Physical Diffusion Cures the Carbuncle Phenomenon

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Motivation

We are investigating the supersonic flow of a calorically perfect ideal gas past a two-dimensional blunt body:

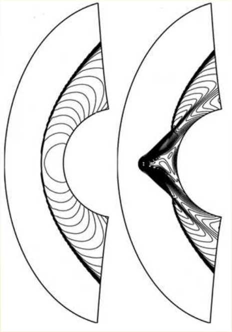
- This study will help to understand and predict the air flow around re-entry vehicles by improving upon prior methods of computation.
- Other strategies for computing such flows have led to an anomalous solution referred to as the “carbuncle phenomenon”.
- We are trying to find a simple antidote that will avoid numerical anomalies in the solution.
- A physically based strategy can also insure fidelity with what is observed in nature.

Flawed solutions in earlier studies

- Anomalous solutions have been predicted by so-called high resolution schemes which employ flux limiters within shock-capturing methods applied to the Euler equations in simulating supersonic flow.
- This unphysical anomaly, described as the "carbuncle phenomenon," was first predicted by Peery and Imlay (1988) and has been widely reported in the literature.
- Although many researchers, such as Quirk (1994), describe complicated methods for eliminating the carbuncle phenomenon, others, such as Elling (2009), describe the phenomenon as "incurable."

What does the carbuncle look like?

- The carbuncle does not appear in nature; it is a spurious solution of numerical origin.
- Dumbser (2004) demonstrated that when simulating the Euler equations with common high resolution methods, above a threshold Mach number M, the solution displays unconditional instability with exponential error growth.



Robinet, et al., *Journal of Fluid Mechanics*, 2000.

Prior solutions

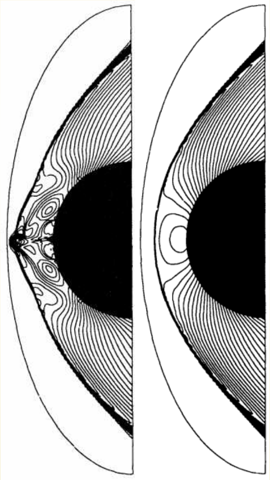
- To remove the carbuncle instability, artificial dissipation is often added to the Euler equations.
  - This is a post-dictive strategy of no use as a predictive tool.
  - This is not a robust approach to a solution as the dissipation added varies from problem to problem and method to method.
- A small fraction of studies have recognized that physical diffusion can be offered as a remedy.
  - Ismail (2009) considered a viscous cure, but discounted it because the carbuncle "disappears only at very low Reynolds number."
  - Ohwada (2013) and Li, et al (2011) have modeled diffusion with a kinetic theory and demonstrated it provides a remedy for carbuncles.
  - Such calculations can be prohibitively expensive.

Quirk's cure to the carbuncle

- Quirk (1994) looked into the carbuncle phenomenon brought upon by the subtle flaws of the Godunov-type methods.
- He offered a solution of using artificial dissipation to remove the carbuncle, specifically showing that Einfeldt's HLLE (Harten, Lax, van Leer and Einfeldt) scheme cured the carbuncle.
- He also used an adaptive Riemann solver, which would choose the type of upwinding scheme that matched the local flow data so that the Riemann solver would give reliable results.

Quirk's results

- Quirk's strategy cures the carbuncle phenomena by adding artificial dissipation.
- Although this solution is sufficient for this case, Quirk's strategy is not robust.
- His solution strategy must be altered for different geometries and flow conditions, and different upwinding schemes must be chosen for different conditions to maintain reliability.



Quirk, *International Journal for Numerical Methods in Fluids*, 1994.

Improved numerical strategy

- We will solve two problems to demonstrate the carbuncle phenomenon and its rectification.
  - We will neglect physical diffusion for the first problem.
  - Then, we will introduce physical momentum and energy diffusion via discretization of the ordinary Navier-Stokes equations, employed on a sufficiently fine grid to capture viscous shocks.
  - Both problems employ the same geometry, initial conditions, computational grid, numeric flux model, and time-advancement scheme.
  - The Riemann solver used in numeric flux formulation is based on Roe flux difference splitting without entropy fix.
- Re-introduction of physical diffusion will provide a damping mechanism to suppress spurious solutions which we believe to be of numerical origin.
- It will be seen that physical diffusion, appropriately resolved, cures the carbuncle problem.

