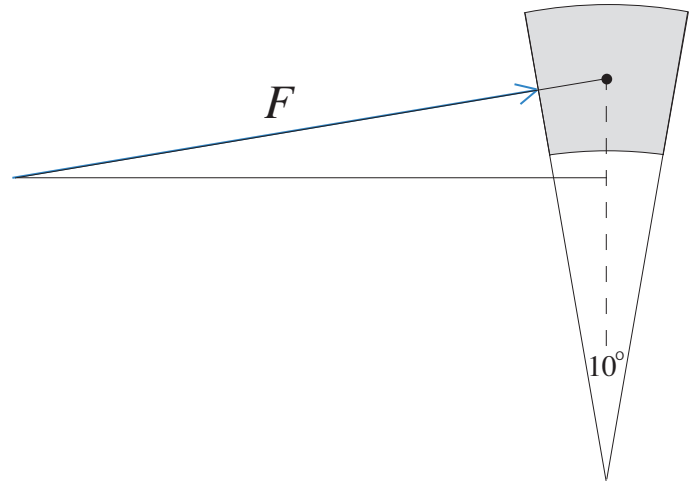


Final Exam Math 10270. May 5, 08. Name

Please Note: For full credit your arguments need to be complete and precise. They also need to be presented in a well organized and lucid way. Suggestion: First collect your thoughts on the scrap paper provided.

1. (15 pts) The figure below depicts a keystone of a stable semicircular Roman arch. The keystone weighs 300 pounds. Compute the magnitude of the force F needed to keep it in place. (It is understood that a force of the same magnitude also acts on the other side of the keystone.)

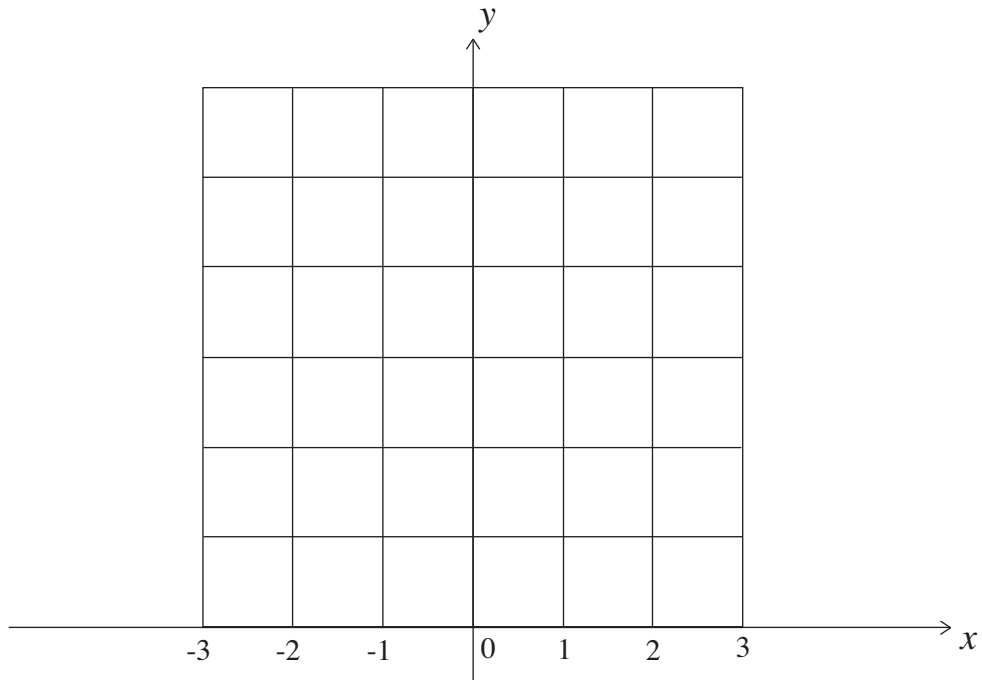


2. (15 pts) Describe the essential structural features of a Gothic cathedral from the vault downward and explain how one depends on the next.

