

Math 10360, Final Exam.
December 15, 2007.

Name: _____

Instructor: _____

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for 2 hours.
- This exam is conducted under the Honor Code.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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Multiple Choice

1.(5 pts.) The solution of the equation $\ln(4x - 3) = 2$ is

(a) $x = \frac{e^3 + 4}{2}$

(b) $x = \frac{e^2 + 4}{3}$

(c) $x = \frac{e^2 + 3}{4}$

(d) $x = \frac{e^3 + 2}{4}$

(e) $x = \frac{e^4 + 3}{2}$

2.(5 pts.) If $f(x) = e^x \cos x$ which of the following is equal to the second derivative $f''(x)$ of $f(x)$?

(a) $2e^x \sin x$

(b) $e^x \sin x + e^x \cos x$

(c) $-2e^x \sin x$

(d) $-2e^x \cos x$

(e) $2e^x \cos x$

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3.(5 pts.) The integral

$$\int e^{\tan x} \sec^2 x \, dx$$

is equal to

(a) $e^{\sec x} + C$

(b) $e^{|\sec x + \tan x|} + C$

(c) $e^{\tan x} \sec x + C$

(d) $e^{\tan x} + C$

(e) $e^{-\ln |\sec x|} + C$

4.(5 pts.) Let $f(x) = \frac{1}{4x+3}$ for $x > 0$? Which of the following is the inverse function $f^{-1}(x)$ of $f(x)$?

(a) $\frac{3-3x}{x}$

(b) $\frac{1-3x}{4x}$

(c) $\frac{1}{4}x - 3$

(d) $4x + 3$

(e) $\frac{4x}{1-3x}$

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5.(5 pts.) A colony of bacteria grows at a rate proportional to the population at any given time. After 3 hours there are 400 bacteria, and after 5 hours there are 1600 bacteria. How many bacteria were there initially?

(a) $400e^{-3}$

(b) 100

(c) 50

(d) $\ln 100$

(e) 25

6.(5 pts.) Which of the following is the equation of the tangent line to the curve

$$\ln(y) \ln(x) = 2$$

at the point where $x = e$?

(a) $2ex + y = e^2$

(b) $y = ex$

(c) $x + y = e^2 + e$

(d) $2ex + y = 3e^2$

(e) $2ex - y = e^2$

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7.(5 pts.) Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx.$$

(a) $-\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) $-\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

(e) 0

8.(5 pts.) Which of the following is the general solution of the differential equation

$$y' + y = x^2 ?$$

(a) $y = Ce^{-x}$

(b) $y = x^2 - Cx + 2 + e^x$

(c) $y = x^2 - 4x + 2 + 3e^{-x}$

(d) $y = x^2 - 2x + 2 + Ce^{-x}$

(e) $y = x^2 + e^{-x}$

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9.(5 pts.) Compute the indefinite integral

$$\int e^x \sin(x) dx$$

(a) $\frac{e^x \sin(x) - e^x \cos(x)}{2} + C$

(b) $e^x \cos(x) + e^x \sin(x) + C$

(c) $\frac{e^x \cos(x) + e^x \sin(x)}{2} + C$

(d) $e^x \cos(x) + C$

(e) $-e^x \cos(x) + C$

10.(5 pts.) The series $\sum_{n=1}^{\infty} \frac{3^n}{7^n}$ is

(a) Convergent and we cannot find the sum

(b) Convergent and the sum is $\frac{7}{4}$

(c) Convergent and the sum is $\frac{7}{3}$

(d) Convergent and the sum is $\frac{3}{4}$

(e) Divergent

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11.(5 pts.) The sequence $\{a_n\} = \frac{2n^2 + 3n + 1}{n(n + 1)} + \frac{\pi}{n}$

- (a) Converges to 2
- (b) Converges to 1
- (c) Diverges
- (d) Converges to $1 + \pi$
- (e) Converges to $2 + \pi$

12.(5 pts.) The series $\sum_{n=1}^{\infty} \frac{\sqrt{6n^4 + 3n^3 + 2n^2 + 6}}{n^4 + 3}$ is

- (a) Divergent by the n 'th term test for divergence
- (b) Convergent by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^4}$
- (c) Divergent by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$
- (d) Convergent by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$
- (e) Convergent because $\lim_{n \rightarrow \infty} \frac{\sqrt{6n^4 + 3n^3 + 2n^2 + 6}}{n^4 + 3} = 0$

