REVIEW FOR EXAM 2

Partial Fractions.
1. Divide $\dfrac{P(x)}{Q(x)} = S(x) + \dfrac{R(x)}{Q(x)}$
2. Factor $Q(x) = (x - a_1)\cdots(x - a_n)$, rewrite $\dfrac{R(x)}{Q(x)} = \dfrac{A_1}{x - a_1} + \cdots + \dfrac{A_n}{x - a_n}$
3. An irreducible quadratic factor $x^2 + bx + c$ contributes $\dfrac{Ax + B}{x^2 + bx + c}$
4. A repeated factor $(x - a)^k$ contributes $\dfrac{A_1}{x - a} + \cdots + \dfrac{A_k}{(x - a)^k}$
5. A repeated irr quadratic factor $(x^2 + bx + c)^k$ contributes $\dfrac{A_1 x + B_1}{x^2 + bx + c} + \cdots + \dfrac{A_k x + B_k}{(x^2 + bx + c)^k}$

Strategy for Integration.
1. Know standard "targets"
2. Algebraic or trig simplifications
3. Simple substitutions
4. Classify by form
   a. trig functions: rewrite as $f(\cos(x))\sin(x)$, $f(\sin(x))\cos(x)$, $f(\tan(x))\sec^2(x)$, $f(\sec(x))\sec(x)\tan(x)$
   b. rational functions: use partial fractions
   c. products of different types, or inverse functions: use integration by parts $\displaystyle\int u\,dv = uv - \int v\,du$.
   d. $\sqrt{\pm x^2 \pm a^2}$: use a trig substitution (draw a triangle).

Approximate Integration.
1. Midpoint Rule: $\displaystyle\int_a^b f(x)\,dx \approx M_n = \Delta x[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)]$ ($\Delta x = (b - a)/n$, $\overline{x}_i$ is the midpoint of the i-th interval) Error bound: $|E_M| \le \dfrac{K_2(b - a)^3}{24n^2}$ ($|f''(x)| \le K_2$)
2. Simpson's Rule: $\displaystyle\int_a^b f(x)\,dx \approx S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$ (n even, x_i is an endpoint of the i-th interval) Error bound: $|E_S| \le \dfrac{K_4(b - a)^5}{180n^4}$ ($|f^{(4)}(x)| \le K_4$)

Improper Integrals.
1. $\displaystyle\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$, etc.
2. If $f(x)$ not defined at $x = b$, $\displaystyle\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$, etc.
3. Comparison Theorem: If $0 \le f(x) \le g(x)$, then $\displaystyle\int_a^\infty f(x)\,dx \le \int_a^\infty g(x)\,dx$. So $\displaystyle\int_a^\infty g(x)\,dx < \infty$ (converges) $\Rightarrow \displaystyle\int_a^\infty f(x)\,dx < \infty$ and $\displaystyle\int_a^\infty f(x)\,dx = \infty$ (diverges) $\Rightarrow \displaystyle\int_a^\infty g(x)\,dx = \infty$.

Arc Length and Surface Area. A piece of arc length is $ds = \sqrt{dx^2 + dy^2}$.
1. The total arc length of a graph $y = f(x)$, $a \le x \le b$, or $x = g(y)$, $c \le y \le d$, is
   $L = \displaystyle\int ds = \int_a^b \sqrt{1 + f'(x)^2}\,dx$ or $L = \displaystyle\int_c^d \sqrt{g'(y)^2 + 1}\,dy$
2. The area of a surface of revolution is $A = \displaystyle\int 2\pi r\,ds$. If rotated about the x-axis, then $r = y = f(x)$ and
   $A = \displaystyle\int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx$ or $A = \displaystyle\int_c^d 2\pi y\sqrt{[g'(y)]^2 + 1}\,dy$.
3. If rotated about the y-axis, then $r = x = g(y)$ and
   $A = \displaystyle\int_a^b 2\pi x\sqrt{1 + [f'(x)]^2}\,dx$ or $A = \displaystyle\int_c^d 2\pi g(y)\sqrt{[g'(y)]^2 + 1}\,dy$.

Moments and Centroids. Let R be the region $g(x) \le y \le f(x)$, $a \le x \le b$, with density ρ and area A.

Mass: $m = \displaystyle\int_a^b \rho[f(x) - g(x)]\,dx = \rho A$, Centroid: $(\overline{x}, \overline{y})$ where $\overline{x} = \dfrac{M_y}{m}$, $\overline{y} = \dfrac{M_x}{m}$

Moments: $M_x = \rho \displaystyle\int_a^b \frac{1}{2}[f(x)^2 - g(x)^2]\,dx$, $M_y = \rho \displaystyle\int_a^b x[f(x) - g(x)]\,dx$

Theorem of Pappus: The volume of a solid of revolution is $V = A \cdot d$, where d is the distance traveled by the centroid of R. Also, $V = 2\pi M_x$ (about x-axis) or $V = 2\pi M_y$ (about y-axis) with $\rho = 1$.

Differential Equations.

1. Know what a differential equation is, how to check a function is a solution, how to use initial conditions to solve for constants, how to derive general properties of a solution from the equation.
2. To solve a differential equation: separate the variables, integrate, and solve for y.
3. Orthogonal Trajectories: Derive a differential equation for the given family of curves, $y' = F(x, y)$, without unknown constants. The orthogonal trajectories must satisfy $y' = -1/F(x, y)$.
4. Mixing Problems: Let y be the amount of a substance in a solution. If the substance flows in at a constant rate a, and flows out at a rate proportional to y, then $\dfrac{dy}{dt} = a - by$. Often, $a = r \cdot c$ and $b = r/s$ where r is the constant rate of flow in and out, c is the concentration of the input, and s is the amount of solution.

Sequences. A sequence is an ordered list of numbers $a_1, a_2, \ldots, a_n, \ldots$, i.e., a function defined for positive integers, $a_n = f(n)$, $n = 1, 2, \ldots$. It converges if the limit $\lim\limits_{n\to\infty} a_n$ exists. Know how to compute such limits. A sequence a_n is monotonic if it is always increasing or always decreasing. It is bounded if there is a constant M such that $|a_n| \le M$ for all n. A bounded monotonic sequence always converges.

Series. $s = a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \displaystyle\sum_{n=1}^{\infty} a_n = \lim_{m\to\infty} s_m$, where s_m is the partial sum,

$s_m = a_1 + a_2 + \cdots + a_m = \displaystyle\sum_{n=1}^{m} a_n$. The remainder is $R_m = a_{m+1} + a_{m+2} + \cdots = \displaystyle\sum_{n=m+1}^{\infty} a_n$.

s_m approximates $s = s_m + R_m$ with error R_m.

Geometric Series: $\displaystyle\sum_{n=k}^{\infty} ar^n = \frac{ar^k}{1-r}$ if $|r| < 1$, diverges otherwise.

Telescoping Series: Cancel terms in the partial sum s_m (possibly after using partial fractions on a_n) before taking limit.