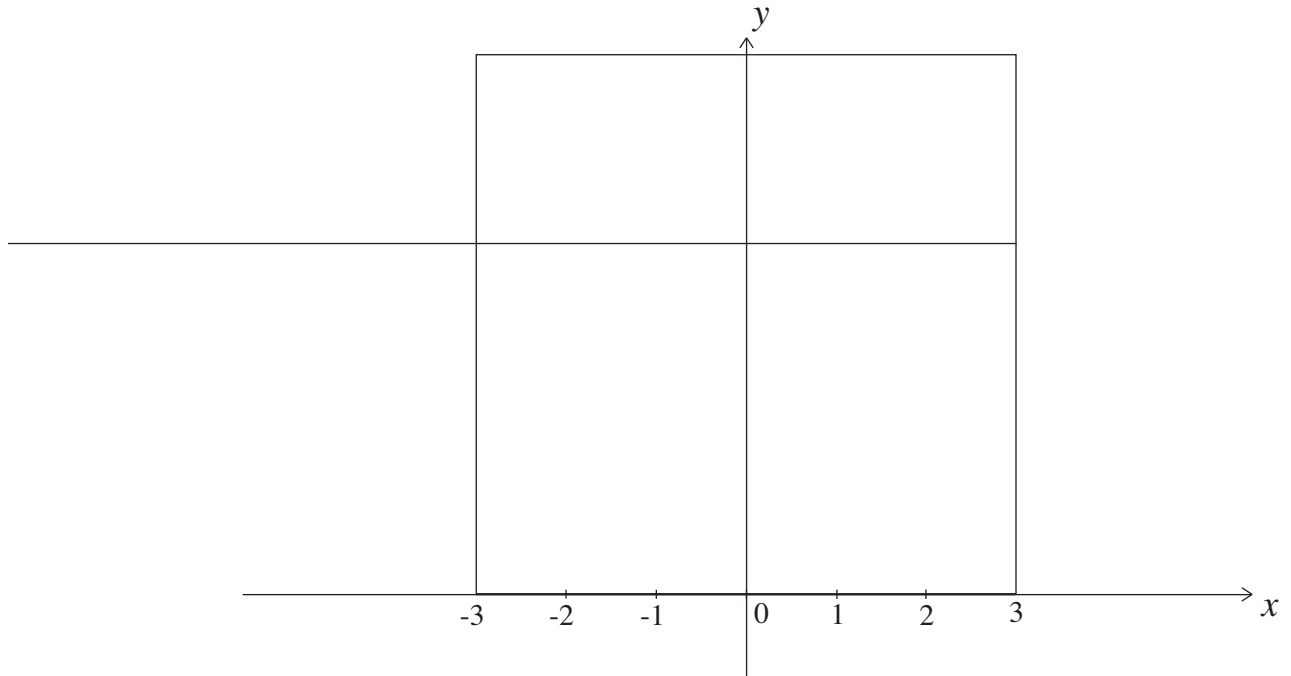
3. (15 pts) A coordinate plane is given. Consider the circle with center $(1, -2)$ and radius 5 as well as the line through the center with slope $\frac{1}{3}$. Determine the points of intersection of the circle and the line.

4. (10 pts) What were the two unprecedented demands - one having to do with the stability of the structure of the dome and the other with its construction - that Brunelleschi faced in the building of the dome of the Santa Maria del Fiore.

5a. (10 pts) Consider a floor of 36 identical square tiles along with a coordinate system as the figure



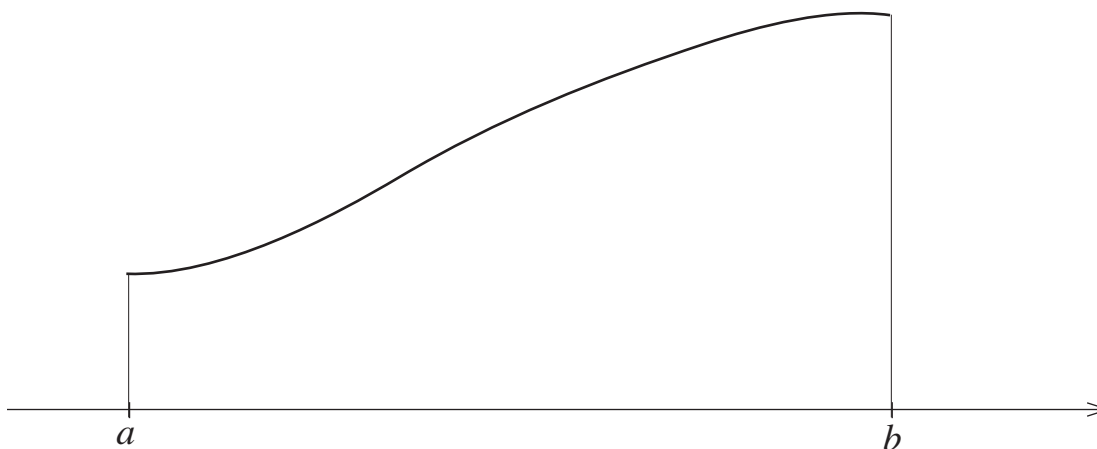
above provides it. In the figure below reproduce Alberti's perspective drawing of the floor. Include the vanishing point $C = (0, c)$ and the secondary vanishing point $D = (-d, c)$.



