Resolved Viscous Detonation in $H_2/O_2/Ar$ Using Intrinsic Low Dimensional Manifolds and Wavelet Adaptive Multilevel Representation

by

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Outline

- Motivation for Intrinsic Low Dimensional Manifold (ILDM) technique (Maas & Pope, 1992) for spatially homogeneous systems (ODEs, no convection-diffusion)
- Compressible Reactive Navier-Stokes equations
- Details of ILDM implementation in operator splitting method
- Wavelet Adaptive Multilevel Representation (WAMR) technique (Paolucci & Vasilyev) for spatial discretization
- Results for one-dimensional viscous $H_2/O_2/Ar$ detonation with detailed kinetics (Singh, Rastigejev, Paolucci, and Powers, *Combustion Theory and Modeling*, submitted 2001) using operator splitting for convection-diffusion
- Work in progress: systematic, center manifold-motivated correction for convection-diffusion using ILDMs
- Conclusions

Some Important Questions

- Do we have resolved, accurate solutions for systems with detailed kinetics?
- How can ILDM improve the calculation of systems with detailed kinetics?
- How can ILDM, derived for spatially homogeneous systems, be used rationally in systems in which convection and diffusion are important?

Motivation for ILDM

- Detailed finite rate kinetics critical in reactive fluid mechanics
- Common detailed kinetic models are computationally expensive.
- Expense increases with
  - number of species and reactions modeled (linear effect),
  - *stiffness*–ratio of slow to fast time scales, (geometric effect).
- chemical time scales typically more demanding than convection-diffusion
- Reduced kinetics necessary given current computational resources.

Intrinsic Low-Dimensional Manifold Method (ILDM)

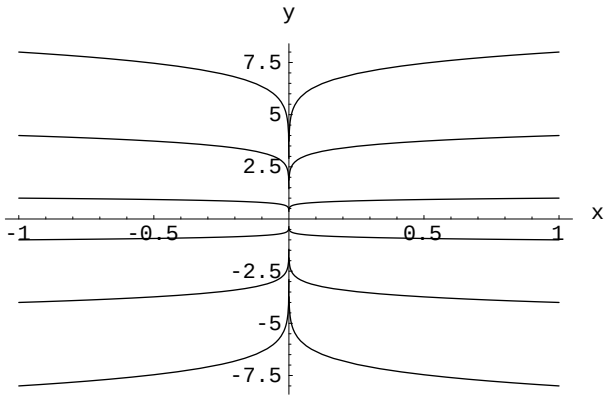
- Uses a dynamical systems approach,
- Most appropriate for spatially homogeneous systems (ODEs)
- Does not require imposition of *ad hoc* partial equilibrium or steady state assumptions,
- Fast time scale phenomena are systematically equilibrated,
- Slow time scale phenomena are resolved in time,
- Computation time reduced by factor of ~ 3 for non-trivial combustion problem considered here; manifold gives much better roadmap to find solution relative to general implicit solution techniques (Singh, *et al.* 2001)
- Speed up factor depends on
  - initial conditions,
  - stiffness ratio
  - dimension of ILDM

Simplest Example

$$\frac{dx}{dt} = -10x, \qquad x(0) = x_o,$$
$$\frac{dy}{dt} = -y, \qquad y(0) = y_o.$$

- Stable equilibrium at $(x, y) = (0,0)$; stiffness ratio = 10.

- ILDM is $x = 0$



- Parameterization of manifold: $x(s) = 0; y(s) = s$.

$$\frac{dy}{dt} = \frac{dy}{ds}\,\frac{ds}{dt}, \qquad \text{chain rule}$$
$$-y(s) = \frac{dy}{ds}\,\frac{ds}{dt}, \qquad \text{substitute from ODE and manifold}$$
$$-s = (1)\frac{ds}{dt}, \qquad \text{no longer stiff!}$$
$$s = s_o e^{-t},$$
$$x(t) = 0; \qquad y(t) = s_o e^{-t}.$$

- Projection onto manifold for s_o, induces small phase error.

Compressible Reactive Navier-Stokes Equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0, \qquad \text{mass}$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}\left(\rho u^2 + p - \tau\right) = 0, \qquad \text{momentum}$$

$$\frac{\partial}{\partial t}\left(\rho\left(e + \frac{u^2}{2}\right)\right) + \frac{\partial}{\partial x}\left(\rho u\left(e + \frac{u^2}{2}\right) + u\left(p - \tau\right) + q\right) = 0, \qquad \text{energy}$$

$$\frac{\partial}{\partial t}(\rho y_l) + \frac{\partial}{\partial x}(\rho u y_l + j_l) = 0, \qquad (l = 1, \ldots, L-1), \qquad \text{elements}$$

$$\frac{\partial}{\partial t}(\rho Y_i) + \frac{\partial}{\partial x}(\rho u Y_i + J_i^m) = \dot{\omega}_i M_i, \qquad (i = 1, \ldots, N-L), \qquad \text{species}$$

$$\tau = \frac{4}{3}\mu\frac{\partial u}{\partial x}, \qquad \text{Newtonian gas with Stokes' assumption}$$

$$J^q = -k\frac{\partial T}{\partial x} + \sum_{i=1}^{N} J_i^m\left(h_i^o + \int_{T_o}^{T} c_{pi}(\hat{T})d\hat{T}\right) - \Re T\sum_{i=1}^{N}\frac{\mathcal{D}_i^T}{M_i}\left(\frac{1}{\chi_i}\frac{\partial \chi_i}{\partial x} + \left(1 - \frac{M_i}{M}\right)\frac{1}{p}\frac{\partial p}{\partial x}\right), \qquad \text{Fourier's law}$$

$$J_i^m = \rho\sum_{j=1, j\neq i}^{N}\frac{M_i}{M}Y_j\mathcal{D}_{ij}\left(\frac{1}{\chi_j}\frac{\partial \chi_j}{\partial x} + \left(1 - \frac{M_j}{M}\right)\frac{1}{p}\frac{\partial p}{\partial x}\right) - \mathcal{D}_i^T\frac{1}{T}\frac{\partial T}{\partial x}, \quad (i = 1, \ldots, N), \qquad \text{Fick's law}$$

$$y_l = m_l\sum_{i=1}^{N}\frac{\phi_{il}}{M_i}Y_i, \qquad (l = 1, \ldots, L-1), \qquad \text{element mass fraction}$$

$$M = \sum_{i=1}^{N} M_i\chi_i, \qquad \text{mean molecular mass}$$

$$\chi_i = \frac{M}{M_i}Y_i, \qquad (i = 1, \ldots, N), \qquad \text{mole fraction}$$

$$j_l = m_l\sum_{i=1}^{N}\frac{\phi_{il}}{M_i}J_i, \qquad (l = 1, \ldots, L-1), \qquad \text{element mass flux}$$

$$\sum_{i=1}^{N} Y_i = 1, \qquad \text{mass fraction constraint}$$

$$\sum_{l=1}^{L} y_l = 1, \qquad \text{element mass fraction constraint}$$

$$\dot{\omega}_i = \sum_{j=1}^{J} a_j T^{\beta_j}\exp\left(\frac{-E_j}{\Re T}\right)\left(\nu_{ij}'' - \nu_{ij}'\right)\prod_{k=1}^{N}\left(\frac{\rho Y_k}{M_k}\right)^{\nu_{kj}'}, \qquad (i = 1, \ldots, N-L) \qquad \text{law of mass action}$$

$$p = \rho\Re T\sum_{i=1}^{N}\frac{Y_i}{M_i}, \qquad \text{thermal equation of state}$$

$$e = \sum_{i=1}^{N} Y_i\left(h_i^o + \int_{T_o}^{T} c_{pi}(\hat{T})d\hat{T} - \frac{\Re T}{M_i}\right). \qquad \text{caloric equation of state}$$

