Name: ____________________

Department of Mathematics
University of Notre Dame
Math 20580 – Spring 2012

Instructor:
& Section ____________

Final Exam

May 7, 2012

This exam has 13 pages and contains 24 problems. Each problem is worth 6 points and 6 points will be given for following instructions. You have 2 hours to work on it. No calculators, books, notes, or other aids are allowed. Be sure to write your name on this page and to put your initials at the top of every page in case pages become detached. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in nor tolerate academic dishonesty.

Signature:________________________________

Mark your answers to the multiple choice problems here.
Place an $\times$ through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)	13.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	14.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
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11.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)

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The next four problems refer to the two matrices

$$A = \begin{bmatrix} 1 & 13 & 2 & -2 & 4 & 3 \\ -1 & -13 & 5 & 9 & 10 & -2 \\ -2 & -26 & 3 & 11 & 6 & -5 \\ 7 & 91 & 2 & -26 & 4 & 11 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 13 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced row echelon form of A.

1.(6pts) Which set of vectors below is a basis for the column space of A?

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 10 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -5 \\ 11 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -5 \\ 11 \end{bmatrix} \right\}$

2.(6pts) Which set of vectors below is a basis for the row space of A?

(a) $\left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 7 \\ 7 \\ 14 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 7 \\ 7 \\ 14 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 13 \\ 9 \\ 5 \\ 18 \\ 4 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -13 \\ 5 \\ 9 \\ 10 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -26 \\ 3 \\ 11 \\ 6 \\ -5 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 1 \\ 13 \\ 0 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -13 \\ 5 \\ 9 \\ 10 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -26 \\ 3 \\ 11 \\ 6 \\ -5 \end{bmatrix}, \begin{bmatrix} 7 \\ 91 \\ 2 \\ -26 \\ 4 \\ 11 \end{bmatrix} \right\}$

For your convenience, here are the two matrices from the previous page:

$$A = \begin{bmatrix} 1 & 13 & 2 & -2 & 4 & 3 \\ -1 & -13 & 5 & 9 & 10 & -2 \\ -2 & -26 & 3 & 11 & 6 & -5 \\ 7 & 91 & 2 & -26 & 4 & 11 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 13 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced row echelon form of A.

3.(6pts) Which set of vectors below is a basis for the null space of A?

(a) $\left\{ \begin{bmatrix} 13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

(e) The null space has dimension 0.

4.(6pts) Which set of vectors below is a basis for (row $A)^\perp$, the orthogonal complement to the row space of the matrix A above?

(a) $\left\{ \begin{bmatrix} 13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

(e) The space (row $A)^\perp$ has dimension 0.

5.(6pts) Which number below is the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$?

(a) 5 (b) 16 (c) -5 (d) -16 (e) 0

6.(6pts) If $\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$ is the inverse to $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 4 \\ -1 & -1 & 1 \end{bmatrix}$ find b_{31}.

(a) $b_{31} = -0.6$ (b) $b_{31} = 0.6$ (c) $b_{31} = -1$ (d) $b_{31} = -0.2$ (e) $b_{31} = 1$

7.(6pts) Suppose **a**, **b**, **x** and **y** are n-vectors and suppose the various dot products are as follows: $\mathbf{a}\bullet\mathbf{a} = -1$, $\mathbf{a}\bullet\mathbf{b} = 1$, $\mathbf{a}\bullet\mathbf{x} = 3$, $\mathbf{a}\bullet\mathbf{y} = -3$, $\mathbf{b}\bullet\mathbf{b} = 7$, $\mathbf{b}\bullet\mathbf{x} = 4$, $\mathbf{b}\bullet\mathbf{y} = -4$, $\mathbf{x}\bullet\mathbf{x} = 7$, $\mathbf{x}\bullet\mathbf{y} = 2$ and $\mathbf{y}\bullet\mathbf{y} = -2$. The number $(2\mathbf{a}+\mathbf{x})\bullet(-\mathbf{y}+\mathbf{b})$ is which answer below?

(a) -5 (b) 10 (c) 3 (d) 0

(e) Can not be computed from the given data.

8.(6pts) Let $W = \text{Span}\left\{\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}, \begin{bmatrix}2\\2\\-1\\3\end{bmatrix}\right\}$. Which vector below is the projection of $\begin{bmatrix}21\\-3\\9\\-3\end{bmatrix}$ into W?

