

Final, Math 10270, May 11, 2011. NAME (10 pts.): _____

Please Note: *Use calculators only in elementary computational mode. Please be aware that if your solutions are not precisely formulated and presented in a well organized, legible way, points will be deducted.*

1. (15 pts) Let $P_1 = (1, -2, -1)$ and $P_2 = (2, -1, -2)$ be two points in coordinate three space.

1a. Use the " P_1 " and " $P_2 - P_1$ " strategy to find a set of parametric equations for the line that the two points determine.

1b. Consider the sphere with center the origin O and radius $2\frac{1}{2}$. Determine whether the line goes through the circle or not.

2. (10 pts) What were the two unprecedented demands – one having to do with the stability of the structure of the dome and the other with its construction – that Brunelleschi faced when he built the dome of the Santa Maria del Fiore.

3. (20 pts) Describe with precision and in detail the meaning of the equations $x_1 = \frac{x_0 d}{d+y_0}$ and $z_1 = \frac{e y_0}{d+y_0}$ (in the context of Alberti's floor and the perspective drawing of it on a canvas) and explain the strategy that was used to derive them. Include explanations of the constants d and e . Use a diagram in three dimensional coordinate space to illustrate your description.

4. (15 pts.) Describe Michelangelo's most important contributions to the design of Saint Peter's.

5. (20 pts.) Let $f(x) = x^3$ and compute $f'(x)$.

5a. Explain why $3x^2 dx \approx (x + dx)^3 - x^3$ for any x and any small dx .

5b. How good is the above approximation for $x = 2$ and $dx = 0.001$?

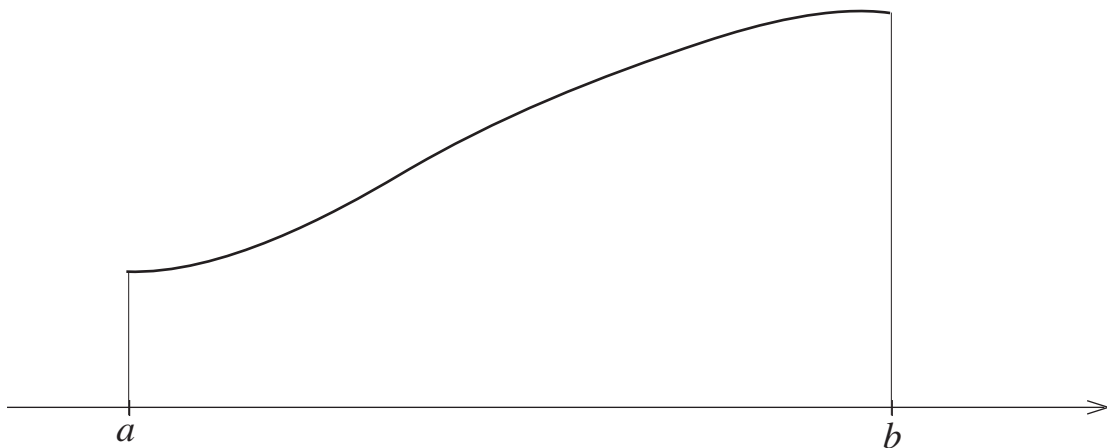
6. (20 pts.) Consider the function $f(x) = 4 - x^2$ with $0 \leq x \leq 2$.

6a. Take $n = 4$ and compute the sum that arises in the definition of the integral $\int_0^2 (4 - x^2) dx$.
Do so with two decimal place accuracy.

6b. Why is this sum only a very rough approximation of $\int_0^2 (4 - x^2) dx$? How can this approximation be made very precise?

6c. Use the Fundamental Theorem of Calculus to find the precise value of this integral.

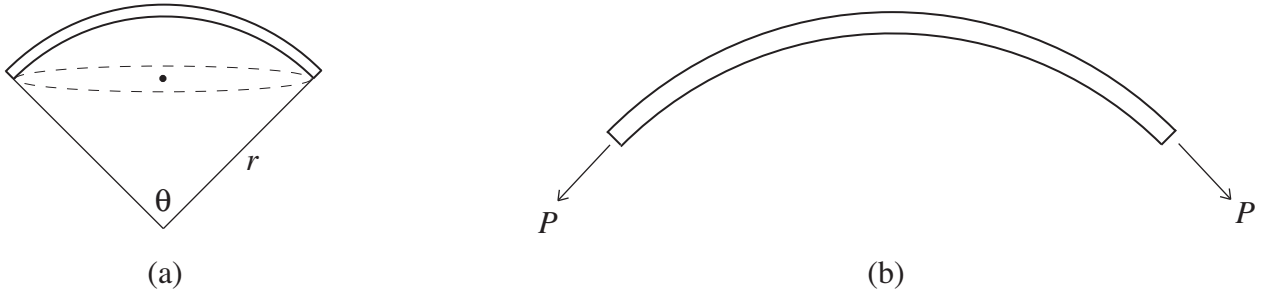
7. (20 pts) Let $y = f(x)$ be a continuous function defined on the interval $a \leq x \leq b$ and assume that $f(x) \geq 0$ for all such x . The graph of the function is depicted in the diagram below.