N species, L elements, J reactions

$4N + L + 7$ equations in $4N + L + 7$ unknowns

Focus on element conservation

- $L-1$ element equations formed; $N-L$ species equations, instead of the typical $N-1$ species equations,
- facilitates a proper use of ILDM in upcoming operator splitting,
- In general element mass fractions change due to mass diffusion

$$\rho \frac{dy_l}{dt} = -\frac{\partial j_l}{\partial x}.$$

- Assuming
  - Soret and DuFour effects are negligible, $D_i^T = 0$,
  - Deviations of molecular mass not large $M_i \sim M$,
  - Diffusion coefficients are equal, $\mathcal{D}_{ij} \sim \mathcal{D}$,
- Molecular mass diffusion reduces to $J_i^m = -\rho \mathcal{D} \frac{\partial Y_i}{\partial x}$.
- Elemental mass diffusion reduces to $j_l = -\rho \mathcal{D} \frac{\partial y_l}{\partial x}$.
- Evolution equation for element mass fraction becomes

$$\rho \frac{dy_l}{dt} = \mathcal{D} \frac{\partial}{\partial x} \left(\rho \frac{\partial y_l}{\partial x} \right).$$

- In uniformly premixed problem with no boundary influences then, all element concentrations are constant for all time:

$$\frac{dy_l}{dt} = 0.$$

Operator Splitting Technique for Convection-Diffusion

- Equations are of form

$$\frac{\partial}{\partial t}\mathbf{q}(x,t) + \frac{\partial}{\partial x}\mathbf{f}(\mathbf{q}(x,t)) = \mathbf{g}(\mathbf{q}(x,t)), \qquad \mathbf{q},\mathbf{f},\mathbf{g} \in \Re^{N+2}.$$

  where

$$\mathbf{q} = \left(\rho, \rho u, \rho\left(e + \frac{u^2}{2}\right), \rho y_l, \rho Y_i\right)^T.$$

- $\mathbf{f}$ models convection-diffusion

- $\mathbf{g}$ models reaction source terms

- Splitting

  1. Inert convection-diffusion step:

$$\frac{\partial}{\partial t}\mathbf{q}(x,t) + \frac{\partial}{\partial x}\mathbf{f}(\mathbf{q}(x,t)) = 0,$$
$$\frac{d}{dt}\mathbf{q}_i(t) = -\Delta_x \mathbf{f}(\mathbf{q}_i(t)).$$

     Δ_x is any spatial discretization operator, here a wavelet operator.

  2. Reaction source term step:

$$\frac{\partial}{\partial t}\mathbf{q}(x,t) = \mathbf{g}(\mathbf{q}(x,t)),$$
$$\frac{d}{dt}\mathbf{q}_i(t) = \mathbf{g}(\mathbf{q}_i(t)).$$

- Operator splitting with implicit stiff source solution can induce non-physical wave speeds! (LeVeque and Yee, *JCP* 1990)

ILDM Implementation in Operator Splitting

- Form of equations in source term step:

$$\frac{d}{dt}\begin{pmatrix} \rho \\ \rho u \\ \rho\left(e+\frac{u^2}{2}\right) \\ \rho y_l \\ \rho Y_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dot{\omega}_i M_i \end{pmatrix}.$$

$$l = 1, \ldots, L-1, \qquad i = 1, \ldots, N-L.$$

- Equations reduce to

$$\rho = \rho_o, \qquad u = u_o, \qquad e = e_o, \qquad y_l = y_{lo},$$

$$\frac{dY_i}{dt} = \frac{\dot{\omega}_i M_i}{\rho_o}, \qquad i = 1, \ldots, N-L$$

- $\dot{\omega}_i$ has dependency on ρ, e, y_l, and Y_i

- ODEs for Y_i are stiff, usually solved with implicit methods.

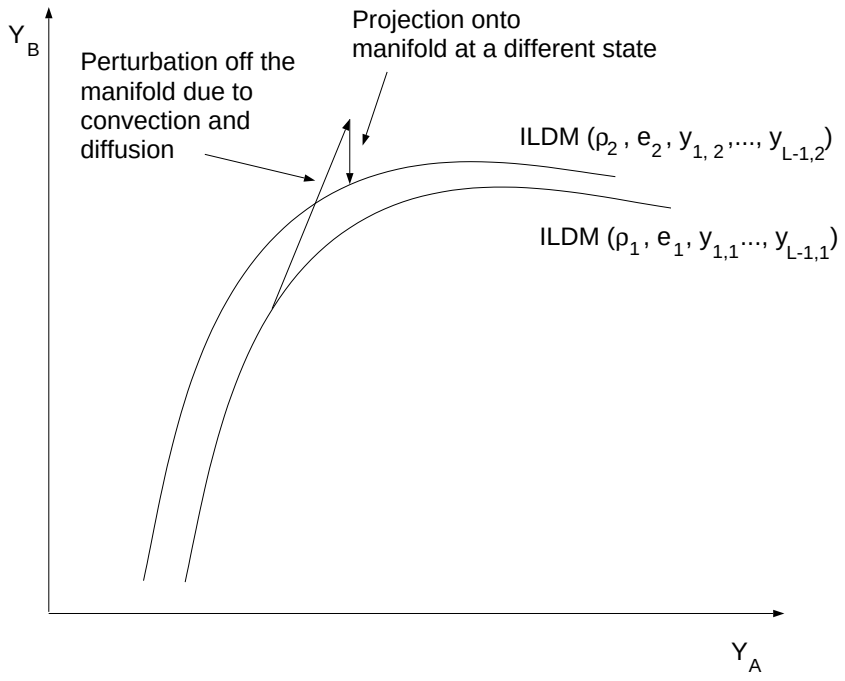
- ODEs for Y_i can be attacked with manifold methods to remove stiffness with ILDM with ρ, e, $y_l, \ldots, y_{L-1}$ parameterization.

Necessary Dimension of ILDM

- Spatial discretization of PDEs results in a set of adiabatic, isochoric spatially homogeneous reactors,
- N species with L elements at constant e and ρ gives rise to a $(N-L)$-dimensional phase space (same as composition space),
- To resolve M slow time scales, we identify M-dimensional subspaces (manifolds), $M < (N-L)$, embedded within the $(N-L)$-dimensional phase space on which the M slow time scale events evolve,
  - Fast time scale events rapidly move to the manifold,
  - Slow time scale events move on the manifold,
  - Because of convection-diffusion, e, ρ, y_l vary, requiring a $K = M + L + 1$-dimensional manifold.
  - If y_l conserved (premixed with no preferential diffusion), dimension of manifold is reduced by $L - 1$.
  - e.g., for $M = 1$ in premixed $H_2/O_2/Ar$ with no preferential diffusion, small molecular mass deviation, and negligible Soret/DuFour diffusion, we need $K = 3$.

