Math 20580 Name:____________________

Practice Midterm 2 Instructor:____________________

March 5, 2015 Section:____________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

---

1. a b c d e

2. a b c d e

3. a b c d e

4. a b c d e

5. a b c d e

6. a b c d e

7. a b c d e

8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Let V be the vector space of all functions $f(x)$ where $f : \mathbb{R} \to \mathbb{R}$.

   Which of the following are subspaces of V?

   A. the constant functions;

   B. functions with $\lim_{x\to\infty} f(x) = 3$;

   C. functions with $f(1) = 1$;

   D. functions with $f(0) = 0$.

   (a) A, B, C and D (b) A, B and C only (c) B, C and D only
   (d) B and D only (e) A and D only.

2. Let $\mathbb{P}_2$ be the vector space of polynomials of degree at most 2, and let $\mathcal{B}$ be the basis

$$\mathcal{B} = \{1, 1 + x, 1 + x^2\}.$$

   Find the $\mathcal{B}$-coordinates $[p]_{\mathcal{B}}$ of the polynomial $p(x) = (1 - x)^2$.

   (a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

3. Let H be a subspace of a vector space V, and suppose that V has dimension d. Which of the following statements are true?

A. $\dim(H) \leq \dim(V)$;

B. a linearly independent set of vectors in H is also linearly independent in V;

C. d vectors which span V will be linearly independent;

D. d vectors which span H will also span V.

(a) A, B, C and D (b) A, B and C only (c) B, C and D only
(d) B and D only (e) A and D only.

4. A linear transformation

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

has outputs $T \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Find $T \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4/5 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

5. The vector $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is an eigenvector for the matrix

$$\begin{bmatrix} 2 & 5 & 1 \\ 1 & 7 & -1 \\ 1 & 0 & 2 \end{bmatrix}$$

What is the corresponding eigenvalue?

(a) 3 (b) 1 (c) 0 (d) -1 (e) 2

6. What are the eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ -2 & -1 \end{bmatrix}$?

(a) $0, 1$ (b) $5, -1$ (c) $1, 3$ (d) $0, 2$ (e) $-1, -3$

7. Suppose A is a 3×3 matrix, that has $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ as an eigenvector with eigenvalue 2, and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ as an eigenvector with eigenvalue -1. Compute $A^3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 8 \\ 16 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 8 \\ 16 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

(e) cannot be computed from the given information

8. Let P_3 be the vector space of polynomials of degree at most 3. Find the dimension of the subspace of P_3 spanned by $1 + x^2$, $x + 2x^2 + x^3$ and $1 + x + x^3$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

Part II: Partial credit questions (11 points each). Show your work.

9. Find a basis for the Row space, Row(A), of the matrix

$$A = \begin{bmatrix} 0 & 1 & 3 & 2 \\ 1 & 0 & 2 & 3 \\ 1 & 1 & -5 & 6 \\ 1 & -1 & -1 & 2 \end{bmatrix}.$$

10. Let $\mathcal{B} = \{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}\}$ and $\mathcal{C} = \{\begin{bmatrix}2\\-1\end{bmatrix}, \begin{bmatrix}-1\\0\end{bmatrix}\}$ be two bases of $\mathbb{R}^2$.

Find the change of coordinate matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{\mathcal{P}}$ sending a $\mathcal{B}$ coordinate vector $[\vec{x}]_{\mathcal{B}}$ to the $\mathcal{C}$ coordinate vector $[\vec{x}]_{\mathcal{C}}$.

11. Is the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. If A is not diagonalizable, explain why not.

(Hint: The eigenvalues of A are 1 and 3.)

12. Let P_3 be the vector space of polynomials of degree at most 3 and P_2 be the space of polynomials of degree at most 2.

Consider the linear transformation

$$T : P_3 \to P_2$$

given by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_3x^2$.

(a) Write down bases $\mathcal{B}_3$ and $\mathcal{B}_2$ of P_3 and P_2 respectively.

(b) Find the matrix of T relative to the bases $\mathcal{B}_3$ and $\mathcal{B}_2$.