Math 20580 Name:______________________

Practice Midterm 1 Instructor:______________________

February 12, 2015 Section:______________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

____

1. a b c d e

2. a b c d e

3. a b c d e

4. a b c d e

5. a b c d e

6. a b c d e

7. a b c d e

8. a b c d e

Part I: Multiple choice questions (7 points each)

1. Consider the linear system

$$\begin{aligned} 2x_1 + 3x_2 - 2x_3 &= 1 \\ x_1 + 4x_2 &= 5 \end{aligned}$$

Which of the following (x_1, x_2, x_3) is a solution?

(a) $(-3/5, 7/5, 1)$ (b) $(2/5, 3/5, 1)$ (c) $(7/5, 3/5, 1)$ (d) $(4/5, -3/5, 1)$
(e) $(2/5, 7/5, 1)$

2. For which constants t do the vectors $\begin{bmatrix} 1 \\ 0 \\ t \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ span all of $\mathbb{R}^3$?

(a) $t = 1$ only (b) all $t \neq 1$ (c) $t = -1/5$ only (d) all $t \neq -1/5$
(e) there are no t

3. Which column in the matrix below is the first from the left which is a linear combination of the previous ones?

$$\begin{bmatrix} 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & -1 \\ 1 & 0 & 2 & 0 & 3 \end{bmatrix}$$

(a) $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$

4. Is the linear transformation corresponding to the matrix below one-to-one or onto?

$$\begin{bmatrix} 2 & 1 & 3 & 2 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

(a) both one-to-one and onto (b) one-to-one but not onto
(c) onto but not one-to-one (d) neither one-to-one nor onto

5. Find the determinant of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 8 & 2 & 9 \\ 0 & 3 & 0 & 4 \\ -1 & 10 & 20 & 30 \end{bmatrix}$$

(a) 2 (b) 4 (c) -4 (d) -2 (e) 0

6. Which of the following matrices are invertible?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 5 & 2 \\ 3 & 6 & 3 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 10 & 20 \\ 0 & 0 & 9 \end{bmatrix}$$

(a) only D (b) they are all invertible (c) C and D only
(d) B and C only (e) only C

7. If B is the matrix below, and $C = (B^T)^5$, compute $\det(C)$.

$$B = \begin{bmatrix} 0 & 6 & 2 \\ 2 & 8 & 7 \\ 0 & 2 & 1 \end{bmatrix}$$

(a) 0 (b) 2^{10} (c) -2^{10} (d) 6^5 (e) -6^5

8. Compute the dimension of the Null-space of the matrices below.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$(\dim \mathrm{Nul}(A), \dim \mathrm{Nul}(B), \dim \mathrm{Nul}(C)) =$

(a) $(0, 0, 2)$ (b) $(2, 2, 2)$ (c) $(1, 0, 3)$ (d) $(0, 1, 1)$ (e) $(2, 1, 1)$

Part II: Partial credit questions (11 points each). Show your work.

9. Find a solution to the linear system

$$\begin{aligned} x_1 + x_3 &= 1 \\ 2x_1 + 2x_3 + x_4 &= 1 \\ x_1 + x_2 + 2x_3 &= 2 \\ 2x_2 + x_3 + x_4 &= 1 \end{aligned}$$

10.

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & -1 \end{bmatrix}$$

(a) A gives a linear transformation $T_A : \mathbb{R}^p \to \mathbb{R}^q$. What are the numbers p and q?

(b) Find a nonzero vector x in $\mathbb{R}^p$ which is a solution of the homogeneous equation $Ax = 0$ (or explain why there are none).

(c) Find a vector b in $\mathbb{R}^q$ which does not lie in the image of T_A (or explain why there are none).

11. Find the inverse of A, where

$$A = \begin{bmatrix} 2 & -1 & 4 \\ -2 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

12. Consider the matrix A below.

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 4 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

(a) Find a basis for the column space $\text{Col}(A)$.

(b) Let v be the last column of A. Find the coordinates of v relative to the basis found in part (a).