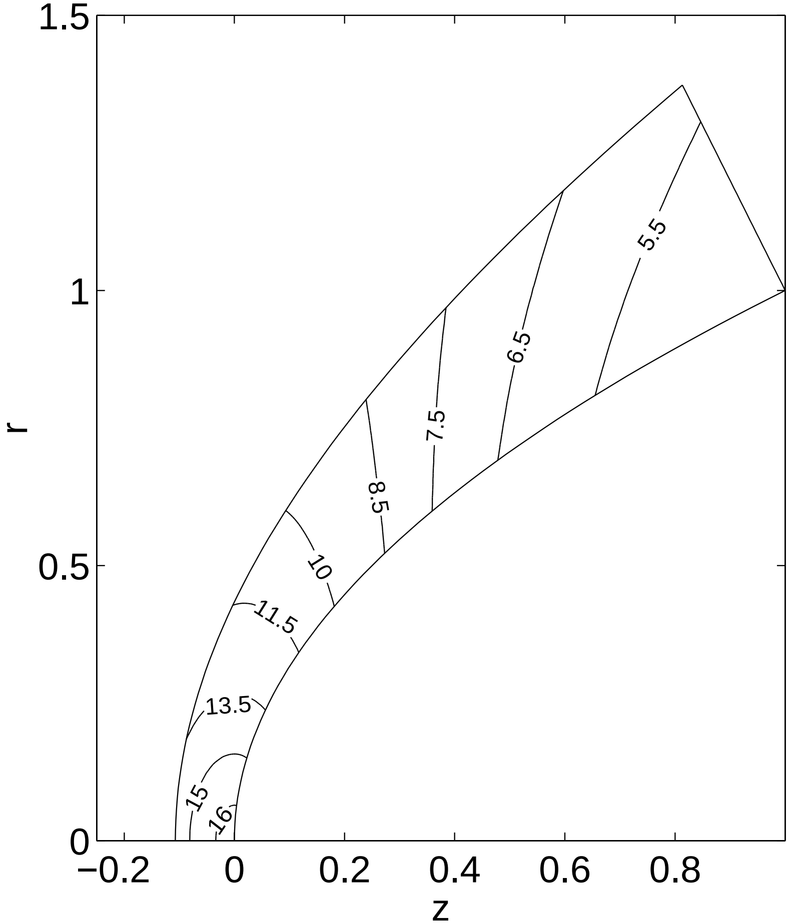
- spectral convergence
- roundoff error realized at coarse resolution, 5×17
- run time $\sim 10^2\ s$; $800\ MHz$ machine

Blunt Body Flow: Mach Number Field



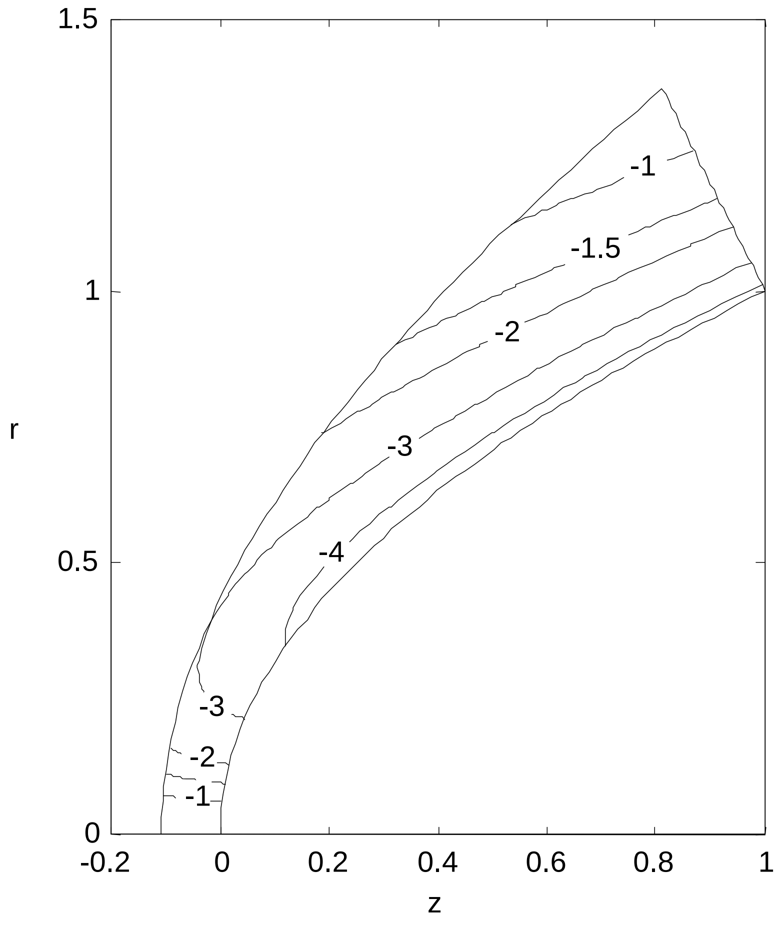
- $R = \sqrt{Z}$
- $M_\infty = 3.5$
- 17×9 grid
- transonic flow field predicted
- qualitatively correct
- not a verification

