Math 20580 Name:____________________________

Practice Midterm 3 Instructor:____________________________

April 16, 2015 Section:____________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

---

1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Find the closest point to $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ in the subspace of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.

(a) $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 8/5 \\ 1 \\ 6/5 \end{bmatrix}$ (e) $\begin{bmatrix} -3/5 \\ 1 \\ 6/5 \end{bmatrix}$

2. Which of the following is a least square solution $\hat{\mathbf{x}}$ to the equation

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 1 & -2 \\ 2 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 3 \end{bmatrix}?$$

(a) $\begin{bmatrix} 11/9 \\ 1/9 \end{bmatrix}$ (b) $\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}$ (c) $\begin{bmatrix} 7/5 \\ 1/5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

3. Which of the following functions is a solution to the initial value problem

$$\frac{dy}{dt} = (y-t)^2 + 1; \qquad y(0) = -1?$$

(a) $y = \dfrac{1}{t+1} - 2$ (b) $y = t$ (c) $y = \dfrac{-1}{t+1} + t$

(d) $y = t - 1$ (e) $y = \dfrac{-2}{t+1} + 1$

4. Let A be an $m \times n$ matrix. Which of the following may be *false*?

(a) The equation $A^T A\mathbf{x} = A^T\mathbf{b}$ is always consistent for any $\mathbf{b}$ in $\mathbb{R}^m$.

(b) $A^T A$ is invertible.

(c) A solution to $A^T A\mathbf{x} = A^T\mathbf{b}$ is a least squares solution of $A\mathbf{x} = \mathbf{b}$.

(d) The columns of A^T lie in the column space of $A^T A$.

(e) If $A^T A\mathbf{x} = A^T\mathbf{b}$ then $A\mathbf{x} - \mathbf{b}$ is orthogonal to $\mathrm{Col}(A)$.

5. Which of the following is a general solution to the differential equation

$$1 + (\frac{x}{y} - \sin y)\frac{dy}{dx} = 0?$$

(a) $xy + y\sin y - \sin y = c$ (b) $xy + y\cos y - \sin y = cy$
(c) $xy + y\sin y - \cos y = c$ (d) $xy + y\cos y - \sin y = c$
(e) $xy + y\cos y - \cos y = c$

6. Consider the initial value problem

$$\sin(2x) + \cos(3y)\frac{dy}{dx} = 0 \qquad y(\pi/2) = \pi/3$$

Which of the following implicitly defines the solution?

(a) $\dfrac{-\cos(2x)}{2} + \dfrac{\sin(3y)}{3} = \dfrac{-1}{2}$ (b) $-\cos(2x) + \sin(3y) = \dfrac{1}{2}$
(c) $\sin(2x) + \cos(3y) = 1$ (d) $-\cos(2x) + \sin(3y) = \dfrac{-1}{2}$
(e) $\dfrac{-\cos(2x)}{2} + \dfrac{\sin(3y)}{3} = \dfrac{1}{2}$

7. Let $y(t)$ be the unique solution of the initial value problem

$$(t^2 - t)\frac{dy}{dt} + \cos(\pi t)y = \frac{t^2 - t}{t - 2} \qquad y(3/2) = 0$$

What is the largest interval where y is defined?

(a) $t > 0$ (b) $0 < t < 2$ (c) $1 < t < 2$ (d) $t < 1/2$ (e) $t < 2$

8. A tank initially contains $100l$ of pure water. Then, at $t = 0$, a sugar solution with concentration of $4g/l$ starts being pumped into the tank at a rate of $5l/\text{min}$. The tank is kept well mixed, and the solution is being pumped out at the rate of $4l/\text{min}$. Which of the following is the initial value problem for $y(t) =$ quantity of sugar, in grams, in the tank at time t?

(a) $\dfrac{dy}{dt} = 5y - 4(100 + t)$ $\qquad y(0) = 0$

(b) $\dfrac{dy}{dt} = 20 - 4y$ $\qquad y(0) = 0$

(c) $\dfrac{dy}{dy} = 4$ $\qquad y(0) = 100$

(d) $\dfrac{dy}{dt} = 20 - \dfrac{4y}{100 + t}$ $\qquad y(0) = 0$

(e) $\dfrac{dy}{dt} = 20 - \dfrac{y}{(100 + t)^2}$ $\qquad y(0) = 100$

Part II: Partial credit questions (11 points each). Show your work.

9. Using the Gram-Schmidt Process, find an orthonormal basis of the subspace of $\mathbb{R}^4$ spanned by the vectors $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $\begin{bmatrix}1\\2\\1\\2\end{bmatrix}$ and $\begin{bmatrix}1\\3\\1\\3\end{bmatrix}$.

10. By drawing a direction field, sketch two solutions to the ODE

$$\frac{dy}{dt} = t^2 y^2 (y-2)$$

with initial conditions $y(0) = 1$ and $y(0) = 3$.

Indicate clearly the limiting behavior $\lim_{t\to\infty} y(t)$ and $\lim_{t\to-\infty} y(t)$.

11. Find the function $y(t)$, for $t > 0$, which solves the initial value problem

$$t\frac{dy}{dt} + 4y = \frac{e^{-t}}{t^2} \quad , \quad y(1) = 0$$

12. Consider the differential equation

$$2y\frac{dy}{dx} = -e^x$$

(a) Find the general solution.
(b) Find the solution with $y(0) = 1$.
(c) What is the largest interval in which the solution in part (b) is defined?