Goals for this study

We thus aim to show that:

- *The carbuncle phenomenon, induced by many high resolution, nominally high order, shock-capturing schemes for Euler equations applied to supersonic flow over a blunt body, is cured by inclusion of properly resolved physical diffusion in a verified and validated Navier-Stokes model, and*
- *When fine scale physical diffusion structures are resolved, simple low order discretization schemes are sufficient to capture the continuum flow physics of supersonic flow over a blunt body.*

Navier-Stokes Equations

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0, \\
\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u}\mathbf{u}^T\right) &= -\nabla p + \nabla \cdot \boldsymbol{\tau}, \\
\frac{\partial}{\partial t}\left(e + \frac{1}{2}\mathbf{u}\cdot\mathbf{u}\right) + \nabla \cdot \left(\rho \mathbf{u}\left(e + \frac{1}{2}\mathbf{u}\cdot\mathbf{u}\right)\right) &= -\nabla \cdot \mathbf{q} - \nabla (p\mathbf{u}) + \nabla \cdot (\boldsymbol{\tau}\cdot\mathbf{u}), \\
\mathbf{q} &= -k\nabla T, \\
\boldsymbol{\tau} &= 2\mu\left(\frac{\nabla \mathbf{u} + (\nabla \mathbf{u})^T}{2} - \frac{1}{3}(\nabla \cdot \mathbf{u})\mathbf{I}\right), \\
p &= \rho R T, \\
e &= c_v T.
\end{aligned}
$$

Parameters

parameter	value	units
R	287.7	J/kg/K
c_v	719.3	J/kg/K
c_p	1007	J/kg/K
p_1	12.4272	Pa
T_1	39.667	K
u_1	724.293	m/s
M_1	5.73	
γ	7/5	
μ	2.3648×10^{-6}	Pa s
k	0.003093	W/m/K
a	0.00015	m
ρ_1	0.001088	kg/m^3
c_1	126.404	m/s
ν_1	0.002174	m^2/s
Re	50	
Pr	0.77	

Numerical Method: OpenFOAM

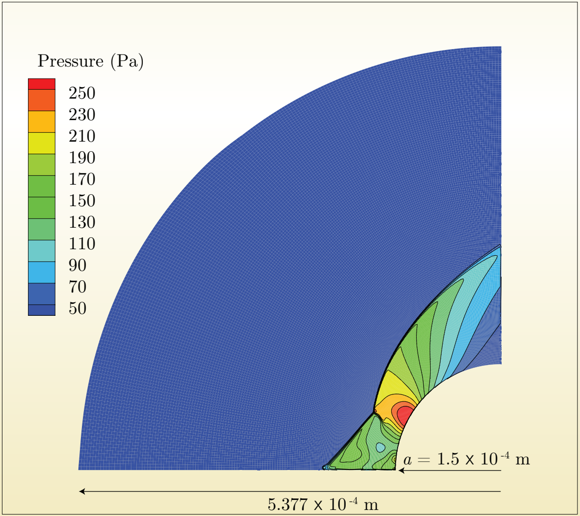
- A solver based on a reconstruction-evolution-projection algorithm was developed using the public domain software OpenFOAM ™.
- The time-advancement scheme was a fourth-order Runge-Kutta method.
- A first order spatial discretization was employed.
- Numerous grids were employed, most studies used approximately 120,000 cells.
  - A typical cell length scale was $\sim$5 microns.
  - The estimated shock thickness of $\sim \frac{\nu_1}{c_1} = 17.2$ microns, showing that a typical cell length is small enough to resolve the shock.

Results

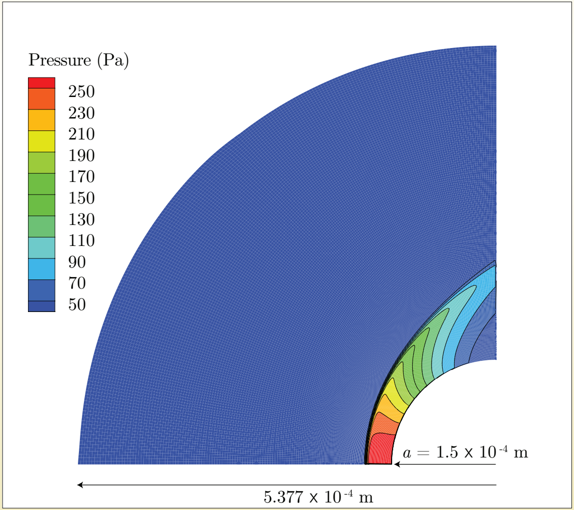
When calculating the solution of the blunt body re-entry problem without physical diffusion, the carbuncle appears. When physical diffusion is present, the carbuncle is cured.

