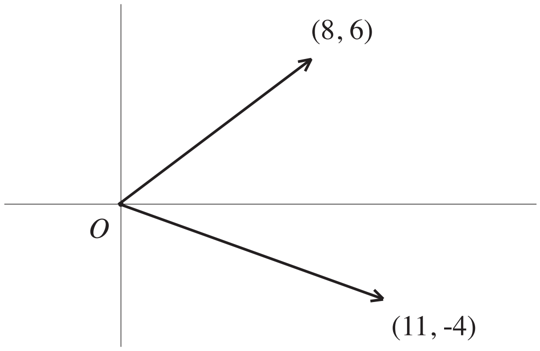
MidTerm Exam Math 10-270, March 10, 2011. Name

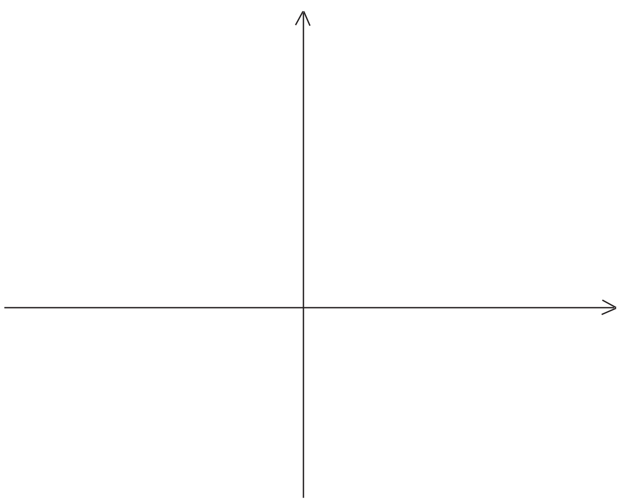
1. (12 pts) Make a careful drawing of a semicircular Roman arch that has 15 identical voussoirs. Assuming that each voussoir weights 350 pounds, compute the outward thrust generated by the top three voussoirs (in each direction).

2. ($12 = 6 + 6$ pts) In the arch considered above, fuse the voussoirs together three at a time to form an arch that has 3 identical voussoirs. Compute the outward thrust that the keystone (it consists of three voussoirs of the previous arch) of this arch generates (in each direction). Explain why your answer is different from the one you derived in Problem 1.

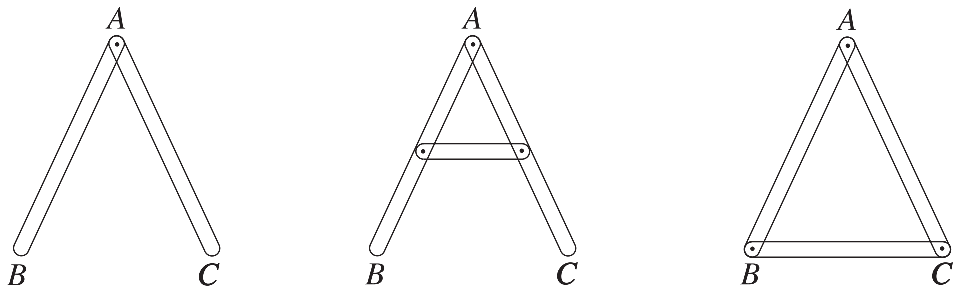
3. (10 pts) The two vectors in the diagram below represent forces. Draw in the resultant of the two forces by using the parallelogram law. Next draw in the horizontal and vertical components of the two original forces. Use the information that they provide to determine the resultant again and to compute its magnitude.



4. (12 pts) Draw the line $x = -5$ into the coordinate plane below and then put in the point $F = (2, 4)$. Find an equation in x and y that a point $P = (x, y)$ has to satisfy so that it lies on the parabola that has the line $x = -5$ as its directrix and the point F as its focus.



Problems 5 and 6 deal with the following: Suppose that the load L on the arch depicted in the figure below consists only of the combined weight of the two slanting members. Let α be the indicated angle, let d be half the span of the arch, let h be the height, and let l be the length of the slanting elements of the arch. Let w be the weight per unit length of these elements.



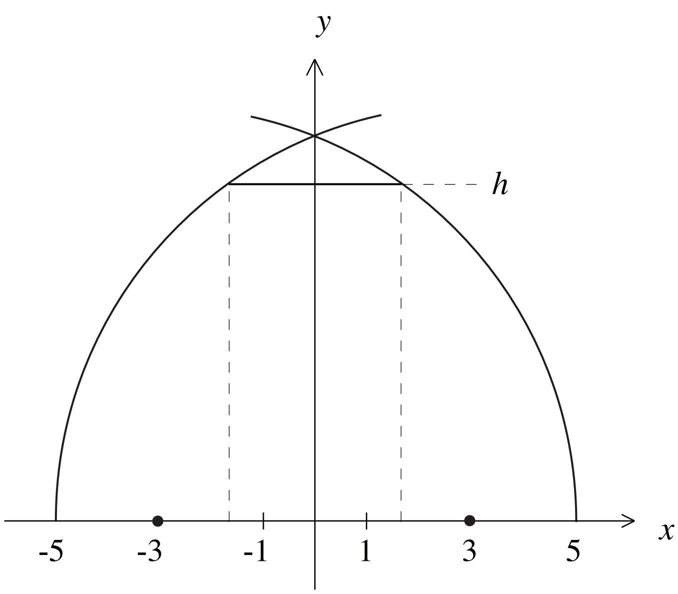
5. (7 pts) Explain why $L \approx 2w\sqrt{d^2 + h^2}$.

6. (15 pts) Let H be the horizontal thrust that the load generates and show that $H \approx wd\sqrt{1 + \frac{d^2}{h^2}}$. Now let w and d be fixed, discuss what happens to both L and H as h varies. Why is this relevant to architecture?

7. (15 pts) What constructional strategy did Brunelleschi deploy that made it possible to build the dome of the Santa Maria del Fiore without using a massive centering structure that reached from

the floor of the cathedral to support the dome.

8. (15 pts) The figure shows an abstract vertical section of the outer surface of a dome in an x-y coordinate plane. It is an arch in the shape of the Gothic fifth with a span of 10 units. What are the equations of the two circles on which the two arcs of the arch lie? The segment with y-coordinate h represents the base of the lantern of the dome. It is $3\frac{1}{3}$ units long. Show that the ratio $\frac{h}{10} \approx 0.65$.



Formulas: $H_0 = \frac{W}{2} \cdot \frac{1}{\tan\frac{\alpha}{2}}$, $H_1 = W \cdot \frac{1}{\tan\frac{3\alpha}{2}}$, $H_2 = W \cdot \frac{1}{\tan\frac{5\alpha}{2}}$, $P_0 = \frac{W}{2} \cdot \frac{1}{\sin\frac{\alpha}{2}}$, $P_1 = W \cdot \frac{1}{\sin\frac{3\alpha}{2}}$, $P_2 = W \cdot \frac{1}{\sin\frac{5\alpha}{2}}$ $P_1P_2 = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ $P_1P_2 = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$.