(a) $\begin{bmatrix}11\\12\\-2\\14\end{bmatrix}$ (b) $\begin{bmatrix}24\\1\\11\\-1\end{bmatrix}$ (c) $\begin{bmatrix}5\\8\\8\\0\end{bmatrix}$ (d) $\begin{bmatrix}3\\4\\2\\2\end{bmatrix}$ (e) $\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

9.(6pts) If $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 10 \\ 0 \\ 2 \end{bmatrix} \right\}$, which set of vectors below is an orthogonal basis for W?

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -3 \\ 0 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 0 \\ -7 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \\ -7 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} \right\}$

10.(6pts) Find the least squares solution to the equation $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -2 \\ 3 \\ -2 \\ 10 \end{bmatrix}$.

(a) $\begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ 1 \\ 10 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

11.(6pts) Let U be any $n \times n$ matrix with orthonormal columns. Which of the following are sometimes false?

(1) The rows of U are orthonormal.
(2) $U^T U$ is the identity.
(3) For any n-vectors $\mathbf{x}$ and $\mathbf{y}$, $U\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot \mathbf{y}$
(4) The determinant of U is ± 1.
(5) UU^T has a non-trivial kernel.

(a) (1) and (4) (b) (1) and (5) (c) (2) and (3)

(d) (3) and (5) (e) All five statements are always true.

12.(6pts) Let y be the solution to $(4xy + 1)dx + (2x^2 + \cos y)dy = 0$ which goes through the point $(0, \pi/2)$. Which implicit equation below is satisfied by this solution?

(a) $2x^2y + x + \sin y = 1$ (b) $2x^2y + x - \sin y = 0$ (c) $2x^2y + y + \sin y = \pi/2 + 1$

(d) $2x^2y + x - \sin y = -1$ (e) $2x^2y + y - \sin y = \pi/2 - 1$

13.(6pts) Let $\mathbb{P}_2$ be the vector space of polynomials of degree 2 or less. Let $L\colon \mathbb{P}_2 \to \mathbb{P}_2$ be the linear transformation defined by $L[y] = t^2y'' + ty' + y$. Determine the matrix of L with respect to the basis $\{1, t, t^2\}$ for $\mathbb{P}_2$.

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & -1 \\ 5 & -1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 2 & 5 \\ 1 & 5 & 0 \end{bmatrix}$

14.(6pts) Suppose Y is the solution to the equation

$$y' + \tan(t)y = \frac{t}{t^2+1} \quad \text{and} \quad y(\pi) = 3$$

Which interval below is the largest interval over which Y is guaranteed to exist?

(a) $0 < t < \pi$ (b) $\pi/2 < t < 3\pi/2$ (c) $0 < t < 2\pi$ (d) $-\pi/2 < t < \pi/2$

(e) $-3\pi/2 < t < 3\pi/2$

15.(6pts) If y is the solution to

$$y'' - 2y' - 8y = 9e^t \quad \text{and} \quad y(0) = y'(0) = 0$$

which number below is $y(1)$?

(a) $\dfrac{e^{-2} - 2e + e^4}{2}$ (b) $\dfrac{e^3 - 6e^{-1} + 3}{2}$ (c) $\dfrac{e^4 + 6e + 5}{6e^2}$

(d) $\dfrac{e^3 + 6e^{-1} + 3}{2}$ (e) $\dfrac{e^6 + 6e + 5}{6e^2}$

16.(6pts) If y is the solution to

$$y' + 4xy = x \quad \text{with} \quad y(1) = 2$$

find $y(0)$.

(a) $\dfrac{1}{4} - \dfrac{7e^2}{4}$ (b) $\dfrac{1}{4} - \dfrac{4e^2}{7}$ (c) $\dfrac{1}{4} + \dfrac{7e^2}{4}$

(d) $\dfrac{1}{7} - \dfrac{4e^2}{7}$ (e) $\dfrac{7e^2}{4} - \dfrac{1}{4}$

17.(6pts) Consider the equation

$$y' = (y^3 - 4y)(9 - y^2)$$

with initial value $y(-1) = 1$. Assuming $\lim_{t\to\infty} y(t)$ exists, find it without solving the equation.

(a) -2 (b) 3 (c) 0 (d) -3 (e) 2

18.(6pts) An object of mass m is released from rest from a boat into the water and allowed to sink. Gravity is pulling the object down and distance is to be measured upwards. By Archimedes's Principle, there is a buoyant force pushing up equal to bmg where b is a constant. Resistance imparts a force on the object proportional to its velocity but in the opposite direction with proportionality constant $k > 0$. Determine a differential equation for the velocity v of the object.