Blunt Body Flow: Pressure Field



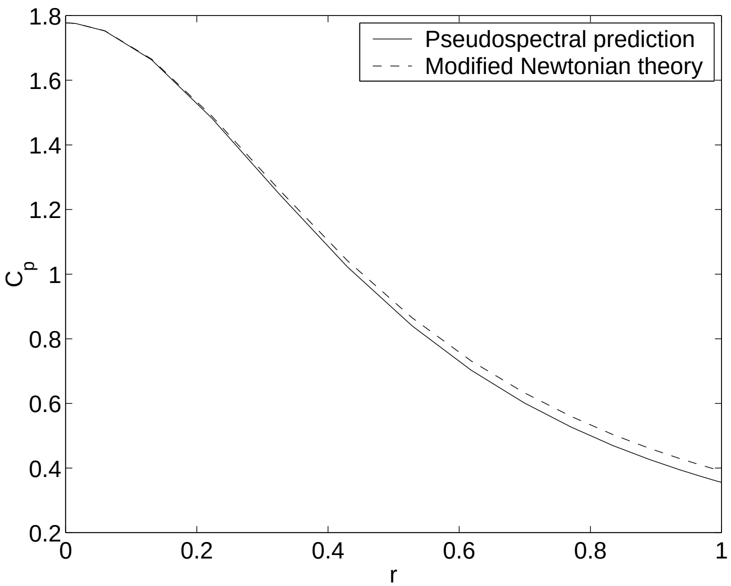
- high pressure at nose
- qualitatively correct
- not a verification