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13.(5 pts.) Find the following limit:

$$\lim_{x \rightarrow 0^+} \frac{xe^x}{\sin(x)}$$

(a) e

(b) ∞

(c) 0

(d) 1

(e) -1

14.(5 pts.) Compute the integral:

$$\int \frac{2x - 14}{(x - 3)(x + 1)} dx$$

(a) $5 \ln |x + 1| - 3 \ln |x - 3| + C$

(b) $4 \ln |x + 1| - 2 \ln |x - 3| + C$

(c) $4 \ln |x - 3| - 2 \ln |x + 1| + C$

(d) $2 \ln |x - 3| - 4 \ln |x + 1| + C$

(e) $3 \ln |x - 3| - 5 \ln |x + 1| + C$

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15.(5 pts.) The series

$$\sum_{n=1}^{\infty} \frac{(-5)^n}{n!}$$

- (a) Converges conditionally by the alternating series test
- (b) Converges absolutely by the ratio test
- (c) Diverges by the ratio test
- (d) Diverges by the n 'th term test for divergence
- (e) Diverges by comparison to $\sum_{n=1}^{\infty} 5^n$

16.(5 pts.) The series $3 + \frac{6}{2} + \frac{9}{6} + \frac{12}{24} + \frac{15}{120} + \cdots$ has closed form

- (a) $\sum_{n=1}^{\infty} \frac{3n}{n!}$
- (b) $\sum_{n=1}^{\infty} \frac{3n}{(n+1)!}$
- (c) $\sum_{n=0}^{\infty} \frac{3n}{n!}$
- (d) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$
- (e) $\sum_{n=1}^{\infty} \frac{3(n+1)}{(n+1)!}$

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17.(5 pts.) What is the volume of the solid obtained by rotating the region between the graphs of $y = x^2$, $y = 0$ and $x = 1$ about the y -axis?

- (a) 2π (b) π (c) 1 (d) $\frac{\pi}{2}$ (e) $\frac{\pi}{4}$

18.(5 pts.) Find the arclength of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ for $1 \leq x \leq 3$.

- (a) $\frac{32}{3}$ (b) $\frac{15}{2}$
(c) $\frac{22}{3}$ (d) 5
(e) 11

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19.(5 pts.) A force of 16 pounds is needed to compress a spring 4 *in* from its natural length of 20 *in*. Find the work done in compressing the spring an additional 2 *in*.

- (a) $64 \text{ lb} \cdot \text{in}$ (b) $40 \text{ lb} \cdot \text{in}$ (c) $20 \text{ lb} \cdot \text{in}$
(d) $72 \text{ lb} \cdot \text{in}$ (e) $32 \text{ lb} \cdot \text{in}$

20.(5 pts.) What is the area between the graphs of the curves $y = x^2 - 4x + 1$ and $y = -x + 1$?

- (a) $\frac{9}{2}$ (b) 12 (c) 9 (d) $\frac{\sqrt{13}}{2}$ (e) 6

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21.(5 pts.) Use the Disk Method to determine what is the correct expression that computes the volume of the solid obtained by rotating the region between the curves $y = 2 - x^2$ and $y = 1$ about the x -axis.

- (a) $2\pi \int_{-1}^1 (x(2 - x^2))dx$ (b) $\int_0^1 ((2 - x^2)^2 - 1)dx$ (c) $\pi \int_{-1}^0 (1 - x(2 - x^2)^2)dx$
(d) $2\pi \int_0^1 ((1 - x^2)^2)dx$ (e) $\pi \int_{-1}^1 ((2 - x^2)^2 - 1)dx$

22.(5 pts.) The solution of the differential equation $(x + 1)y = -y'$ with initial condition $y(0) = 1$ is:

- (a) $y(x) = e^{x^2+x}$ (b) $y(x) = 1$
(c) $y(x) = e^{-x^2+1} - e + 1$ (d) $y(x) = e^{-x-x^2/2}$
(e) $y(x) = -e^{x^2} + 2e^x$

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23.(5 pts.) The 3th Taylor polynomial centered at $c = 1$ of $f(x) = \ln x$ is:

(a) $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{6}(x - 1)^3$

(b) $\ln x^3$

(c) $(x - 1) + \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3$

(d) $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3$

(e) $(x - 3) - \frac{1}{2}(x - 3)^2 + \frac{1}{3}(x - 3)^3$

24.(5 pts.) The interval of the convergence of the power series $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$ is:

(a) $-\infty < x < 0$

(b) $0 \leq x < \infty$

(c) all x

(d) $-1 < x < 1$

(e) $x = 0$

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25.(5 pts.) The power series for the function $f(x) = \frac{3}{x+2}$ centered at $c = 0$ is:

- (a) $\sum_{n=0}^{\infty} \frac{3}{2} \left(-\frac{x}{2}\right)^n$ (b) $\sum_{n=0}^{\infty} \frac{3}{2} \left(-\frac{x}{2}\right)^{2n}$ (c) $\sum_{n=0}^{\infty} \frac{1}{2} \left(-\frac{x}{2}\right)^n$
- (d) $\sum_{n=0}^{\infty} \frac{2}{3} \left(-\frac{x}{2}\right)^n$ (e) $\sum_{n=0}^{\infty} \frac{3}{2} \left(\frac{x}{2}\right)^n$

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