Math 10360, Exam 1  
September 25, 2007

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Please mark your answers with an X, not a circle!

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Multiple Choice  

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Total
Multiple Choice

1. (5 pts.) If \( x > 3 \), which of the following is equal to
\[ \ln \frac{\sqrt{x^2 - 9}}{(x^2 + 4)^{2}} \]?

(a) \( 4 \ln(x - 3) + 4 \ln(x + 3) - 2 \ln(x^2 + 4) \)
(b) \( \ln \left( \frac{1}{4}(x^2 - 9) - 2(x^2 + 4) \right) \)
(c) \( \frac{2 \ln(x - 3) \ln(x + 3)}{\ln(x^2 + 4)} \)
(d) \( \frac{1}{4} \ln(x - 3) + \frac{1}{4} \ln(x + 3) - 2 \ln(x^2 + 4) \)
(e) \( \left( \ln(x - 3) + \ln(x + 3) \right)^{1/4} \frac{1}{(\ln(x^2 + 4))^{2}} \)

2. (5 pts.) Which of the following is the equation of the tangent line to the curve \( \ln(y) \ln(x) = 1 \) at the point \((e^{1/2}, e^{2})\)?

(a) \( y + e^{3/2}x = 2e^{2} \)  (b) \( y + 3e^{3/2}x = 4e^{2} \)  (c) \( y + 4e^{3/2}x = 5e^{2} \)
(d) \( y + 2e^{3/2}x = 3e^{2} \)  (e) \( y + 5e^{3/2}x = 6e^{2} \)
3. (5 pts.) If \( f(x) \) is a differentiable function with inverse function \( f^{-1}(x) \) satisfying 
\[
  f(1) = 2, \quad f(2) = 3, \quad f'(1) = 4, \quad f'(2) = 3, \quad f'(3) = 2
\]
which of the following is equal to \( (f')^{-1}(2) \)?

(a) \( \frac{1}{3} \)  \quad (b) \( \frac{1}{2} \)  \quad (c) \( 3 \)  \quad (d) \( 4 \)  \quad (e) \( \frac{1}{4} \)

4. (5 pts.) Which of the following functions is equal to the derivative \( f'(x) \) of the function
\[
  f(x) = e^{\sin x}
\]

(a) \( \cos x \, e^{\cos x} \)  \quad (b) \( \cos x \, e^{\sin x} \)  \quad (c) \( \cos x \, e^{-\sin x} \)

(d) \( -\sin x \, e^{\cos x} \)  \quad (e) \( \sin x \, e^{\cos x} \)
5. (5 pts.) Evaluate the integral
\[ \int \frac{3 + 2e^x}{e^{3x}} \, dx. \]

(a) \( 3e^{-2x} + 2e^{-2x} + C \)  
(b) \( \frac{3 + 2e^x}{e^{3x}} + C \)  
(c) \(-3e^{-2x} - 2e^{-2x} + C\)

(d) \(-e^{-3x} - e^{-2x} + C\)  
(e) \(e^{-3x} + e^{-2x} + C\)

6. (5 pts.) Evaluate the integral
\[ \int \frac{\sin x}{2 + \cos x} \, dx. \]

(a) \( \ln(2 + \sin x) + C \)  
(b) \(-\ln(2 + \cos x) + C \)  
(c) \(\ln(2 - \sin x) + C\)

(d) \(\ln(2 + \cos x) + C\)  
(e) \(-\ln(2 + \sin x) + C\)
7. (5 pts.) If 
\[ \log_3(x - 5) = 3 \]
then

(a) \( x = e^3 + 5 \)
(b) \( x = 18 \)
(c) \( x = 32 \)
(d) \( x = \ln 3 + 5 \)
(e) \( x = 14 \)

8. (5 pts.) Which of the following is the value of

\[ \sin \left( \arctan \left( \frac{x}{\sqrt{3}} \right) \right) \]?

(a) \( \frac{x}{\sqrt{x^2 + 3}} \)
(b) \( \frac{\sqrt{3}}{\sqrt{x^2 - 3}} \)
(c) \( \frac{x}{\sqrt{x^2 - 3}} \)
(d) \( \frac{\sqrt{x^2 + 3}}{\sqrt{3}} \)
(e) \( \frac{\sqrt{x^2 - 3}}{x} \)
9. (5 pts.) The equation of the tangent line to the graph of the function

\[ f(x) = \arcsin(3x) \]

at the point where \( x = \frac{1}{6} \) is given by

(a) \( y = \frac{\pi}{6} + \frac{2}{\sqrt{3}} \left( x - \frac{1}{6} \right) \)

(b) \( y = \frac{\pi}{3} + \frac{2}{\sqrt{3}} \left( x - \frac{1}{6} \right) \)

(c) \( y = \frac{\pi}{6} + 2\sqrt{3} \left( x - \frac{1}{6} \right) \)

(d) \( y = \frac{\pi}{6} + \sqrt{3} \left( x - \frac{1}{6} \right) \)

(e) \( y = \frac{\pi}{6} + 2\sqrt{3} \left( x - \frac{1}{6} \right) \)

10. (5 pts.) Evaluate

\[ \int_{0}^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} \, dx. \]

(a) 0  (b) 1  (c) \( \frac{\pi}{2} \)  (d) \( \frac{\pi}{4} \)  (e) \( \frac{1}{2} \ln 2 \)
11. (5 pts.) $2000 is invested in a savings account in which the interest is compounded continuously. What is the annual interest rate (as a percentage) if it takes 20 years for the balance to be $4000?

(a) \( r = 20 \ln 5 \% \) \hspace{1cm} (b) \( r = 2 \ln 5 \% \) \hspace{1cm} (c) \( r = 2 \ln 20 \% \)

(d) \( r = 20 \ln 2 \% \) \hspace{1cm} (e) \( r = 5 \ln 2 \% \)

12. (5 pts.) Which of the following is a solution of the differential equation \( y' + y = x \) ?

(a) \( y = x - 1 + 2e^{-x} \) \hspace{1cm} (b) \( y = 1 + e^{-x} \)

(c) \( y = e^{-x} \) \hspace{1cm} (d) \( y = x - 1 + 2e^{x} \)

(e) \( y = x - 3e^{-x} \)
Partial Credit
You must show your work on the partial credit problems to receive credit!

13. (10 pts.) Use the substitution $u = \sqrt{x} + 1$ to evaluate the integral

$$\int \frac{x}{\sqrt{x} (\sqrt{x} + 1)} \, dx.$$
14. (10 pts.) If

\[ F(x) = \int_0^{e^{2x}} \ln(t^2 + 1) \, dt, \]

evaluate the derivative \( F'(x) \).
15. (10 pts.) Use logarithmic differentiation to find $dy/dx$ where

$$y = x^{\ln x}.$$
16. (10 pts.) Evaluate
\[ \int_{0}^{\sqrt{2}} \frac{x}{x^4 + 4} \, dx. \]
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