Asymptotic and Numerical Predictions of Oblique Detonations with Simple Finite-Rate Chemistry[1]

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Prediction of the high-speed non-equilibrium flow of a combustible gas over a sharp-edged wedge has been the subject of numerous studies over the past decades. In addition to its basic research value, this problem has gained recent attention because of its applicability to the oblique detonation wave engine [1] and the ram accelerator [2], a ramjet-in-tube concept. Two-dimensional inviscid cases of these flows have the following features. An incoming supersonic uniform flow of premixed combustible gases encounters a straight wedge. The resulting shock compression initiates non-equilibrium processes which relax to equilibrium in a finite length zone roughly parallel to the lead shock. Due to the heat release, the attached shock becomes curved. Near the wedge tip the shock inclination is nearly that of an inert shock; far from the wedge tip, the flow is accelerated by combustion in a direction normal to the shock. As a result, the shock must rotate and increase its inclination in order to satisfy the wall boundary condition. Consequently, vorticity which is proportional to the magnitude of the curvature, is generated at the shock front and is convected in a layer near the wedge surface. The non-equilibrium and vorticity layers both relax to an equilibrium, irrotational uniform state far from the shock and wedge surface.

The objective of this study is to compare asymptotic and numerical predictions of oblique detonation waves. We model simple finite-rate chemical reactions which give rise to thick reaction and vorticity layers. The recent paper of Powers and Stewart [3] gives new asymptotic oblique detonation solutions. The governing equations were the steady two-dimensional reactive Euler equations. The reaction was assumed to be one-step ($A \rightarrow B$) and irreversible with Arrhenius kinetics. Both reactants and products were modelled as calorically perfect ideal gases with identical material properties. This model was studied in the hypersonic limit, linearized about the inert oblique shocked state. In this limit the kinetic energy of the flow is much greater than the heat release from chemical reaction. The leading order solution is the inert shock, and the linear asymptotic theory corrects for the effects of small heat release. Also in this limit the induction zone length is effectively zero; the assumption of large activation energy, which gives rise to a thick induction zone and thin reaction zone is not made here. Consequently, a simple leading order solution of the kinetic rate law is available. At the following order, acoustic equations with chemical reaction forcing terms generated at leading order are solved to determine the pressure and velocity fields. Two classes of solutions are obtained. The first is irrotational and characterized by a straight shock attached to a curved wedge. The second is rotational and characterized by a curved shock attached to a straight wedge. For the irrotational solutions, it was demonstrated that the differential equations predicted a path to the equilibrium point which was previously identified by a Rankine-Hugoniot analysis.

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The solution procedure of Ref. 3 is as follows. To simplify application of the boundary conditions, the Euler equations were first transformed to a non-orthogonal coordinate system fixed to the shock and wedge. The equations and shock conditions were then written as linear equations in the hypersonic limit. Assuming the oblique shock was weak and the trailing flow was supersonic allowed the equations to be written in characteristic form. In an analytic shock-fitting technique, the equations were solved with the shock position function as a parameter. The shock position function was then specified such that a downstream boundary condition on the wedge surface was met.

In this study, we compare weak rotational solutions of Ref. 3 with numerical solutions of their non-linear counterparts obtained by two different numerical methods. These comparisons represent an extension of those presented by Powers and Grismer [4]. The two CFD codes solve the unsteady Reynolds-averaged Navier-Stokes equations and species continuity equations in a fully coupled manner, and are based on the LU-SSOR implicit factorization scheme. The first code (RPLUS) [5] utilizes a finite volume method in which the spatial discretization is achieved with central differencing. To avoid oscillations near shock waves, a combined second-order and fourth-order artificial dissipation term is added. The second code, developed by Yungster [6], is based on the finite-difference technique, and employs a second-order symmetric total variation diminishing (TVD) differencing scheme.

We find for sufficiently high Mach number and heat release that the two methods predict qualitatively similar behavior. Quantitatively, for fixed heat release, the numerical solution has less error than the asymptotic solution at low supersonic Mach number while the asymptotic solution has less error than the numerical solution at high Mach number. For high fixed Mach number, as heat release is lowered, residues come to obscure the effects of heat release in the numerical solution while the asymptotic method predicts the disturbance more accurately. The results show that asymptotic solutions have utility in the hypersonic limit as benchmarks to verify the predictions of numerical models.

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