Implementation of ILDMs with convection-diffusion and operator splitting

- To minimize phase error, must integrate full equations until sufficiently close to ILDM
- When near ILDM, M slow equations are integrated, other variables found by table lookup
- Convection-diffusion step *applied to all variables* perturbs system from ILDM
- In next reaction step, project to ILDM at *different* value of ρ, e, $y_1, \ldots, y_{N-1}$.



Formulation of General ILDMs

- A spatially homogeneous adiabatic, isochoric chemically reactive system of N species in L elements is modeled by a set of non-linear ordinary differential equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}), \qquad \mathbf{x}(0) = \mathbf{x}_o,$$

$$\mathbf{x}: \text{ species concentration}; \mathbf{x} \in \Re^{N-L}$$

- Equilibrium points defined by

$$\mathbf{x} = \mathbf{x}_{eq} \text{ such that } \mathbf{F}(\mathbf{x}_{eq}) = 0.$$

- Consider a system near equilibrium (the argument can and must be extended for systems away from equilibrium) with $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_{eq}$.

- Linearization gives

$$\frac{d\tilde{\mathbf{x}}}{dt} = \mathbf{F}_\mathbf{x} \cdot \tilde{\mathbf{x}},$$

where $\mathbf{F}_\mathbf{x}$ is a *constant* Jacobian matrix.

- Schur decompose the Jacobian matrix:

$$\mathbf{F}_\mathbf{x} = \mathbf{Q} \cdot \mathbf{U} \cdot \mathbf{Q}^T$$

$$\mathbf{Q} = \begin{pmatrix} \vdots & \vdots & & \vdots \\ q_1 & q_2 & \cdots & q_{N-L} \\ \vdots & \vdots & & \vdots \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} \lambda_1 & u_{1,2} & \cdots & u_{1,N-L} \\ 0 & \lambda_2 & \cdots & u_{2,N-L} \\ 0 & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda_{N-L} \end{pmatrix}, \quad \mathbf{Q}^T = \begin{pmatrix} \cdots & q_1^T & \cdots \\ \cdots & q_2^T & \cdots \\ & \vdots & \\ \cdots & q_{N-L}^T & \cdots \end{pmatrix}$$

Formulation of General Manifolds (cont.)

- $\mathbf{Q}$ is an orthogonal matrix with real Schur vectors q_i in its columns.
- $\mathbf{U}$ is an upper triangular matrix with eigenvalues of $\mathbf{F_x}$ on its diagonal, sometimes placed in order of decreasing magnitude.
- The Schur vectors q_i form an orthonormal basis which spans the phase space, $\Re^{N-L}$.
- We then define M slow time scales, $M < N - L$.
- Next define a non-square matrix $\mathbf{W}$ which has in its rows the Schur vectors associated with the fast time scales:

$$\mathbf{W} = \begin{pmatrix} \cdots & \cdots & q_{M+1}^T & \cdots & \cdots \\ \cdots & \cdots & q_{M+2}^T & \cdots & \cdots \\ & & \vdots & & \\ \cdots & \cdots & q_{N-L}^T & \cdots & \cdots \end{pmatrix}.$$

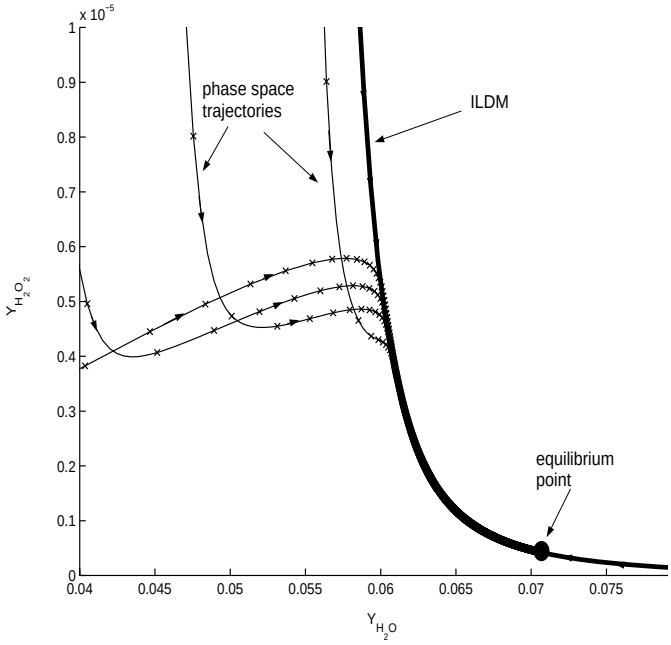
- Letting the fast time scale events equilibrate defines the manifold:

$$\mathbf{W} \cdot \mathbf{F}(\mathbf{x}) = 0.$$

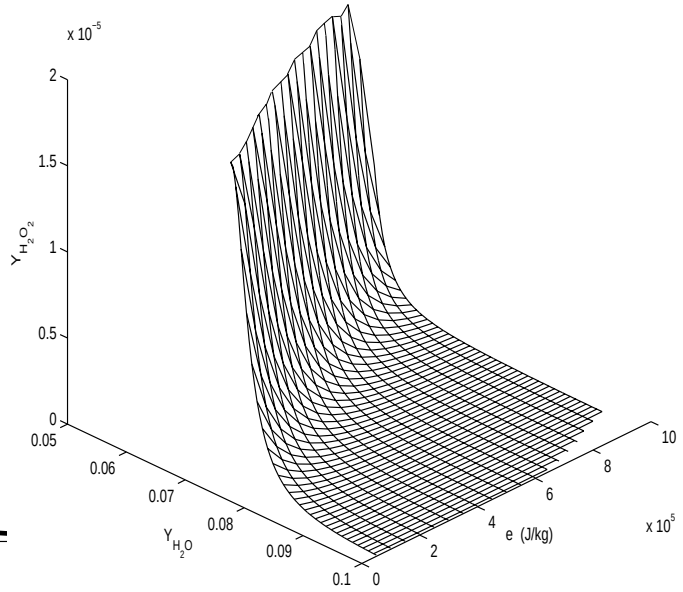
Sample ILDM for $H_2/O_2/Ar$

- Based on $N = 9$, $J = 37$ mechanism of Maas and Warnatz,
- Projection in Y_{H_2O}, $Y_{H_2O_2}$ plane and Y_{H_2O}, $Y_{H_2O_2}$, e space
- Adiabatic ($e = 8 \times 10^5\ J/kg$), isochoric ($\rho = 5.0 \times 10^{-4}\ kg/m^3$), $y_H = 0.01277$, $y_O = 0.10137$, $y_{Ar} = 0.88586$,
- We can get e.g. $p\left(\rho, e, Y_{H_2O}\right), T\left(\rho, e, Y_{H_2O}\right), Y_H\left(\rho, e, Y_{H_2O}\right), \ldots$
- Linear interpolation used for points not in table,
- Captures $\sim 0.1\ \mu s$ reaction events.