Blunt Body Flow: Vorticity Field



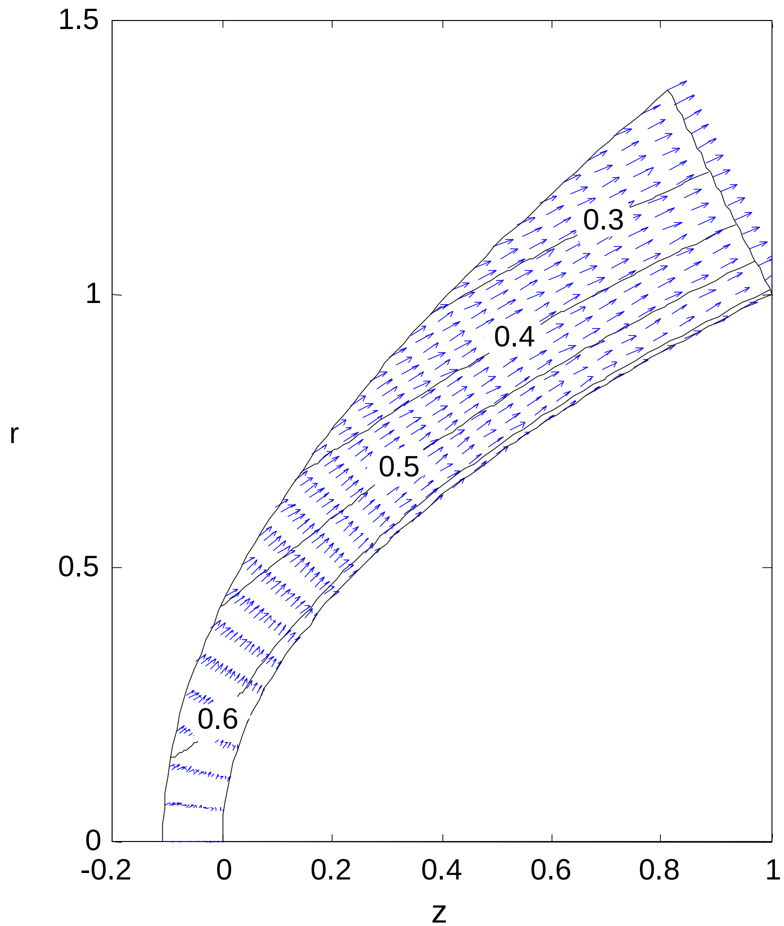
- Helmholtz Theorem: $\frac{d\rho}{dt}$, $\nabla p \times \nabla \rho$, shock curvature, flow divergence induce $\frac{d\omega_\theta}{dt}$
- intuition difficult
- not a verification

Verification: Blunt Body Pressure Coefficient



- $C_p = \frac{2p(\xi,0,\tau)-1}{\gamma M_\infty^2}$
- Newtonian theory gives prediction in high Mach number limit
- comparison quantitatively excellent
- not global

Verification: Blunt Body Entropy Field



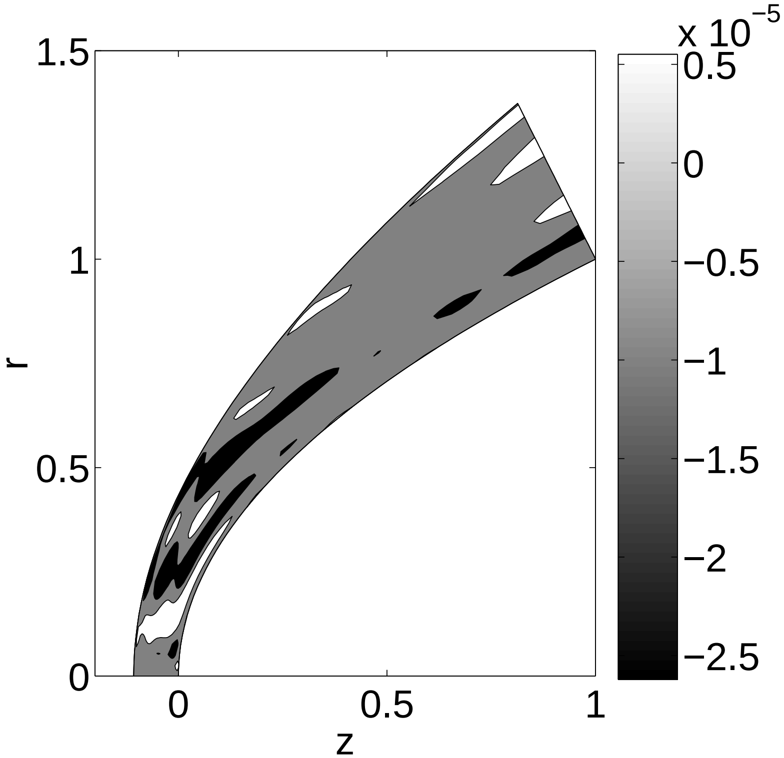
- $\frac{ds}{dt} = \frac{\partial s}{\partial t} + \nabla \cdot \mathbf{v} = 0$
- if stable, $\frac{\partial s}{\partial t} \longrightarrow 0$ as $t \longrightarrow \infty$
- thus, $\mathbf{v} \cdot \nabla s \longrightarrow 0$
- quantitative difference approaches roundoff error

