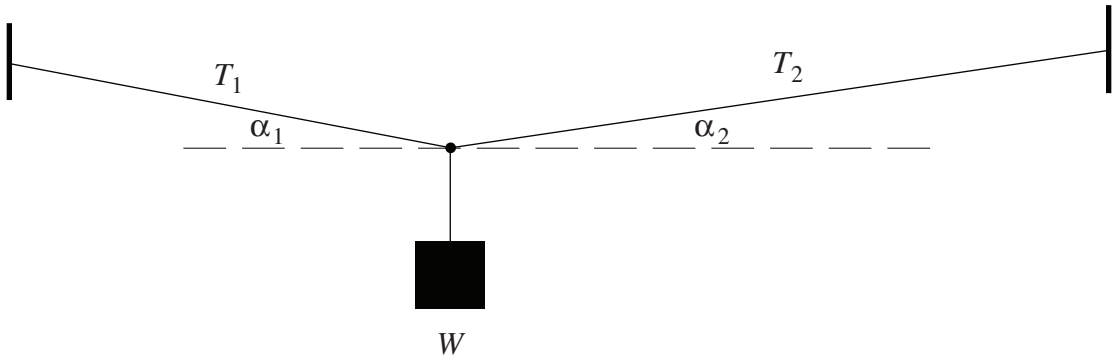
Computing the Tension in a Cable

Turn to the figure below. Assume that a weight W hangs from a cable as shown. The cable is anchored firmly on the left and right. The respective angles the cable makes with the horizontal are α_1 and α_2 respectively. The tensions in the cable—the tension in a cable



is the magnitude with which the cable pulls—are T_1 and T_2 respectively.

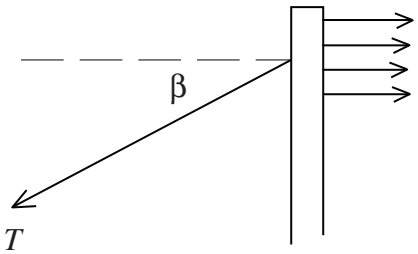
Problem 1. Consider the weight W and the angles α_1 and α_2 as given and assume that the configuration depicted in the figure is stable. Draw a force diagram for the point at which the weight is suspended and use results of the section "Dealing with Forces" of Chapter 2 to express both T_1 and T_2 in terms of W and the angles α_1 and α_2. Conclude that if $\alpha_1 = \alpha_2$, then $T_1 = T_2$.

Problem 2. Look up the addition formula for the sine and use it to simplify the expressions for T_1 and T_2 derived in Problem 1 to

$$T_1 = \frac{W \cos \alpha_2}{\sin(\alpha_1 + \alpha_2)} \quad \text{and} \quad T_2 = \frac{W \cos \alpha_1}{\sin(\alpha_1 + \alpha_2)}.$$

Problem 3. Assume that $W = 500$ pounds, $\alpha_1 = 10°$, and $\alpha_2 = 5°$ and use your the formulas of Problem 2 to compute the tensions T_1 and T_2. Repeat your computation of T_1 and T_2 with $W = 1000$ pounds, $\alpha_1 = 5°$, and $\alpha_2 = 4°$. Finally, repeat the computations once more with $W = 2000$ pounds and the angles $\alpha_1 = 4°$ and $\alpha_2 = 2°$.

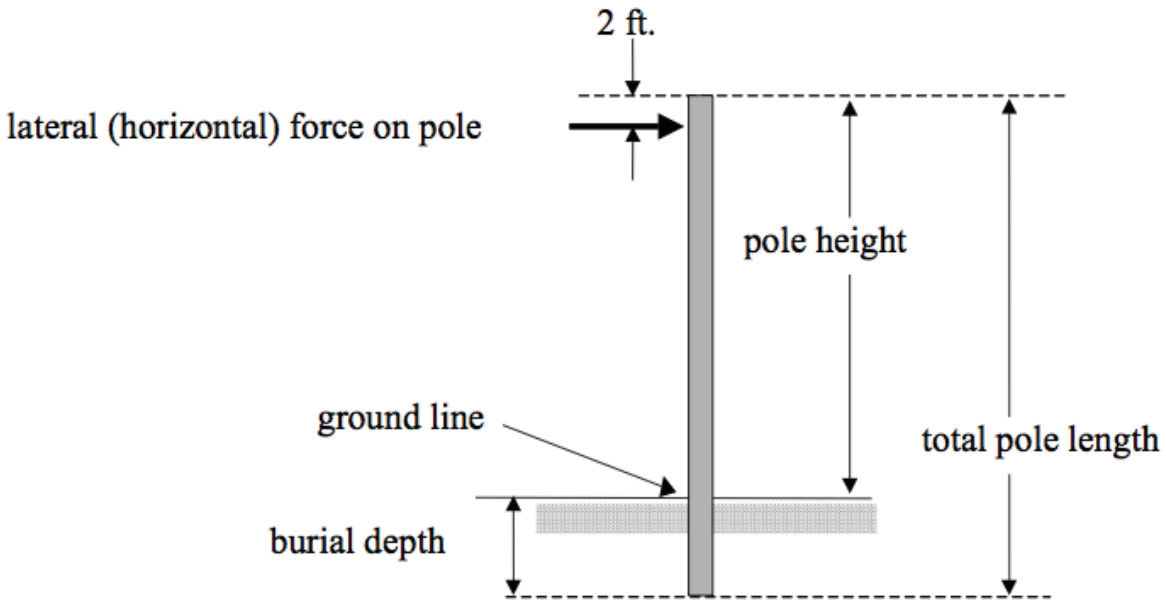
Problem 4. The figure below is an abstraction of Image 6. It shows a cable pulling on a utility pole with a force of magnitude T at an angle β with the horizontal. Provide an



expression for the horizontal component of T. Let $T = 20{,}000$ pounds, take β equal to $40°, 30°$, and $20°$, and compute the magnitude of this horizontal component in each case. Do you think that this force is problematic in the context of the solution of the structural problem?

About Steel Cables. A $\frac{1}{2}$-inch diameter steel cable typically has a minimum breaking strength of about 20,000 to 25,000 pounds and can support a load between 4000 and 5000 pounds safely. For a $\frac{3}{4}$-inch diameter steel cable, these ranges are from about 45,000 to 50,000 pounds and 9,000 to 10,000 pounds.

About Utility Poles. The force that a vertical wooden pole can support depends on a number of factors including the type of wood, the thickness of the pole, the depth and quality of the foundation, and the height and direction at which the force is applied. The figure and table below provide very general guidelines (for applications in the U.S.).



Minimum Diameter of Pole (in inches)	Length Range of Pole (in feet)	Allowable Horizontal Load (in pounds)
12.4	45-125	11,400
11.8	45-125	10,000
11.1	40-125	8,700
10.5	40-125	7,500
9.9	35-125	6,400