

REVIEW FOR EXAM 1

Inverse Functions.

- $y = f^{-1}(x) \iff f(y) = x$
- $(f^{-1})'(x) = 1/f'(f^{-1}(x))$

Logarithms & Exponentials.

- $\ln(x) = \int_1^x \frac{1}{t} dt, x > 0, \ln(xy) = \ln(x) + \ln(y), \ln(x^r) = r \ln(x), \log_a(x) = \ln(x)/\ln(a)$
- $e^x = \ln^{-1}(x), a^x = e^{x \ln(a)}, a^x a^y = a^{x+y}, (a^x)^r = a^{xr}$
- $\frac{d}{dx} \ln|u| = \frac{1}{u} \frac{du}{dx}, \frac{d}{dx} e^u = e^u \frac{du}{dx}, \text{ etc.}$
- Logarithmic differentiation

Exponential Growth and Decay.

- $\frac{dy}{dt} = ky, y(0) = y_0 \Rightarrow y = y_0 e^{kt}$
- Population growth, radioactive decay, heating & cooling, continuously compounded interest

Inverse Trig Functions.

- $\sin^{-1}(x), \cos^{-1}(x), \tan^{-1}(x)$
- $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}, \text{ etc. } \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}, \text{ etc.}$
- To derive, take derivative of $\sin(y) = x$, etc.

Hyperbolic Functions.

- $\cosh(x) = \frac{1}{2}(e^x + e^{-x}), \sinh(x) = \frac{1}{2}(e^x - e^{-x}), \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$
- $\cosh^2(x) - \sinh^2(x) = 1$
- $\sinh^{-1}(x), \cosh^{-1}(x), \tanh^{-1}(x)$
- $\frac{d}{dx} \sinh^{-1}(x) = \frac{1}{\sqrt{1+x^2}}, \frac{d}{dx} \cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}}, \frac{d}{dx} \tanh^{-1}(x) = \frac{1}{1-x^2}$
- To derive, take derivative of $\sinh(y) = x$, etc.

Indeterminate Forms and L'Hospital's Rule.

- $\frac{0}{0}, \frac{\infty}{\infty}: \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$
- $\infty - \infty, 0 \cdot \infty, 0^0, 1^\infty$: convert to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ (possibly in exponent).

Integration by Parts.

- $\int u dv = uv - \int v du$
- Use for $x^n e^{ax}, x^n (\ln(x))^a, e^{ax} \sin(bx), e^{ax} \cos(bx)$, inverse functions

Trig Integrals.

- $\int \sin^n(x) \cos^m(x) dx = \int f(\sin(x))(\cos(x) dx)$ or $\int f(\cos(x))(\sin(x) dx)$,
or use $\cos^2(x) = (1 + \cos(2x))/2, \sin^2(x) = (1 - \cos(2x))/2$
- $\int \tan^n(x) \sec^m(x) dx = \int f(\tan(x))(\sec^2(x) dx)$ or $\int f(\sec(x))(\sec(x) \tan(x) dx)$
- $\sin A \sin B = [\cos(A - B) - \cos(A + B)]/2, \cos A \cos B = [\cos(A - B) + \cos(A + B)]/2$
 $\sin A \cos B = [\sin(A - B) + \sin(A + B)]/2$

Trig Substitutions.

- For expressions in $\sqrt{u^2 + a^2}, \sqrt{u^2 - a^2}, \sqrt{a^2 - u^2}$: draw a triangle!
- May need: $\sin(2x) = 2 \sin(x) \cos(x)$.