2-D projection



3-D projection



Wavelet Adaptive Multilevel Representation (WAMR) Technique

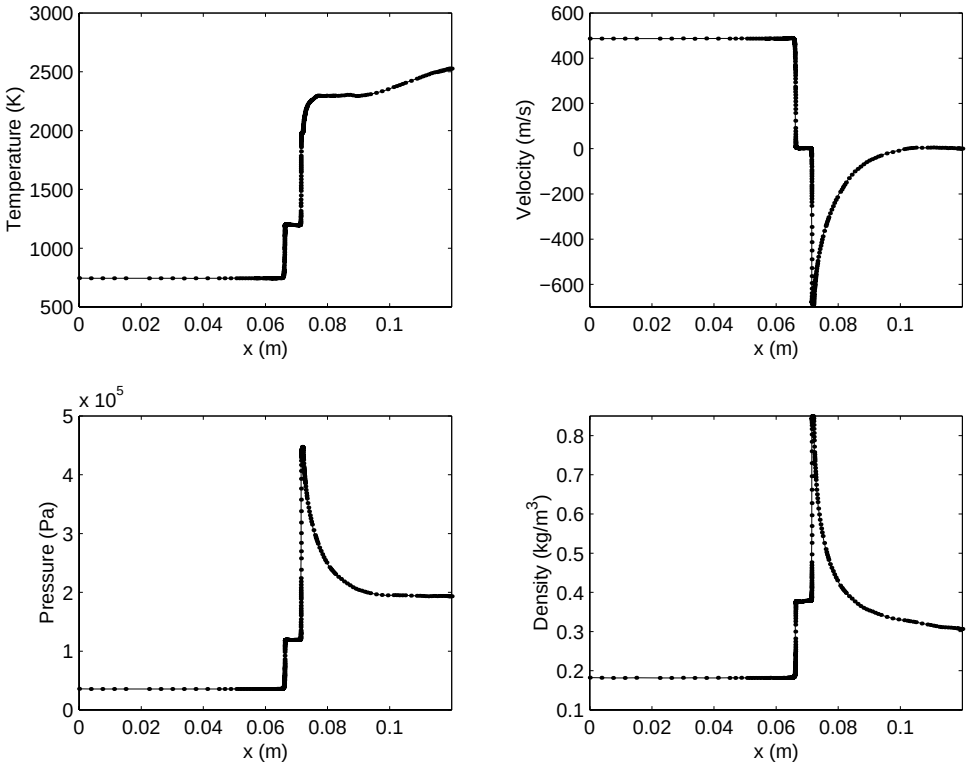
- Summary of standard spatial discretization techniques
  - Finite difference-good spatial localization, poor spectral localization, and slow convergence,
  - Finite element- good spatial localization, poor spectral localization, and slow convergence,
  - Spectral–good spectral localization, poor spatial localization, but fast convergence.
- Wavelet technique
  - See e.g. Vasilyev and Paolucci, "A Fast Adaptive Wavelet Collocation Algorithm for Multidimensional PDEs," *J. Comp. Phys.*, 1997,
  - Basis functions have compact support,
  - Well-suited for problems with widely disparate spatial scales,
  - Good spatial and spectral localization, and fast (spectral) convergence,
  - Easy adaptable to steep gradients via adding collocation points,
  - Spatial adaptation is automatic and dynamic to achieve prescribed error tolerance.

Ignition Delay in Premixed $H_2/O_2/Ar$

- Consider standard problem of Fedkiw, Merriman, and Osher, *J. Comp. Phys.*, 1996,
- Shock tube with premixed H_2, O_2, and Ar in 2/1/7 molar ratio,
- Initial inert shock propagating in tube,
- Reaction commences shortly after reflection off end wall,
- Detonation soon develops,
- Model assumptions
  - One-dimensional,
  - Mass, momentum, and energy diffusion,
  - Nine species, thirty-seven reactions,
  - Ideal gases with variable specific heats.

Viscous $H_2 - O_2$ Ignition Delay with Wavelets and ILDM

- $t = 195\ \mu s$, 300 collocation points, 15 wavelet scale levels
- ILDM gives nearly identical results as full chemistry
- WAMR spatial discretization, implicit linear trapezoidal convection-diffusion time stepping, explicit (ILDM)/implicit (non-ILDM) reaction time stepping
- *Viscous shocks, inductions zones, and entropy layers spatially resolved!*