Proof: Total Enthalpy is Constant

- $H_o \equiv \frac{\gamma}{\gamma-1}\frac{p}{\rho} + \frac{1}{2}\left(u^2 + w^2\right)$ (definition)
- $\rho\frac{dH_o}{dt} = \rho T \underbrace{\frac{ds}{dt}}_{=0} + \underbrace{\frac{\partial p}{\partial t}}_{\to 0}$ (from Euler equations)
- $H_o =$ constant on streamline as $t \to \infty$
- RH shock jump equations admit no change in H_o
- If H_o is spatially homogeneous before the shock, it will remain so after the shock; $H_o = constant$.

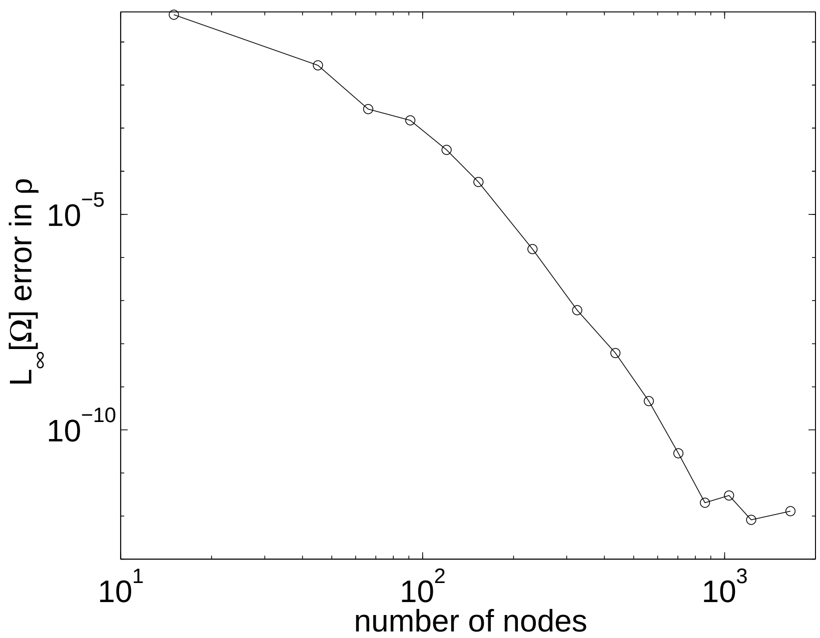
  QED.

Verification: Blunt Body Total Enthalpy



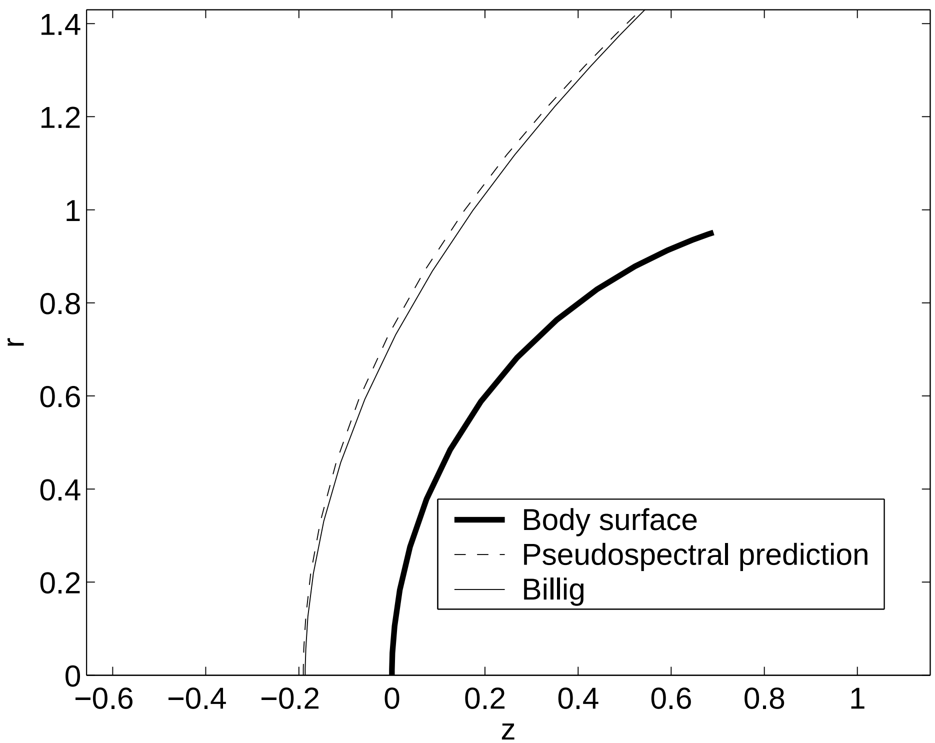
- H_o: a *true* constant
- deviation from freestream value measures error
- 17×9, error $\sim 10^{-5}$
- 29×15, error $\sim 10^{-9}$
- good quantitative verification

