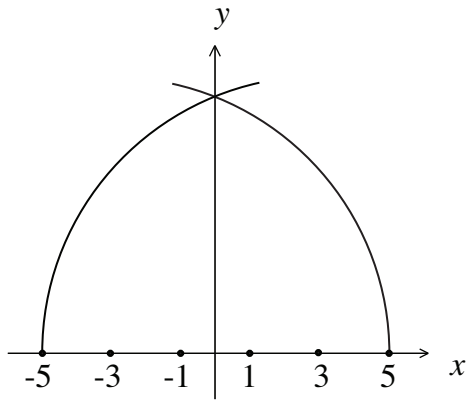**Please Note:** *Calculators may be used only in elementary computational mode. Please note also that for your solutions to receive full credit, they must be complete, precisely formulated, and cast in a well organized, legible way.*

1. (15 pts.) The figure below depicts a pointed *quinto acuto* arch in the coordinate plane.


What are the centers and the radii of the circles on which the two arches lie? Use the equations of these circles to determine the height of the arch.

2. (10 pts) What were the two unprecedented demands - one having to do with the stability of the structure of the dome and the other with its construction - that Brunelleschi faced in the building of the dome of the Santa Maria del Fiore.

3a. (10 pts) Consider a floor of 36 identical square tiles along with a coordinate system as the figure
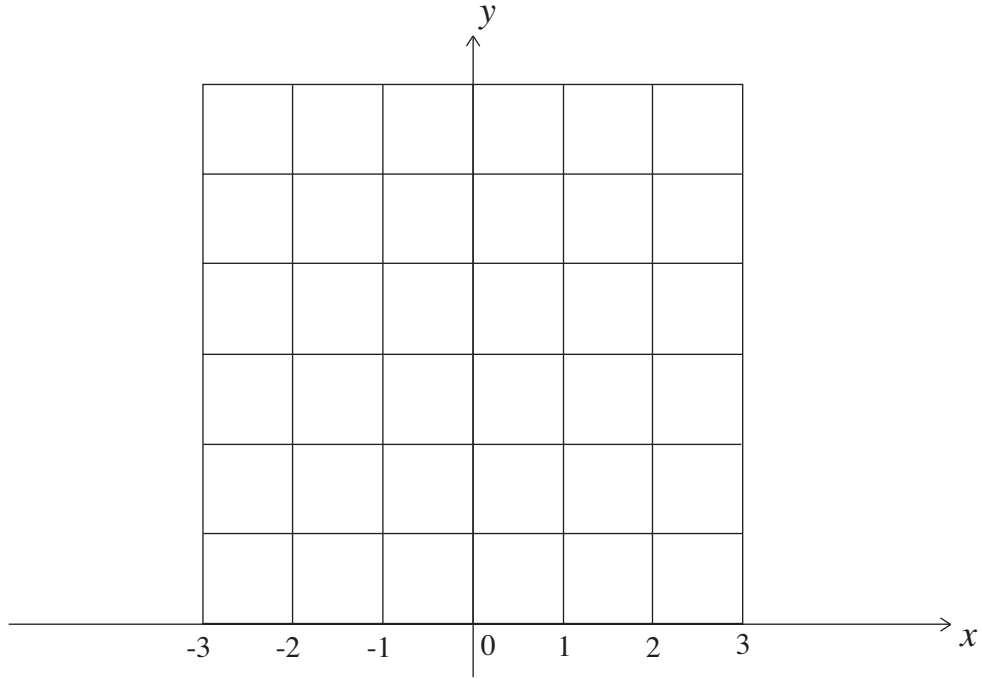
above provides it. In the figure below reproduce Alberti's perspective drawing of the floor. Include the vanishing point $C = (0, c)$ and the secondary vanishing point $D = (-d, c)$.
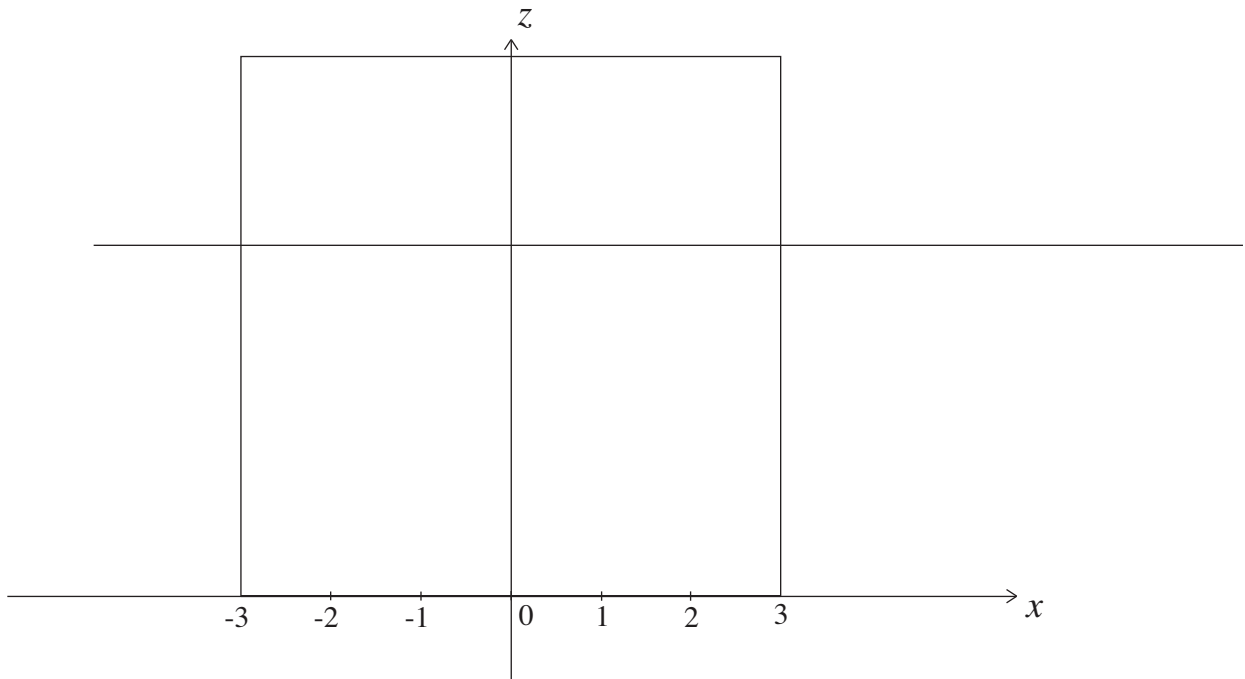
3b. (20 pts) Describe with precision and in detail the meaning of the equations $x_1 = \frac{x_0 d}{d+y_0}$ and $z_1 = \frac{e y_0}{d+y_0}$ *in the context of the floor and the drawing* and *explain the strategy that was used to derive them.* Include explanations of the constants d and e.