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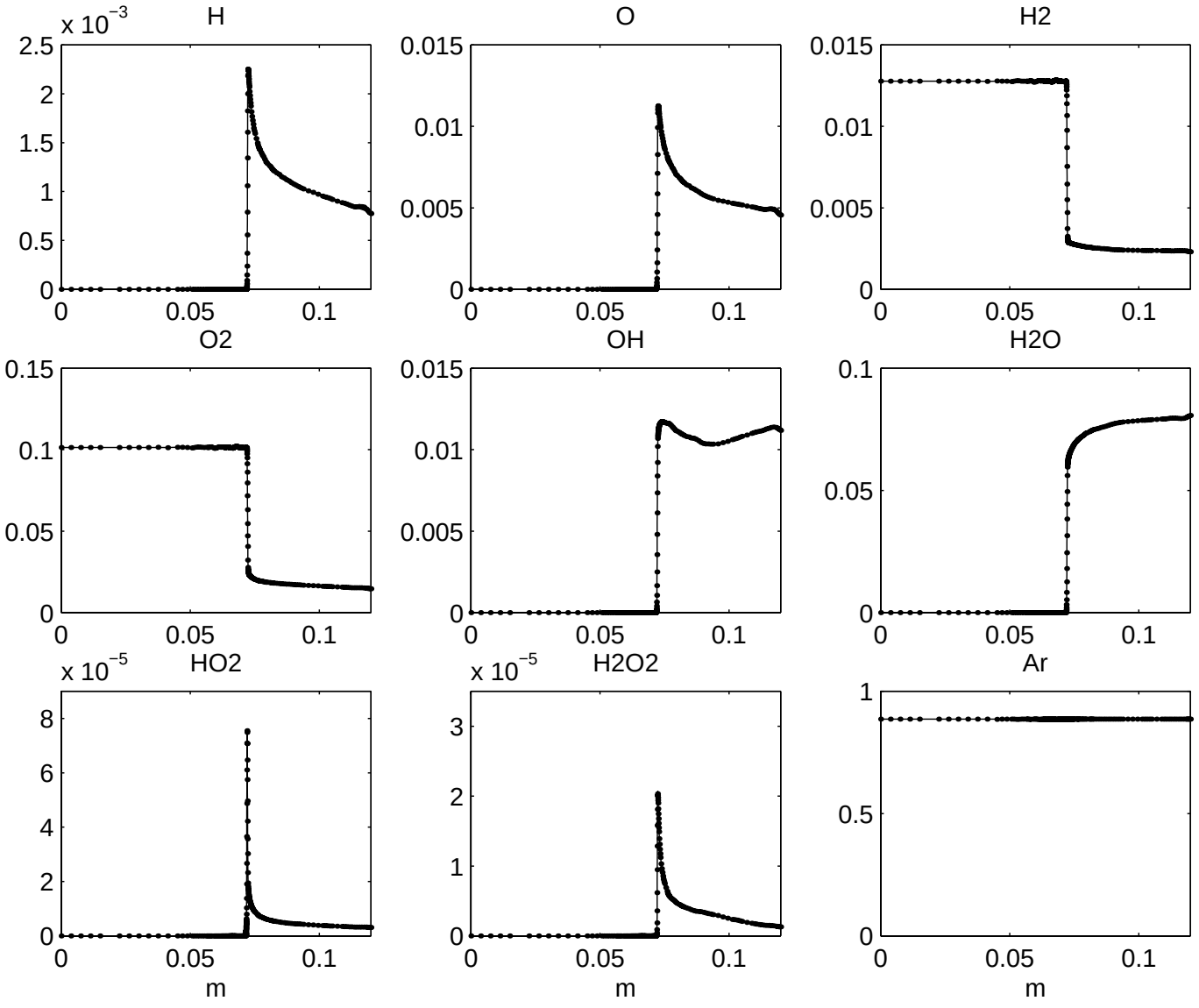
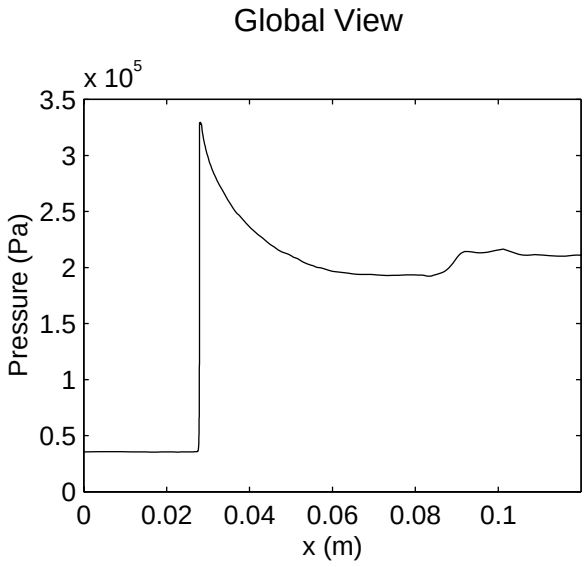
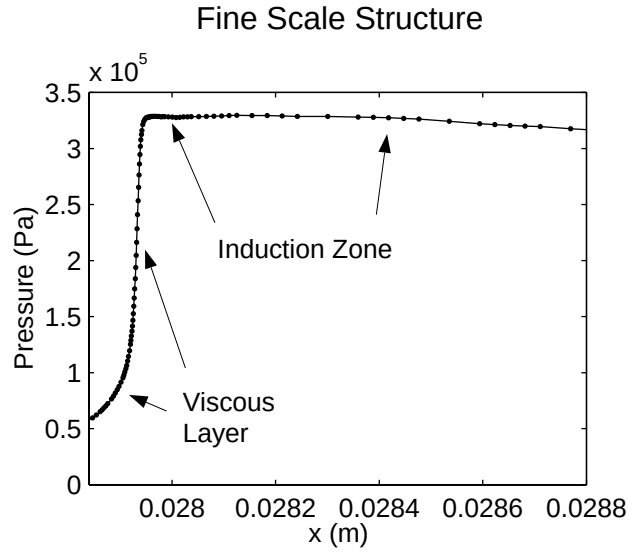


Viscous $H_2 - O_2$ Ignition Delay with Wavelets

Global and Fine Scale Structures

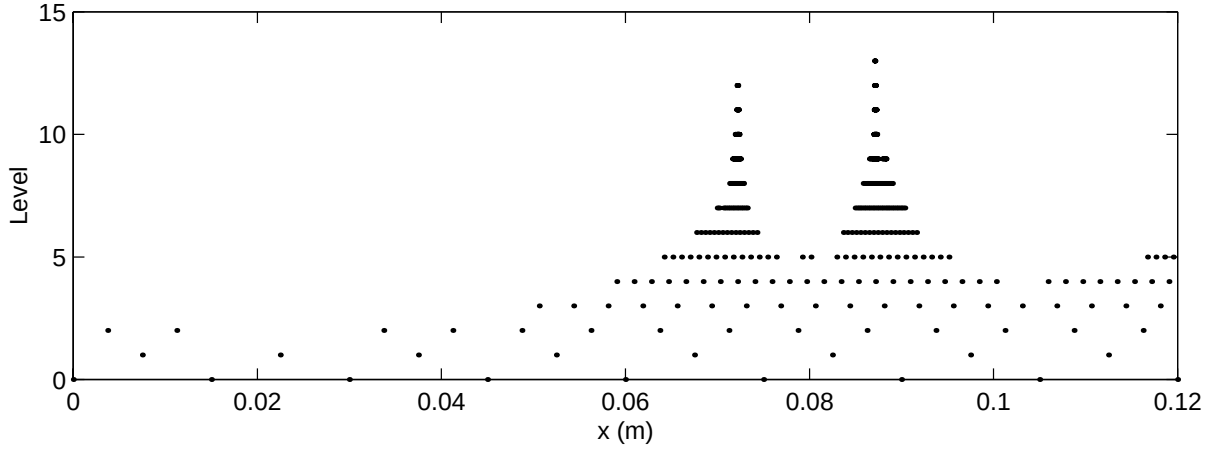
- $t = 230\ \mu s$, Induction zone length: $\sim 470\ \mu m$, Viscous shock thickness: $\sim 50\ \mu m$ (should use smaller μ),
- No significant reaction in viscous shock zone.

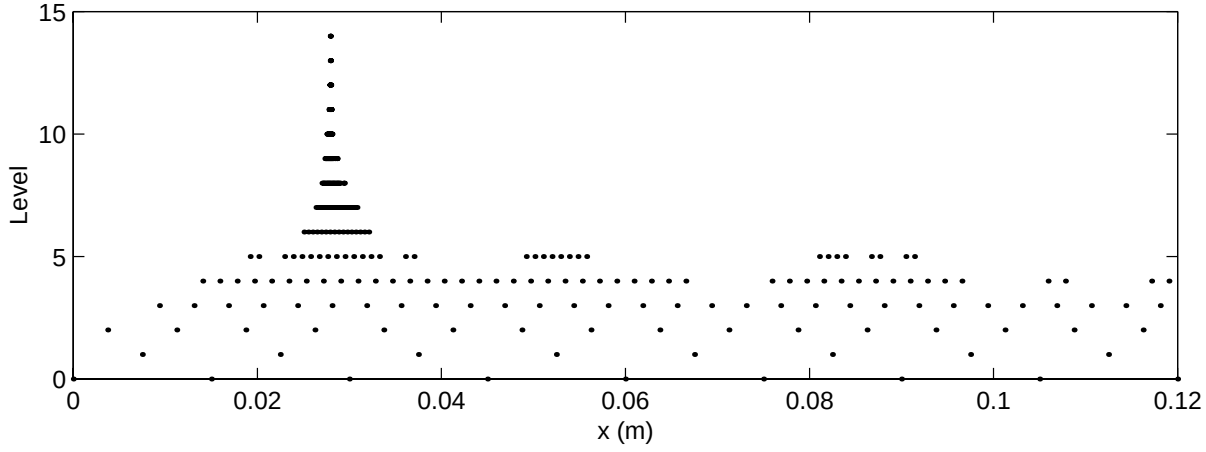




Viscous $H_2 - O_2$ Ignition Delay with Wavelets, Instantaneous Distributions of Collocation Points

