MATH 10560: CALCULUS II
TRIGONOMETRIC FORMULAS

Basic Identities

The functions $\cos(\theta)$ and $\sin(\theta)$ are defined to be the x and y coordinates of the point at an angle of θ on the unit circle. Therefore, $\sin(-\theta) = -\sin(\theta)$, $\cos(-\theta) = \cos(\theta)$, and $\sin^2(\theta) + \cos^2(\theta) = 1$. The other trigonometric functions are defined in terms of sine and cosine:

$$\begin{aligned} \tan(\theta) &= \sin(\theta)/\cos(\theta) & \qquad \cot(\theta) &= \cos(\theta)/\sin(\theta) = 1/\tan(\theta) \\ \sec(\theta) &= 1/\cos(\theta) & \qquad \csc(\theta) &= 1/\sin(\theta) \end{aligned}$$

Dividing $\sin^2(\theta) + \cos^2(\theta) = 1$ by $\cos^2(\theta)$ or $\sin^2(\theta)$ gives $\tan^2(\theta) + 1 = \sec^2(\theta)$ and $1 + \cot^2(\theta) = \csc^2(\theta)$.

Addition Formulas

The following two addition formulas are fundamental:

$$\begin{aligned} \sin(A+B) &= \sin(A)\cos(B) + \cos(A)\sin(B) \\ \cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B) \end{aligned}$$

They can be used to prove simple identities like $\sin(\pi/2 - \theta) = \sin(\pi/2)\cos(\theta) + \cos(\pi/2)\sin(\theta) = \cos(\theta)$, or $\cos(x - \pi) = \cos(x)\cos(\pi) - \sin(x)\sin(\pi) = -\cos(x)$. If we set $A = B$ in the addition formulas we get the double-angle formulas:

$$\sin(2A) = 2\sin(A)\cos(A) \qquad \cos(2A) = \cos^2(A) - \sin^2(A)$$

The formula for $\cos(2A)$ is often rewritten by replacing $\cos^2(A)$ with $1 - \sin^2(A)$ or replacing $\sin^2(A)$ with $1 - \cos^2(A)$ to get

$$\cos(2A) = 1 - 2\sin^2(A) \qquad \cos(2A) = 2\cos^2(A) - 1$$

Solving for $\sin^2(A)$ and $\cos^2(A)$ yields identities important for integration:

$$\sin^2(A) = \frac{1}{2}(1 - \cos(2A)) \qquad \cos^2(A) = \frac{1}{2}(1 + \cos(2A))$$

The addition formulas can also be combined to give other formulas important for integration:

$$\begin{aligned} \sin A \sin B &= \tfrac{1}{2}[\cos(A-B) - \cos(A+B)] \\ \cos A \cos B &= \tfrac{1}{2}[\cos(A-B) + \cos(A+B)] \\ \sin A \cos B &= \tfrac{1}{2}[\sin(A-B) + \sin(A+B)] \end{aligned}$$

Derivatives and Integrals

$$\begin{aligned} \sin'(x) &= \cos(x) & \qquad \sec'(x) &= \sec(x)\tan(x) \\ \cos'(x) &= -\sin(x) & \qquad \csc'(x) &= -\csc(x)\cot(x) \\ \tan'(x) &= \sec^2(x) & \qquad \cot'(x) &= -\csc^2(x) \end{aligned}$$

$$\begin{aligned} \int \sin(x)\,dx &= -\cos(x) + C & \qquad \int \sec(x)\,dx &= \ln|\sec(x) + \tan(x)| + C \\ \int \cos(x)\,dx &= \sin(x) + C & \qquad \int \csc(x)\,dx &= \ln|\csc(x) - \cot(x)| + C \\ \int \tan(x)\,dx &= \ln|\sec(x)| + C & \qquad \int \cot(x)\,dx &= -\ln|\csc(x)| + C \end{aligned}$$