- When physical diffusion is neglected, a triangle forms attached to the front of the cylinder, essentially the same shape as the carbuncle phenomenon predicted in other studies.
- Investigation of the typical fields, such as pressure, temperature, and velocity fields, was completed to check if there were smooth contours and a smooth shock was formed in the viscous solution.
- Plots of the temperature, velocity, and pressure along the centerline show that the viscous solution calculated a smooth and accurate shock.

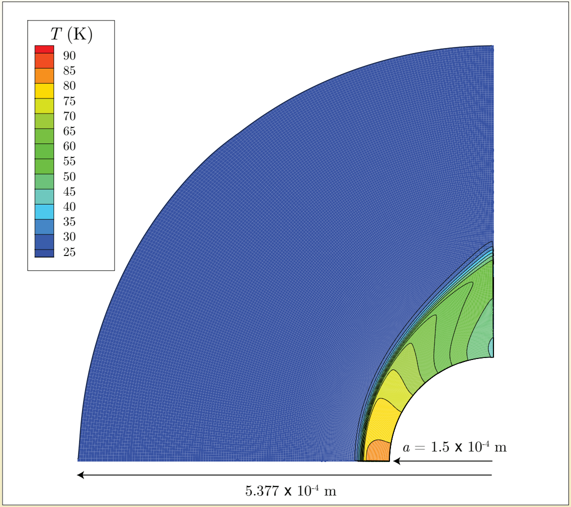
Pressure with physical diffusion neglected