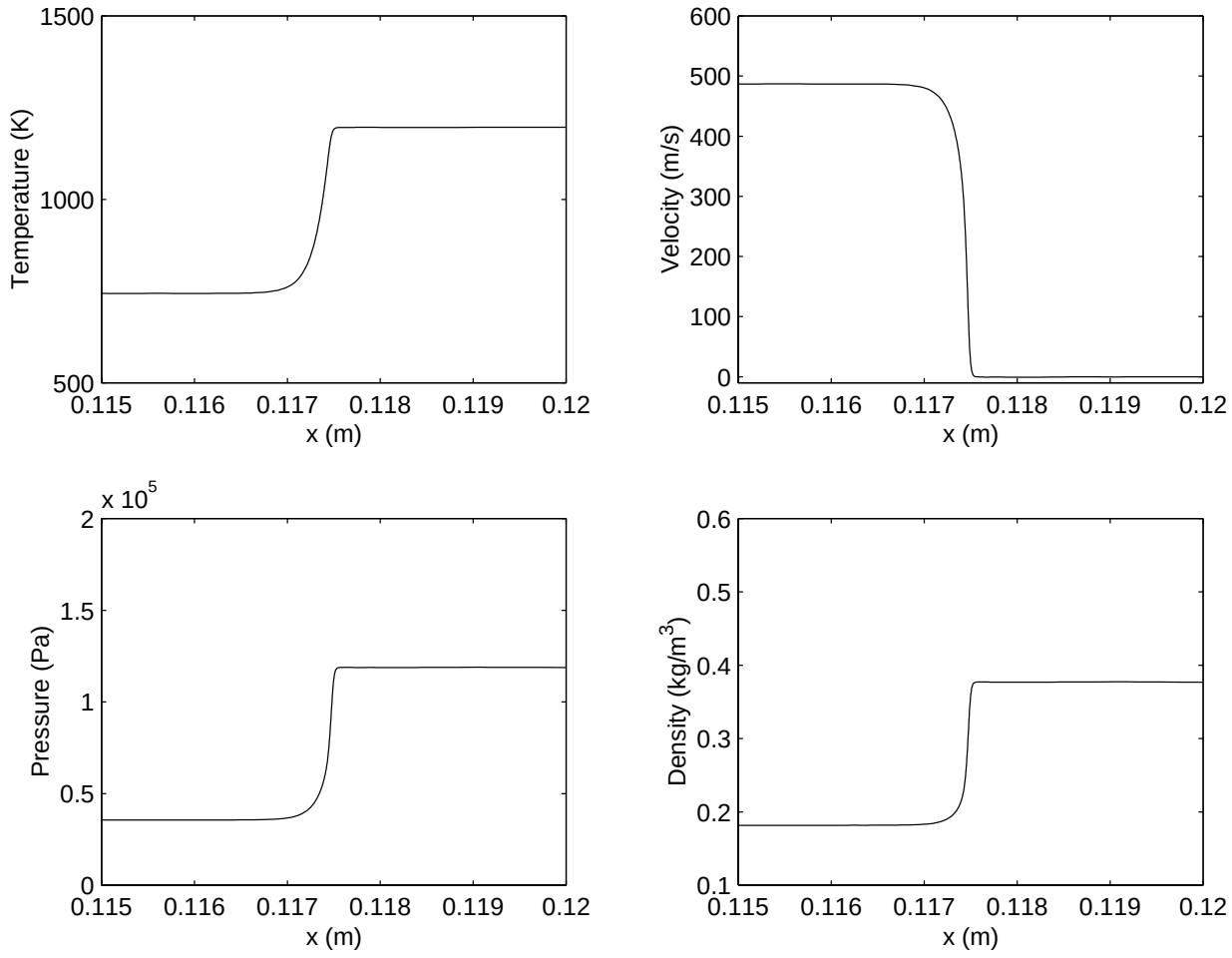
- $t = 180\ \mu s$, two-shock structure with consequent collocation point distribution,
- $t = 230\ \mu s$, one-shock structure with evolved collocation point distribution.





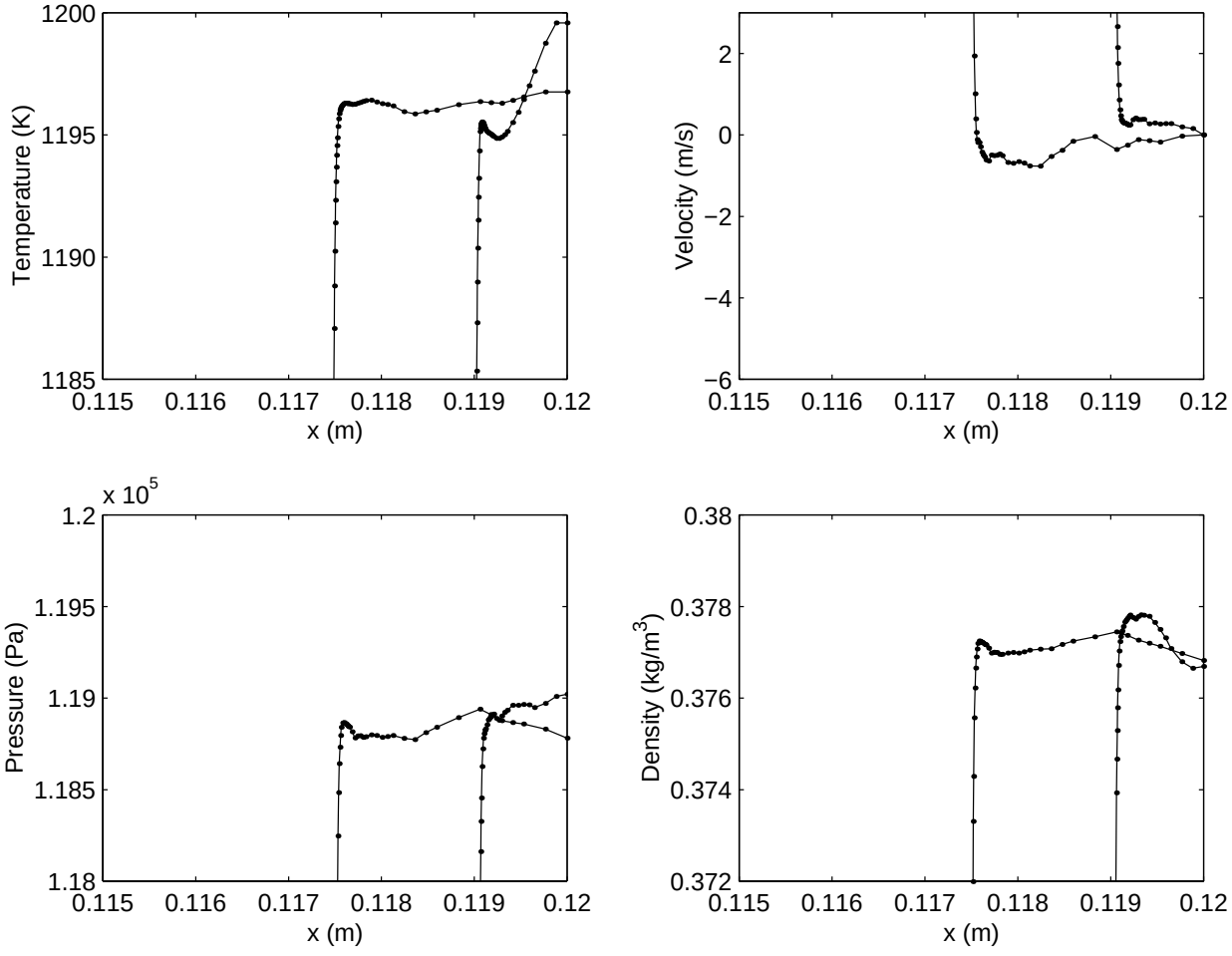
Post Reflection Entropy Layer?: Viscous Wavelet Results

