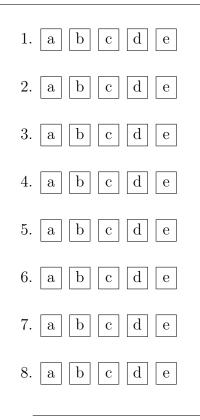
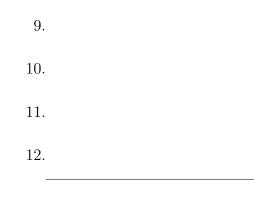
Math 20580	Name:
Practice Midterm 3	Instructor:
April 16, 2015	Section:
Calculators are NOT allowed. Do not remove this answer page – you will return the whole	
exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are	
finished.	

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an  $\times$  through one letter for each problem on this answer sheet.

**Sign the pledge.** "On my honor, I have neither given nor received unauthorized aid on this Exam":



Multiple Choice.



Total.

## Part I: Multiple choice questions (7 points each)

1. Find the closest point to 
$$\begin{bmatrix} 1\\1\\2 \end{bmatrix}$$
 in the subspace of  $\mathbb{R}^3$  spanned by  $\begin{bmatrix} 0\\1\\0 \end{bmatrix}$  and  $\begin{bmatrix} -1\\1\\2 \end{bmatrix}$ .  
(a)  $\begin{bmatrix} -2\\1\\1 \end{bmatrix}$  (b)  $\begin{bmatrix} -1\\1\\2 \end{bmatrix}$  (c)  $\begin{bmatrix} 1\\1\\2 \end{bmatrix}$  (d)  $\begin{bmatrix} 8/5\\1\\6/5 \end{bmatrix}$  (e)  $\begin{bmatrix} -3/5\\1\\6/5 \end{bmatrix}$ 

2. Which of the following is a least square solution  $\mathbf{\hat{x}}$  to the equation

$$\begin{bmatrix} 1 & -2\\ 2 & 1\\ 1 & -2\\ 2 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1\\ 3\\ 1\\ 3 \end{bmatrix}?$$
(a)  $\begin{bmatrix} 11/9\\ 1/9 \end{bmatrix}$  (b)  $\begin{bmatrix} 3/2\\ 1/2 \end{bmatrix}$  (c)  $\begin{bmatrix} 7/5\\ 1/5 \end{bmatrix}$  (d)  $\begin{bmatrix} 1\\ -2 \end{bmatrix}$  (e)  $\begin{bmatrix} 2\\ 1 \end{bmatrix}$ 

3. Which of the following functions is a solution to the initial value problem

$$\frac{dy}{dt} = (y-t)^2 + 1; \qquad y(0) = -1?$$
(a)  $y = \frac{1}{t+1} - 2$  (b)  $y = t$  (c)  $y = \frac{-1}{t+1} + t$   
(d)  $y = t - 1$  (e)  $y = \frac{-2}{t+1} + 1$ 

4. Let A be an  $m \times n$  matrix. Which of the following may be *false*?

- (a) The equation  $A^T A \mathbf{x} = A^T \mathbf{b}$  is always consistent for any  $\mathbf{b}$  in  $\mathbb{R}^m$ .
- (b)  $A^T A$  is invertible.
- (c) A solution to  $A^T A \mathbf{x} = A^T \mathbf{b}$  is a least squares solution of  $A \mathbf{x} = \mathbf{b}$ .
- (d) The columns of  $A^T$  lie in the column space of  $A^T A$ .
- (e) If  $A^T A \mathbf{x} = A^T \mathbf{b}$  then  $A \mathbf{x} \mathbf{b}$  is orthogonal to  $\operatorname{Col}(A)$ .

5. Which of the following is a general solution to the differential equation

$$1 + \left(\frac{x}{y} - \sin y\right)\frac{dy}{dx} = 0?$$
(a)  $xy + y \sin y - \sin y = c$   
(b)  $xy + y \cos y - \sin y = cy$   
(c)  $xy + y \sin y - \cos y = c$   
(d)  $xy + y \cos y - \sin y = c$   
(e)  $xy + y \cos y - \cos y = c$ 

6. Consider the initial value problem

$$\sin(2x) + \cos(3y)\frac{dy}{dx} = 0$$
  $y(\pi/2) = \pi/3$ 

Which of the following implicitly defines the solution?

(a) 
$$\frac{-\cos(2x)}{2} + \frac{\sin(3y)}{3} = \frac{-1}{2}$$
 (b)  $-\cos(2x) + \sin(3y) = \frac{1}{2}$   
(c)  $\sin(2x) + \cos(3y) = 1$  (d)  $-\cos(2x) + \sin(3y) = \frac{-1}{2}$   
(e)  $\frac{-\cos(2x)}{2} + \frac{\sin(3y)}{3} = \frac{1}{2}$ 

7. Let y(t) be the unique solution of the initial value problem

$$(t^{2} - t)\frac{dy}{dt} + \cos(\pi t)y = \frac{t^{2} - t}{t - 2} \qquad y(3/2) = 0$$

What is the largest interval where y is defined?

(a) 
$$t > 0$$
 (b)  $0 < t < 2$  (c)  $1 < t < 2$  (d)  $t < 1/2$  (e)  $t < 2$ 

8. A tank initially contains 100*l* of pure water. Then, at t = 0, a sugar solution with concentration of 4g/l starts being pumped into the tank at a rate of  $5l/\min$ . The tank is kept well mixed, and the solution is being pumped out at the rate of  $4l/\min$ . Which of the following is the initial value problem for y(t) = quantity of sugar, in grams, in the tank at time t?

(a) 
$$\frac{dy}{dt} = 5y - 4(100 + t)$$
  $y(0) = 0$   
(b)  $\frac{dy}{dt} = 20 - 4y$   $y(0) = 0$   
(c)  $\frac{dy}{dy} = 4$   $y(0) = 100$   
(d)  $\frac{dy}{dt} = 20 - \frac{4y}{2}$   $y(0) = 0$ 

(e) 
$$\frac{dt}{dy} = 20 - \frac{100 + t}{(100 + t)^2}$$
  $y(0) = 100$ 

## Part II: Partial credit questions (11 points each). Show your work.

9. Using the Gram-Schmidt Process, find an orthonormal basis of the subspace of  $\mathbb{R}^4$  spanned by the vectors  $\begin{bmatrix} 1\\0\\0\\0\end{bmatrix}, \begin{bmatrix} 1\\2\\1\\2\end{bmatrix}$  and  $\begin{bmatrix} 1\\3\\1\\3\end{bmatrix}$ .

10. By drawing a direction field, sketch two solutions to the ODE

$$\frac{dy}{dt} = t^2 y^2 (y-2)$$

with initial conditions y(0) = 1 and y(0) = 3. Indicate clearly the limiting behavior  $\lim_{t \to \infty} y(t)$  and  $\lim_{t \to -\infty} y(t)$ . 11. Find the function y(t), for t > 0, which solves the initial value problem

$$t\frac{dy}{dt} + 4y = \frac{e^{-t}}{t^2}$$
 ,  $y(1) = 0$ 

12. Consider the differential equation

$$2y\frac{dy}{dx} = -e^x$$

- (a) Find the general solution.
- (b) Find the solution with y(0) = 1.
- (c) What is the largest interval in which the solution in part (b) is defined?