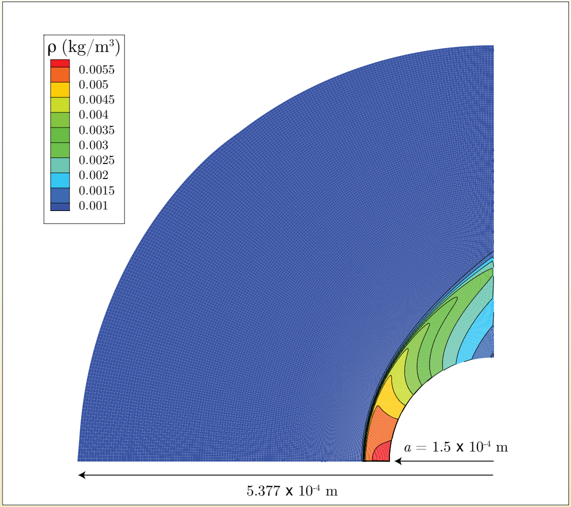
Pressure with physical diffusion



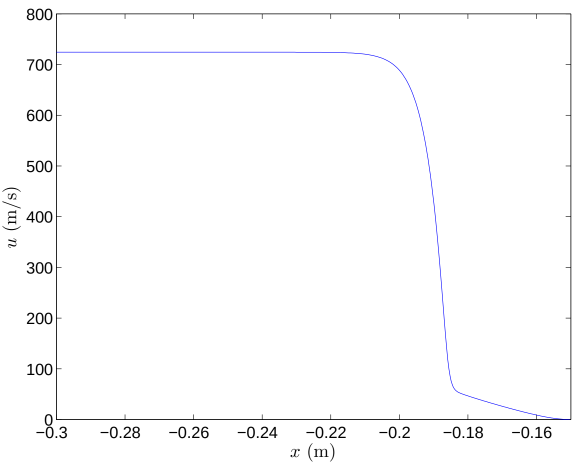
Temperature with physical diffusion



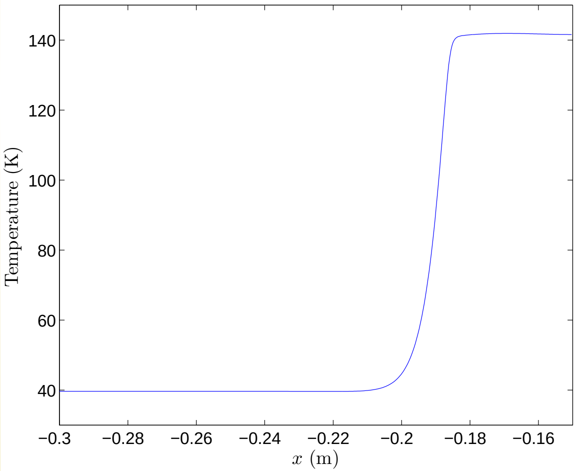
Density with physical diffusion



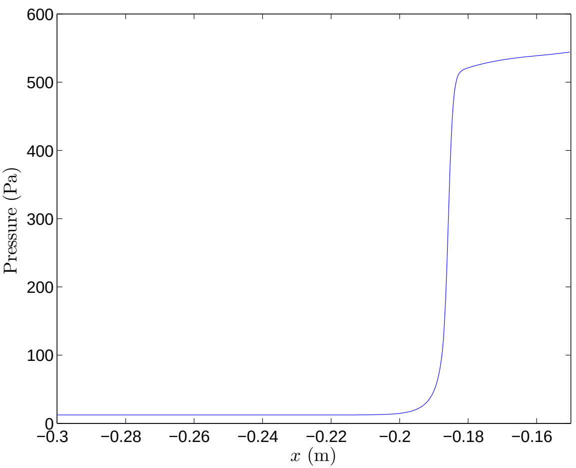
Velocity along the centerline



Temperature along the centerline

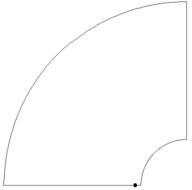


Pressure along the centerline



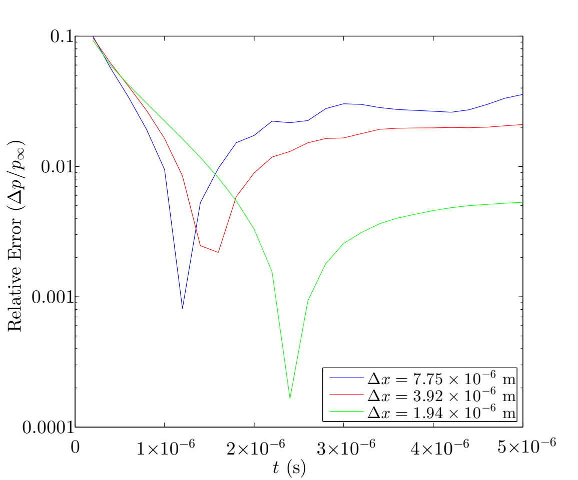
Convergence at a point

The relative error of pressure at a single point for each Navier-Stokes solution was calculated to find when the viscous shock relaxed to a fixed state and the convergence rate of the method.

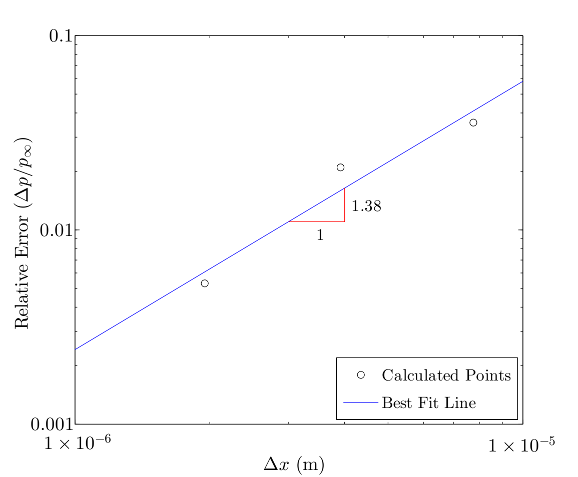


- This point has with coordinates $(-150.3 \times 10^{-6}\ \text{m},\ 0\ \text{m},\ 0\ \text{m})$, directly in front of the cylinder.
- The relative error was found for several grid sizes ranging from 1.95 microns to 7.75 microns using the same time step.

Relaxing of the Viscous Shock



Error Convergence



Conclusions

- When a simple physical diffusion model is introduced into the model of fluid motion and its effects simulated on a sufficiently fine grid, the carbuncle phenomenon is removed.
- The carbuncle may arise due to what amounts to what is sometimes called anti-diffusion, an effect which has been shown to exist via construction of the so-called modified equation for many shock-capturing schemes when exercised on Euler equations.
- The technique of adding physical viscosity is not yet a realistic method for large scale design problems, as the fine grids would make computations expensive.