- No significant entropy layer evident on macroscale after shock reflection when resolved viscous terms considered,
- Inviscid codes with coarse gridding introduce a larger entropy layer due to numerical diffusion,
- Unless suppressed, unphysically accelerates reaction rate.



Post Reflection Entropy Layer: Viscous Wavelet Results

- small entropy layer evident on finer scale,
- temperature rise $\sim 5\ K$; dissipates quickly,
- inviscid calculations before adjustment give persistent temperature rise of $\sim 20\ K$; reaction acceleration small.



Center manifold-motivated correction for small convection-diffusion

- Consider system of Davis and Skodje, 1999, extended for diffusion

$$\begin{aligned}
\frac{\partial y_1}{\partial t} &= \underbrace{-y_1}_{\text{reaction}} + \underbrace{\epsilon \frac{\partial^2 y_1}{\partial x^2}}_{\text{diffusion}}, \\
\frac{\partial y_2}{\partial t} &= \underbrace{-\gamma y_2 + \frac{(\gamma-1)y_1 + \gamma y_1^2}{(1+y_1)^2}}_{\text{reaction}} + \underbrace{\epsilon \frac{\partial^2 y_2}{\partial x^2}}_{\text{diffusion}},
\end{aligned}$$

$$\begin{aligned}
y_1(x,0) &= x, \qquad y_1(0,t) = 0, \qquad y_1(1,t) = 1, \\
y_2(x,0) &= 0.55x \qquad y_2(0,t) = 0, \qquad y_2(1,t) = 0.55.
\end{aligned}$$

- $\gamma >> 1$ for chemical stiffness; $\epsilon << 1$ for small diffusion

- Maas-Pope ILDM:

$$y_2 = \frac{y_1}{1+y_1} + \frac{2y_1^2}{\gamma(\gamma-1)(1+y_1)^3}.$$

- *Purely* reactive system has equilibrium *point* in phase space at $y_1 = 0$, $y_2 = 0$ at $t \to \infty$.

- System with convection-diffusion approaches steady state *manifold*, not ILDM, as $t \to \infty$ given by solution of ODEs:

$$\begin{aligned}
0 &= -y_1 + \epsilon \frac{d^2 y_1}{dx^2}; \quad y_1(0) = 0; \quad y_1(1) = 1, \\
0 &= -\gamma y_2 + \frac{(\gamma-1)y_1 + \gamma y_1^2}{(1+y_1)^2} + \epsilon \frac{d^2 y_2}{dx^2}; \quad y_2(0) = 0; \quad y_2(1) = 0.55.
\end{aligned}$$

Center manifold-motivated correction for small convection-diffusion

- Assume convection-diffusion acts as a small perturbation
- Define fast (w_f) and slow (w_s) variables based on analytic Jacobian of chemical source term:

$$\begin{pmatrix} w_s(x,t) \\ w_f(x,t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\alpha(y_{10}(x)) & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1(x,t) - y_{10}(x) \\ y_2(x,t) - h(y_{10}(x)) \end{pmatrix}$$

- $y_{10}(x)$ is solution for $y_1(x)$ at previous time step,
- $h(y_{10}(x))$ is the ILDM.
- Project original PDEs onto the slow and fast basis near ILDM to get

$$\frac{\partial w_s}{\partial t} = -y_{10}(x) - w_s + \underbrace{\epsilon\left(\frac{d^2 y_{10}}{dx^2} + \frac{\partial^2 w_s}{\partial x^2}\right)}_{\text{convection-diffusion correction}} + H.O.T.,$$

$$\frac{\partial w_f}{\partial t} = \underbrace{\text{Maas-Pope ILDM term}}_{=0} - \gamma w_f + \underbrace{+\epsilon\left(g_1(y_{10}(x)) + w_s g_2(y_{10}(x)) + \frac{\partial w_s}{\partial x} g_3(y_{10}(x)) + \frac{\partial^2 w_f}{\partial x^2}\right)}_{\text{convection-diffusion correction}} + H.O.T.$$