7a. The formula $V = \int_a^b \pi(f(x))^2 dx$ provides a volume. Explain what volume this is.

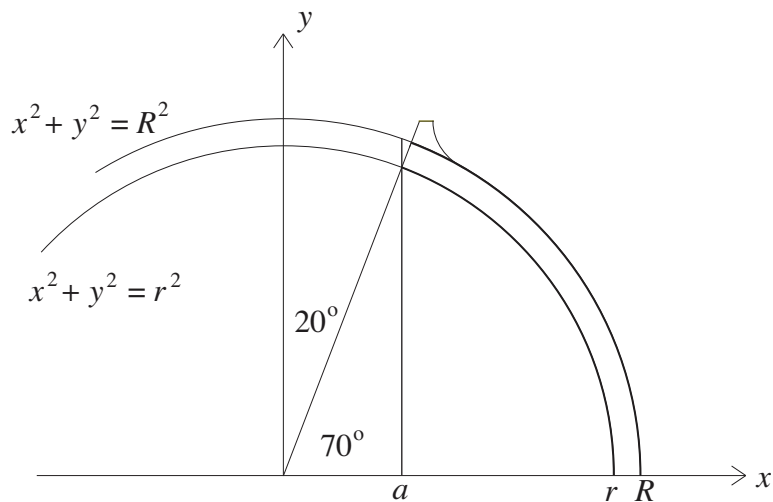
7b. Explain how this volume formula is derived. Start by dividing the interval $[a, b]$ into n equal pieces and make relevant additions to the diagram above. Then develop a certain sum and discuss how it is related both to the volume and to the integral.

8. (20 pts.) Figure a below depicts a model of the shell of the dome of the Hagia Sophia above the gallery of 40 windows. The inner radius of the shell is denoted by r and the angle that determines the extent of the shell is denoted by θ . The horizontal circular cross section of the dome (just above the windows) along with its center are also shown. Figure b depicts a vertical cross section of the shell (the part over the 40 windows) and indicates the thrust P



generated by each of two opposite ribs (of the total of 40).

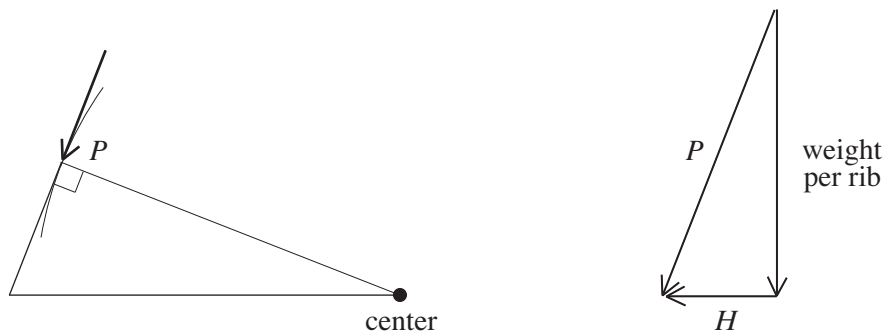
Consider the dome of the Hagia Sophia as it was rebuilt by the year 563 after having been partially destroyed by an earth quake. For it, $r = 50$ feet and $\theta = 140^\circ$. The diagram below and integral calculus was used in the text to derive the estimates of 27,000 cubic feet and of 3,000,000 pounds for, respectively, the volume and the weight of the shell of the rebuilt dome.



Not much is known about the original dome of the Hagia Sophia other than that it was 10 feet lower than the rebuilt second dome. Assume that the basic structure of this original dome was also given by Figure a, that its shell was also $2\frac{1}{2}$ feet thick, that it also weighed 110 pounds per cubic foot, and that the size and configuration of its rib and support structures (including the forty windows) were basically the same as those of the rebuilt dome. Under these assumptions, it can be shown (in reference Figure a), that $r = 60$ feet and $\theta = 104^\circ$ (approximately) for the original dome.

8a. Adapt the figure above and use calculus to estimate first the volume and then the weight of the shell of the original dome.

8b. Use the estimate of 2,560,000 pounds for the weight of the original shell (above the row of windows) and make use of the diagram below to estimate the thrust P of one rib on the original base of the dome as well as its horizontal component H . Compare your results against the



values $P \approx 80,000$ pounds and $H \approx 27,000$ pounds that were derived for the rebuilt dome and explain the differences.