(a) $v' + \dfrac{k}{m}v = bg$ (b) $v' + \dfrac{k}{m}v = (b-1)g$ (c) $v' - \dfrac{k}{m}v = (b-1)g$

(d) $v' - \dfrac{k}{m}v = bg$ (e) $v' - kv = (b-1)g$

19.(6pts) Which function below is the Wronskian of the two functions $y_1 = t^2$ and $y_2 = t^{-1}$?

(a) $\dfrac{t^2 + t^{-1}}{2t - t^{-1}}$ (b) -3 (c) 5 (d) $2t$ (e) $t^2 \ln|t| - \dfrac{t^2}{3}$

20.(6pts) Given that $y = t$ is one solution to the homogeneous equation

$$y'' - \left(1 + \frac{2}{t}\right) y' + \left(\frac{1}{t} + \frac{2}{t^2}\right) y = 0$$

use reduction of order to find a second independent solution. Which function below is such a function?

(a) t^3 (b) e^{-t} (c) te^t (d) $t^2 - 3t$ (e) $t^2 e^{-2t}$

21.(6pts) Find the form of a particular solution to

$$y'' + 49y = t^2 \sin(7t)$$

using the method of Undetermined Coefficients.

(a) $y = (A_0 t + A_1 t^2 + A_2 t^3)\sin(7t) + (B_0 t + B_1 t^2 + B_2 t^3)\cos(7t)$

(b) $y = (A_0 t + A_1 t^2 + A_2 t^3)\sin(7t) + (A_0 t + A_1 t^2 + A_2 t^3)\cos(7t)$

(c) $y = (A_0 + A_1 t + A_2 t^2)\sin(7t) + (B_0 + B_1 t + B_2 t^2)\cos(7t)$

(d) $y = (A_0 t + A_1 t^2 + A_2 t^3)\sin(7t) + (A_2 t + A_1 t^2 + A_0 t^3)\cos(7t)$

(e) $y = (A_0 t^2 + A_1 t^3 + A_2 t^4)\sin(7t) + (B_0 t^2 + B_1 t^3 + B_2 t^4)\cos(7t)$

22.(6pts) Which of these equations is 2nd order and non-linear?

(1) $y'' + (\sin x)y' + (\tan x)y = e^x$
(2) $y'' + (\sin x)y' + \tan(xy) = e^x$
(3) $y' + \tan(xy) = e^x$
(4) $y' + (\tan x)y = e^x$
(5) $y''' + e^{ty} = 0$
(6) $y''' + e^t y = 0$
(7) $y'' + e^t y = \sin(t)$
(8) $y'' + e^{ty} = 0$

(a) (2), (6) and (8)
(b) (1), (3) and (4)
(c) (2) and (7)
(d) (5) and (8)
(e) (2) and (8)

23.(6pts) Suppose t and $\sin t$ are two solutions to the homogeneous system

$$y'' + p(t)y' + q(t)y = 0$$

Use Variation of Parameters to find a solution to the equation

$$y'' + p(t)y' + q(t)y = t\cos(t) - \sin(t)$$

Which function below is a solution?

(a) $\dfrac{t^3 \sin t + 3t \cos t}{3}$ (b) $\dfrac{t^2 \sin t + 2t \cos t}{2}$ (c) $\dfrac{2t \sin t + t^2 \cos t}{2}$

(d) $\dfrac{2t \sin t - 3t^2 \cos t}{2}$ (e) $\dfrac{t^2 \sin t + 3t \cos t}{3}$

24.(6pts) The vector $\begin{bmatrix} 7 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ is an eigenvector for the matrix $\begin{bmatrix} 1 & 2 & 5 & 7 & 10 \\ 0 & -1 & 3 & 4 & 12 \\ 0 & 0 & 2 & 8 & 8 \\ 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix}$. What is the corresponding eigenvalue?

(a) 0 (b) -3 (c) 3 (d) -2 (e) 2

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12.	(•)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(•)

Please do not write below here.

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Solutions

1. A basis for the column space consists of the columns of A in the pivot positions which can be easily seen from B: columns 1, 3 and 6.

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2. The row space of A equals the row space of B and the non-zero rows of B are a basis.

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3. There is an algorithm for finding a basis for the null space of A using B.

---

4. The orthogonal complement to the row space is the null space so the answer is the same as that for the previous problem.

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5. Row reduce:

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & -3 & -2 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & 0 & 4 \end{bmatrix}$$

so the determinant is $1 \cdot (-4) \cdot 4 = -16$.