Verification: Blunt Body Grid Convergence



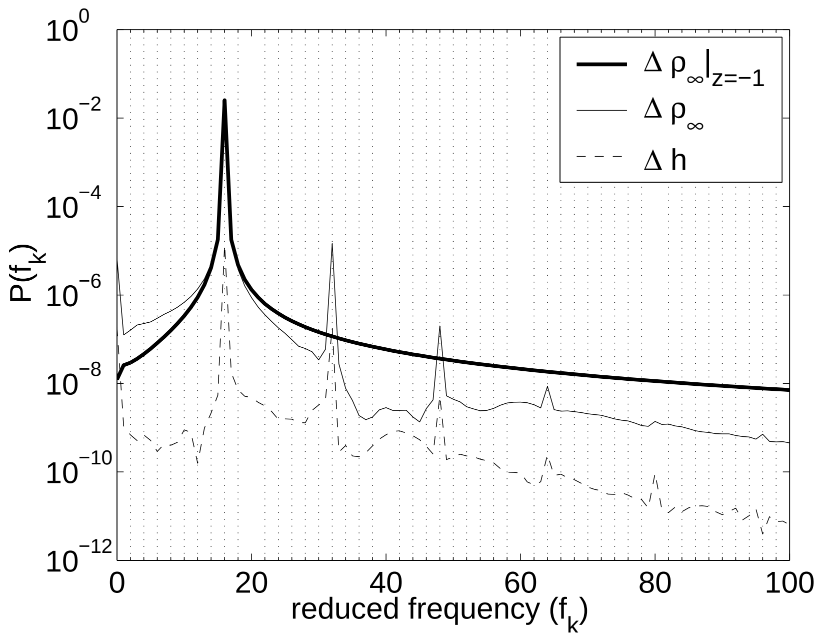
- “exact solution” from 65×33 grid
- spectral convergence
- error $\longrightarrow 10^{-12}$
- best quantitative verification

Validation: Flow over a Sphere



- Shock shape predictions match Billig's (*JSR*, 1967)
- Error $\sim 10^{-2}$

Unsteady Problem: Acoustic Wave/Shock Interaction



- low-frequency freestream input disturbance
- low-amplitude, high-frequency response captured by high accuracy method
- 33×17 grid; run time, $7.5\ hrs.$

Conclusions

- Pseudospectral method coupled with shock fitting gives solutions with high accuracy and spectral convergence rates in space for Euler equations.
- Standardized formulation of $\frac{d\mathbf{x}}{dt} = \mathbf{q}(\mathbf{x})$ allows use of integration methods with high accuracy in time.
- Algorithm has been verified to 10^{-12}.
- Predictions have been validated to 10^{-2}.
- Discrepancy between prediction and experiment is not attributable to truncation error.

- Challenge to determine which factor (*e.g.* neglected physical mechanisms, inaccurate constitutive data, measurement error, *etc.*) best explains the remaining discrepancy between prediction and observation.
- Challenge also to exploit verification and validation for first order shock capturing methods, necessary for complex geometries.