Center manifold-motivated correction for small convection-diffusion

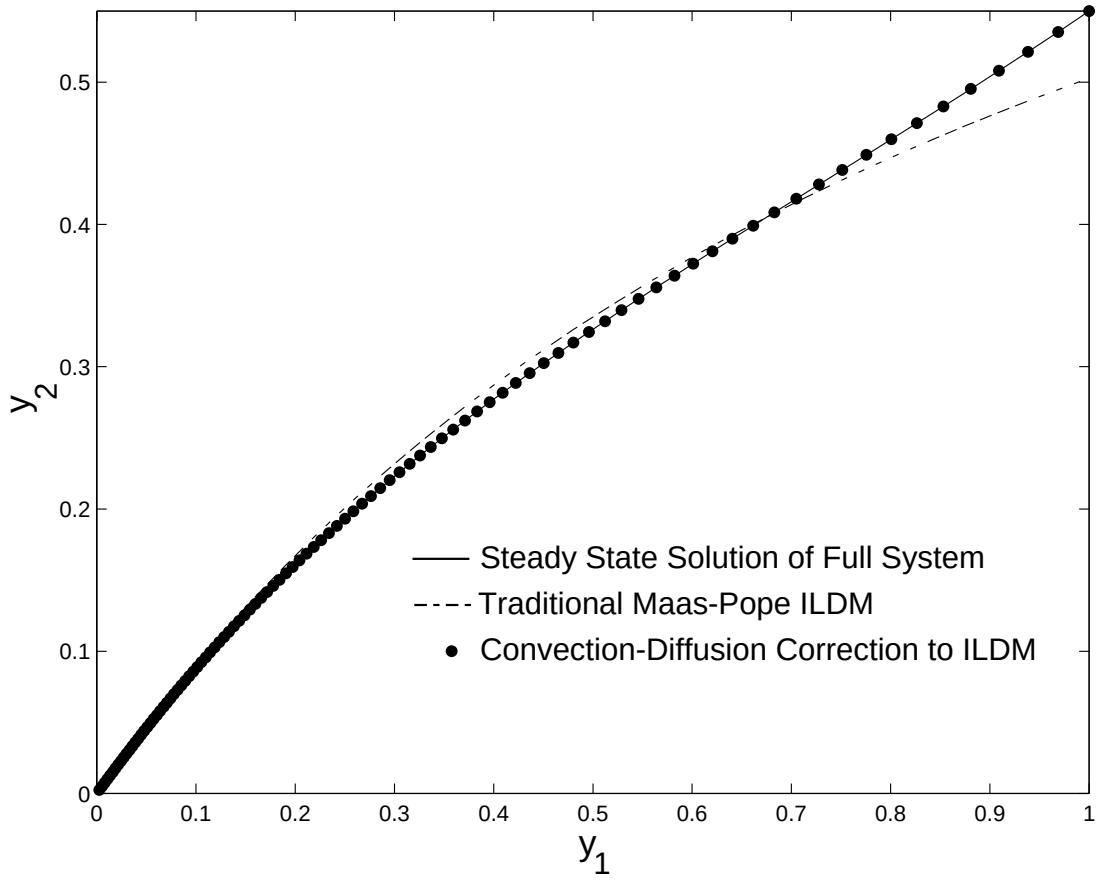
- equilibrate fast variables: $\frac{\partial w_f}{\partial t} = 0$, giving an elliptic equation

$$
\begin{aligned}
0 \;=\; & -\gamma w_f + \\
& + \underbrace{\epsilon \left(g_1(y_{10}(x)) + w_s g_2(y_{10}(x)) + \frac{\partial w_s}{\partial x} g_3(y_{10}(x)) + \frac{\partial^2 w_f}{\partial x^2} \right)}_{\text{convection}-\text{diffusion correction}} \\
& + H.O.T.
\end{aligned}
$$

- Use method of lines, combined with simultaneous solution of elliptic equation, to advance slow variables using large time step,
- Analogous to solving elliptic equation for pressure when time advancing incompressible Navier-Stokes equations.

Center Manifold-Motivated Correction for Convection-Diffusion

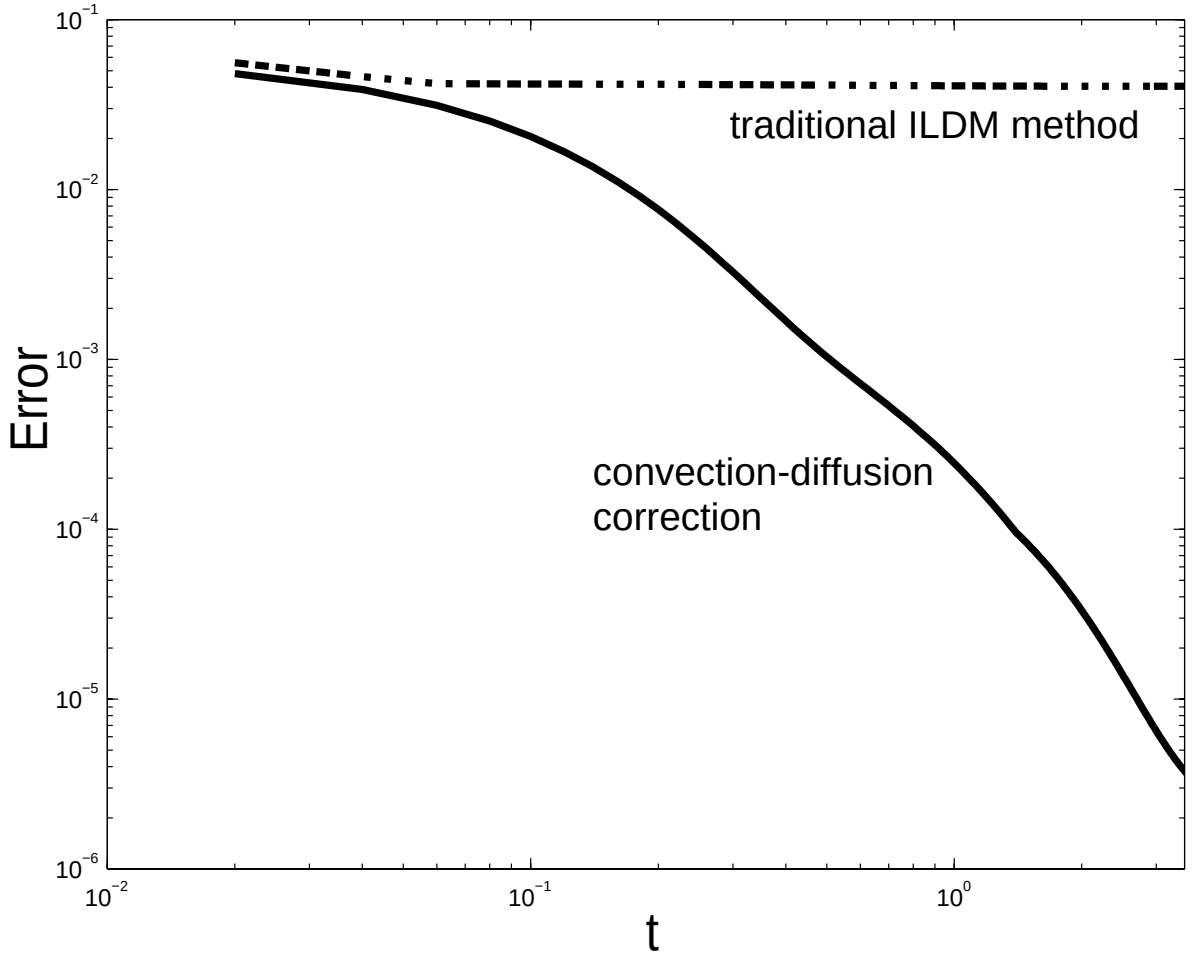
- Long time solution does not approach Maas and Pope ILDM,
- Convection-diffusion correction gives more accurate predictions



- $H_2/O_2/Ar$ ILDM results accurate because restricted to near equilibrium regions
- arbitrary use of ILDM can give inaccurate results

Center Manifold-Motivated Correction for Convection-Diffusion

- The corrected method gives more accurate predictions of intermediate and long times.



Conclusions

- The WAMR method gives dramatic spatial resolution in viscous one-dimensional H_2/O_2 detonations with detailed kinetics; viscous shocks, entropy layers, and induction zones are resolved,
- ILDM method, coupled appropriate use of full integration, with operator spitting accurately recovers most results of full chemistry with decrease (factor of three for our case) in computational time,
- Work needed to better account for projection of initial conditions onto ILDM,
- Center manifold-motivated correction for convection and diffusion a promising improvement to ILDM method in some (not all) problems.