6.

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 5 & 6 & 4 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -11 & -5 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 1 \\ 0 & -4 & -11 & -5 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 1 \\ 0 & 0 & 5 & -1 & 1 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 1 \\ 0 & 0 & 1 & -0.2 & 0.2 & 0.8 \end{bmatrix}$$

From here you can read off $b_{3\,1}$ and $b_{3\,2}$ since further row operations to get to reduced row echelon form will not change the last row. But if you do continue

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 1 \\ 0 & 0 & 1 & -0.2 & 0.2 & 0.8 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1.8 & -0.8 & -2.2 \\ 0 & 0 & 1 & -0.2 & 0.2 & 0.8 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 1.8 & -0.8 & -2.2 \\ 0 & 0 & 1 & -0.2 & 0.2 & 0.8 \end{bmatrix}$$

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7. $(2\mathbf{a} + \mathbf{x}) \bullet (-\mathbf{y} + \mathbf{b}) = -2\mathbf{a} \bullet \mathbf{y} - \mathbf{x} \bullet \mathbf{y} + 2\mathbf{a} \bullet \mathbf{b} + \mathbf{x} \bullet \mathbf{b} = -2(-3) - (2) + 2(1) + 4 = 10.$

8.

$$\frac{\begin{bmatrix}a\\b\\c\\d\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}}{\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}}\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}+\frac{\begin{bmatrix}a\\b\\c\\d\end{bmatrix}\cdot\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}}{\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}\cdot\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}}\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}=\frac{a+2b+3c-d}{15}\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}+\frac{2a+2b-c+3d}{18}\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}=$$

$$\frac{21+2\cdot(-3)+3\cdot 9-(-3)}{15}\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}+\frac{2\cdot 21+2\cdot(-3)-(9)+3\cdot(-3)}{18}\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}=$$

$$\frac{45}{15}\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}+\frac{18}{18}\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}=3\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}+\begin{bmatrix}2\\2\\-1\\3\end{bmatrix}=\begin{bmatrix}5\\8\\8\\0\end{bmatrix}$$

9.

We may take $\begin{bmatrix}1\\2\\3\\1\end{bmatrix}$ as the first vector. The second vector is $\begin{bmatrix}8\\10\\0\\2\end{bmatrix}-\frac{\begin{bmatrix}8\\10\\0\\2\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\\1\end{bmatrix}}{\begin{bmatrix}1\\2\\3\\1\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\\1\end{bmatrix}}\begin{bmatrix}1\\2\\3\\1\end{bmatrix}=$

$$\begin{bmatrix}8\\10\\0\\2\end{bmatrix}-\frac{30}{15}\begin{bmatrix}1\\2\\3\\1\end{bmatrix}=\begin{bmatrix}8\\10\\0\\2\end{bmatrix}-\begin{bmatrix}2\\4\\6\\2\end{bmatrix}=\begin{bmatrix}6\\6\\-6\\0\end{bmatrix}$$

10.

Solve $A^TA\hat{\mathbf{x}}=A^T\mathbf{b}$ or $\begin{bmatrix}2&0&2&0\\-1&2&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}2&-1&0\\0&2&0\\2&1&0\\0&0&1\end{bmatrix}\hat{\mathbf{x}}=\begin{bmatrix}2&0&2&0\\-1&2&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}-2\\3\\-2\\10\end{bmatrix}$

$$\begin{bmatrix}8&0&0\\0&6&0\\0&0&1\end{bmatrix}\hat{\mathbf{x}}=\begin{bmatrix}-8\\6\\10\end{bmatrix}$$

11.
(1) is true
(2) is true
(3) is not always true since the correct equation is $U\mathbf{x} \cdot U\mathbf{y} = \mathbf{x} \cdot \mathbf{y}$
(4) is true
(5) is false since $U^T = U^{-1}$ so UU^T is also the identity matrix and hence has trivial kernel.

12.
$(4xy + 1)dx + (2x^2 + \cos y)dy = 0$ is exact. $\phi_x = 4xy + 1$ so $\phi = 2x^2y + x + g(y)$ so $\phi_y = 2x^2 + g' = 2x^2 + \cos y$ and $g'(y) = \cos y$ so $g(y) = \sin y + C$. $\phi(x, y) = 2x^2y + x + \sin y$ and $\phi(0, \pi/2) = -1$ so the implicit solution is $2x^2y + x + \sin y = 1$.

13.

$L(1) = 1$ so the first column is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$L(t) = t + t$ so the second column is $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$.

