Practice Midterm 2

March 5, 2015

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an × through one letter for each problem on this answer sheet.

**Sign the pledge.** “On my honor, I have neither given nor received unauthorized aid on this Exam”:

1. a b c d e

2. a b c d e

3. a b c d e

4. a b c d e

5. a b c d e

6. a b c d e

7. a b c d e

8. a b c d e

**Multiple Choice.**

9.

10.

11.

12.

Total.
Part I: Multiple choice questions (7 points each)

1. Let $V$ be the vector space of all functions $f(x)$ where $f: \mathbb{R} \to \mathbb{R}$.

Which of the following are subspaces of $V$?

A. the constant functions;
B. functions with $\lim_{x \to \infty} f(x) = 3$;
C. functions with $f(1) = 1$;
D. functions with $f(0) = 0$.

(a) A, B, C and D  (b) A, B and C only  (c) B, C and D only
(d) B and D only  (e) A and D only.

2. Let $\mathbb{P}_2$ be the vector space of polynomials of degree at most 2, and let $\mathcal{B}$ be the basis

$$\mathcal{B} = \{1, 1 + x, 1 + x^2\}.$$ 

Find the $\mathcal{B}$-coordinates $[p]_\mathcal{B}$ of the polynomial $p(x) = (1 - x)^2$.

(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  (b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$  (c) $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$  (d) $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$  (e) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$
3. Let $H$ be a subspace of a vector space $V$, and suppose that $V$ has dimension $d$.
Which of the following statements are true?

A. $\dim(H) \leq \dim(V)$;
B. a linearly independent set of vectors in $H$ is also linearly independent in $V$;
C. $d$ vectors which span $V$ will be linearly independent;
D. $d$ vectors which span $H$ will also span $V$.

(a) A, B, C and D  (b) A, B and C only  (c) B, C and D only
(d) B and D only  (e) A and D only.

4. A linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$

has outputs $T \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Find $T \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  (b) $\begin{bmatrix} 4/5 \\ 1 \end{bmatrix}$  (c) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$  (d) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$  (e) $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$
5. The vector \[
\begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}
\] is an eigenvector for the matrix
\[
\begin{bmatrix}
2 & 5 & 1 \\
1 & 7 & -1 \\
1 & 0 & 2
\end{bmatrix}
\]
What is the corresponding eigenvalue?
(a) 3 (b) 1 (c) 0 (d) -1 (e) 2

6. What are the eigenvalues of the matrix \[
\begin{bmatrix}
5 & 4 \\
-2 & -1
\end{bmatrix}
\]?
(a) 0, 1 (b) 5, -1 (c) 1, 3 (d) 0, 2 (e) -1, -3
7. Suppose \( A \) is a \( 3 \times 3 \) matrix, that has \[
\begin{pmatrix}
1 \\
2 \\
0
\end{pmatrix}
\] as an eigenvector with eigenvalue 2, and
\[
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\] as an eigenvector with eigenvalue \(-1\). Compute \( A^3 \begin{pmatrix}1 \\ 2 \\ 1\end{pmatrix} \).

(a) \[
\begin{pmatrix}8 \\ 16 \\ 1\end{pmatrix}
\]  
(b) \[
\begin{pmatrix}8 \\ 16 \\ -1\end{pmatrix}
\]  
(c) \[
\begin{pmatrix}2 \\ 4 \\ -1\end{pmatrix}
\]  
(d) \[
\begin{pmatrix}1 \\ 2 \\ 0\end{pmatrix}
\]  
(e) cannot be computed from the given information

8. Let \( P_3 \) be the vector space of polynomials of degree at most 3. Find the dimension of the subspace of \( P_3 \) spanned by \( 1 + x^2 \), \( x + 2x^2 + x^3 \) and \( 1 + x + x^3 \).

(a) 0  
(b) 1  
(c) 2  
(d) 3  
(e) 4
Part II: Partial credit questions (11 points each). Show your work.

9. Find a basis for the Row space, \( \text{Row}(A) \), of the matrix

\[
A = \begin{bmatrix}
0 & 1 & 3 & 2 \\
1 & 0 & 2 & 3 \\
1 & 1 & -5 & 6 \\
1 & -1 & -1 & 2
\end{bmatrix}.
\]
10. Let $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix} \}$ and $\mathcal{C} = \{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \end{bmatrix} \}$ be two bases of $\mathbb{R}^2$.

Find the change of coordinate matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$ sending a $\mathcal{B}$ coordinate vector $[\vec{x}]_\mathcal{B}$ to the $\mathcal{C}$ coordinate vector $[\vec{x}]_\mathcal{C}$. 
11. Is the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ diagonalizable? If so, find an invertible matrix $P$ and a diagonal matrix $D$ such that $A = PDP^{-1}$. If $A$ is not diagonalizable, explain why not.

(Hint: The eigenvalues of $A$ are 1 and 3.)
12. Let $P_3$ be the vector space of polynomials of degree at most 3 and $P_2$ be the space of polynomials of degree at most 2.

Consider the linear transformation

$$T : P_3 \rightarrow P_2$$

given by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_3x^2$.

(a) Write down bases $B_3$ and $B_2$ of $P_3$ and $P_2$ respectively.

(b) Find the matrix of $T$ relative to the bases $B_3$ and $B_2$. 