5b. (20 pts) Describe with precision and in detail the meaning of the equations $x_1 = \frac{x_0 d}{d+y_0}$ and $y_1 = \frac{c y_0}{d+y_0}$ as well as $x_0 = \frac{c x_1}{c-y_1}$ and $y_0 = \frac{d y_1}{c-y_1}$ in the context of the floor and the drawing and explain the strategy that was used to derive them. Make use of the previous diagrams to explain things (but don't carry the derivation out).

5c. (15 pts) Place the upper half of the circle $x^2 + y^2 = 1$ into the first figure. The curve that represents this semicircle in perspective is the upper half of an ellipse. Draw the upper half of this ellipse into the second figure. Determine the equation of this ellipse.

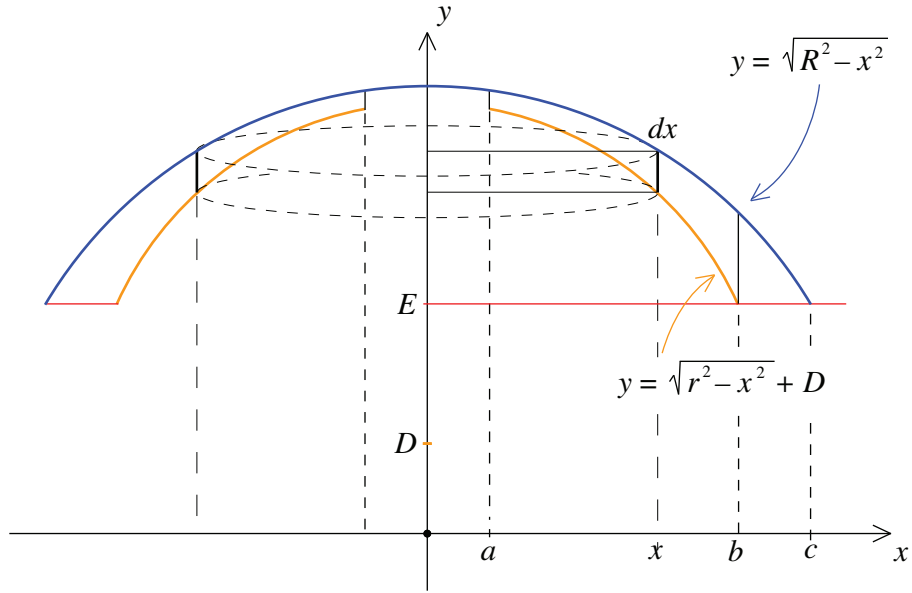
6a. (10 pts) Let $y = f(x)$ be a continuous function defined on the interval $a \leq x \leq b$ and assume that $f(x) \geq 0$ for all such x . Explain what the formula $V = \int_a^b \pi(f(x))^2 dx$ means.



6b. (15 pts) Explain by expanding the diagram above and by *making use of the working definition of the definite integral* how this formula is derived.

7. (Greetings from Professor Crowe). (10 pts) Explain by using a few key words how fundamental ideas of geometry and mathematics entered human consciousness by way of observed celestial phenomena.

Extra Credit. (15 pts) Make use of the diagram below to express the volume of the shell of Pantheon



(above the line $y = E$) as a sum of two definite integrals. Provide an explanation for your answer. Then describe how you would evaluate these integrals (without carrying the evaluation out).