$L(t^2) = 2t^2 + 2t^2 + t^2$ so the third column is $\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$.

14.
The right hand side is continuous everywhere so the only constraint is on the tan. The tangent is continuous except at $\pi/2 + k\pi$ for any integer k. At these points the tangent is undefined. Hence $(\pi/2, 3\pi/2)$ is the largest interval containing π over which the tangent is continuous.

15.
The characteristic equation is $r^2 - 2r - 8 = (r - 4)(r + 2)$. A particular solution is $y_p = -e^t$ so the general solution is $Y = -e^t + ae^{4t} + be^{-2t}$. Then $Y' = -e^t + 4ae^{4t} - 2be^{-2t}$ so $Y(0) = -1 + a + b = 0 = Y'(0) = -1 + 4a - 2b$ or $a + b = 1$ and $4a - 2b = 1$.

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 4 & -2 & 1 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -6 & -3 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 0.5 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 0 & 0.5 \\ 0 & 1 & 0.5 \end{array}\right]$$

so $Y(t) = -e^t + 0.5e^{4t} + 0.5e^{-2t}$

Hence $Y(1) = \dfrac{-2e + e^4 + e^{-2}}{2}$.

16.
Integrating factor: $\mu = e^{2x^2}$ so need $\int xe^{2x^2}\,dx = \frac{1}{4}e^{2x^2} + C$ and general solution is $y = \frac{1}{4} + Ce^{-2x^2}$. Then $y(1) = \frac{1}{4} + Ce^{-2} = 2$ so $C = \frac{7e^2}{4}$ and $y = \frac{1}{4} + \frac{7e^2}{4e^{2x^2}}$. Then $y(0) = \frac{1}{4} + \frac{7e^2}{4}$.

17.
The constant solutions are $y = 0$, $y = \pm 2$, $y = \pm 3$. Since $y(-1) = 1$ the solution lies between the lines $y = 0$ and $y = 2$. Here $f(y) = (y^3 - 4y)(9 - y^2)$ is negative since $f(1) = -3 \cdot 8 = -24$. Hence y is decreasing so the limit is 0.

18.
$F = ma = mv'$. In this case there is a force downward of $-mg$, a force upward due to buoyancy of bmg and a further force of resistance of $-kv$. Hence $mv' = -mg + bmg - kv$ or $v' + \frac{k}{m}v = (b-1)g$.

19.

$$\det \begin{vmatrix} t^2 & t^{-1} \\ 2t & -t^{-2} \end{vmatrix} = -1 - 2 = -3$$

20.
$\frac{v''}{v'} + \left(2\frac{y'}{y} + p\right) = 0$. Since $y = t$ is one solution the equation is $\frac{v''}{v'} + \left(2\frac{1}{t} + \left(-1 - \frac{2}{t}\right)\right) = 0$, or $\frac{v''}{v'} = 1$ and hence $v = e^t$. Hence a second solution is $z = te^t$.

21.
The characteristic equation is $r^2 + 49$ which has roots $\pm 7i$. There are no repeated roots so the formula has the form $t(\textit{generic quadratic polynomial})\cos(7t) + t(\textit{generic quadratic polynomial})\sin(7t)$

22.
(1) is linear
(2) is 2nd order, non-linear
(3) is 1st order, non-linear
(4) is 1st order, linear
(5) is 3rd order, non-linear

(6) is 3rd order, linear
(7) is 2nd order, linear
(8) is 2nd order, non-linear

23.
The Wronskian is $W = \det\begin{vmatrix} t & \sin t \\ 1 & \cos t \end{vmatrix} = t\cos t - \sin t$. Solution is

$$Y = \det\begin{vmatrix} \displaystyle\int \frac{g(t)t}{W}dt & \displaystyle\int \frac{g(t)\sin t}{W}dt \\ t & \sin t \end{vmatrix}$$

where $g(t) = t\cos t - \sin t$ so the integrals are $\displaystyle\int t dt = \frac{t^2}{2}$ and $\displaystyle\int \sin t dt = -\cos t$ so

$$Y = \det\begin{vmatrix} \frac{t^2}{2} & -\cos t \\ t & \sin t \end{vmatrix} = \frac{t^2\sin t + 2t\cos t}{2}$$

24.
$\begin{bmatrix} 1 & 2 & 5 & 7 & 10 \\ 0 & -1 & 3 & 4 & 12 \\ 0 & 0 & 2 & 8 & 8 \\ 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ so the eigenvalue is 2.