4. (20 pts.) Describe Michelangelo's most important contributions to the design of Saint Peter's.

5. (10 pts) Find the two points P on the circle $x^2 + y^2 = 9$ with the property that the slope of the tangent line (to the circle) at P is 2.

6. (20 pts.) Consider $f(x) = \frac{1}{x^2}$.
   6a. Use the *limit definition* of the derivative to show that $f'(x) = -\frac{2}{x^3}$.

   6b. Explain why $\frac{1}{(x+dx)^2} - \frac{1}{x^2} \approx -\frac{2}{x^3} dx$ for any $x \neq 0$ and any very small dx (with $x + dx \neq 0$).

   6c. How good is the approximation of 6b for $x = 3$ and $dx = 0.001$?

7. (25 pts.) Consider the function $f(x) = 9 - x^2$ with $0 \le x \le 3$.

   i. Take $n = 6$ and compute the sum that arises in the definition of the integral $\int_0^3 (9 - x^2)\, dx$. Do so with two decimal place accuracy.

   ii. Why is this sum only a rough approximation of $\int_0^3 (9 - x^2)\, dx$?

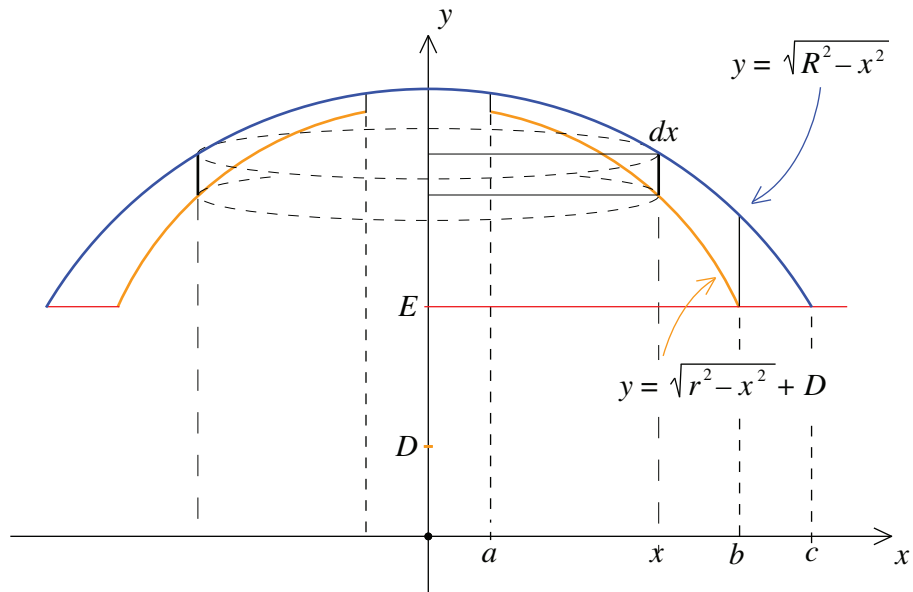   iii. Use the Fundamental Theorem of Calculus to find the precise value of this integral.

   iv. Sketch the area that you computed in iii.

8a. (10 pts) Let $y = f(x)$ be a continuous function defined on the interval $a \leq x \leq b$ and assume that $f(x) \geq 0$ for all such x. Explain the meaning of the formula $V = \int_a^b \pi(f(x))^2 \, dx$.
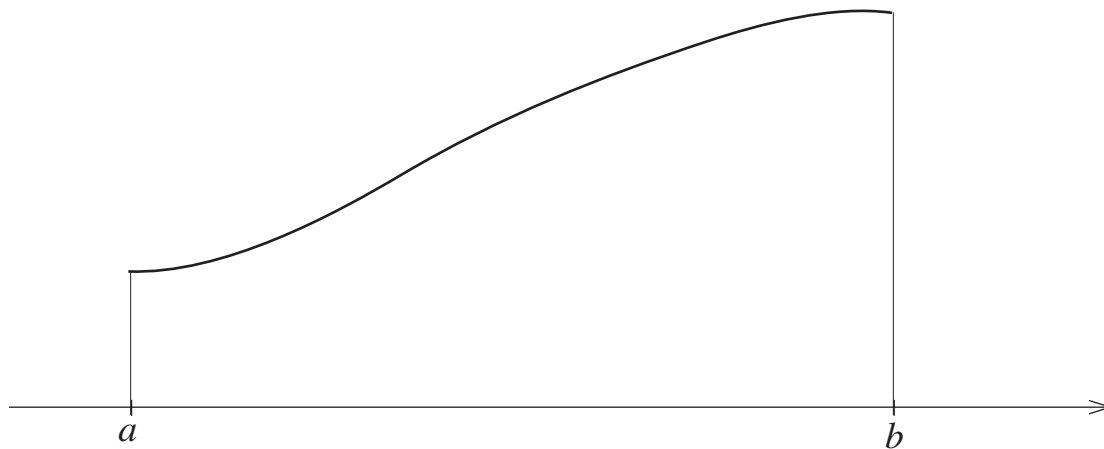
8b. (15 pts) Explain by expanding the diagram above and by *making use of the definition of the definite integral* how this formula is derived.

9. (15 pts) Make use of the diagram below to express the volume of the shell of Pantheon

The diagram shows a coordinate system with y-axis vertical and x-axis horizontal. A blue outer curve labeled $y = \sqrt{R^2 - x^2}$ and an orange inner curve labeled $y = \sqrt{r^2 - x^2} + D$. A horizontal red line is at $y = E$. A shell element of thickness dx is shown. Points on the x-axis are marked a, x, b, c. Values D and E are marked on the y-axis.

(above the line $y = E$) as a sum of two definite integrals. Explain how you got your answer.