

**Math Reasoning, Practice Exam 2.****April 3, 2013**

There are 5 questions worth 10 points each. Please write clearly and give complete proofs.

**Problem 1.**

Find the orders of  $\bar{2}$  and  $\bar{3}$  in  $\mathbb{Z}_7$ .

**Problem 2.**

For each of the relations  $R$  on sets  $S$  defined below, determine whether  $R$  is an equivalence relation.

(a)  $S = \mathbb{Z}$  and  $(x, y) \in R$  if and only if  $x^2 \equiv y^2 \pmod{3}$ .

(b)  $S = \mathbb{Z}_p$  where  $p$  is a prime number and  $(\bar{x}, \bar{y}) \in R$  if and only if  $\bar{y} = \bar{2}^k \cdot \bar{x}$  for some  $k \geq 0$ .

**Problem 3.**

Find all integer solutions  $x, y$  to the equation

$$159x + 225y = 9$$

or show that no solutions exist.

**Problem 4.**

Find all solutions  $x$  to the following system of equations.

$$x \equiv 2 \pmod{5}; x \equiv 2 \pmod{9}; x \equiv 3 \pmod{7}.$$

**Problem 5.**

Compute

$$2^{508} \bmod 499.$$

You may assume without proof that 499 is a prime number.