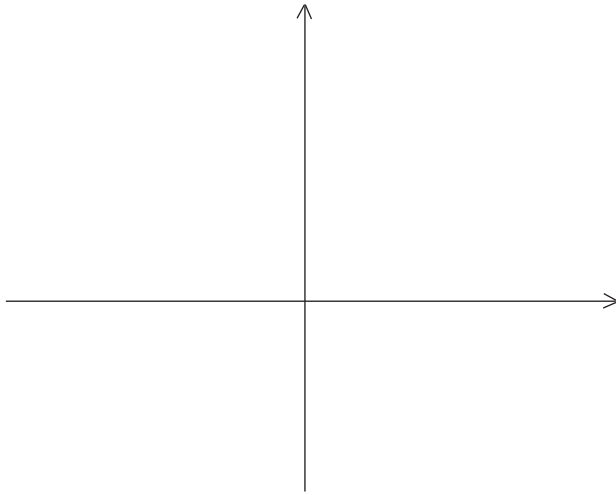


MidTerm Exam Math 10-270, March 7, 2012. Name

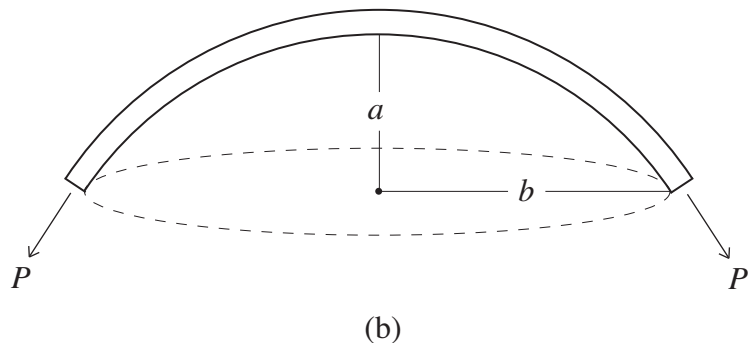
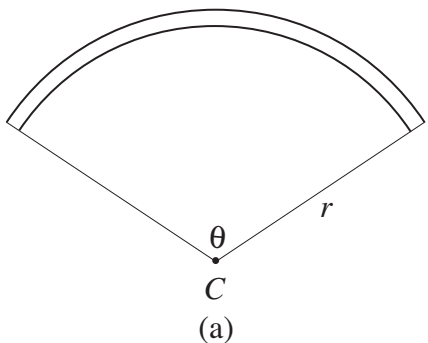
1. (15 pts) Make a careful drawing of a semicircular Roman arch that has 9 identical voussoirs. Assuming that each voussoir weights 350 pounds, compute the outward thrust generated by the top three voussoirs. (It suffices to consider just one direction.)

2. (15 pts) In the arch considered above, fuse the voussoirs together three at a time to form an arch that has 3 identical voussoirs. Compute the outward thrust that the keystone (it consists of the three top voussoirs of the previous arch) of this arch generates. (One direction is sufficient.) Explain by commenting on the difference between the structure of the two arches why your answer is less than the one you derived in Problem 1.

3. (15 pts) Draw the line $x = -4$ into the coordinate plane below and then put in the point $(3, 1)$. Sketch a graph of the parabola in the xy -plane below that has $x = -4$ as directrix and the point $F = (3, 1)$ as focus. Determine an equation in x and y that a point $P = (x, y)$ has to satisfy so that it lies on the parabola. Determine the points of the parabola that lie on the vertical line through F .



About the original dome of the Hagia Sophia. History tells us that an earthquake caused the partial collapse of the original dome of the Hagia Sophia soon after its construction was completed and that it was rebuilt soon thereafter. Some historians assert that the original dome was lower and flatter than the rebuilt dome, but that it was similar in design to the rebuilt dome. Accordingly, we will assume that the figure below depicts the essential structure of the original dome, that the radius of its circular base was the same $b = 47$ as that of the rebuilt dome, and that the distance from the circular base to the top of the inside of the shell was $a = 13$ feet, or 20 feet less than that of the rebuilt dome. We also assume that the original dome had a rib structure with 40 ribs.



4. (12 pts) Given the assumptions that have been made, show that for the original dome of the Hagia Sophia $r = 91$ feet and $\theta = 62^\circ$ (both approximately). [Hint: Use the Pythagorean Theorem to find r .]

Using the fact that the shell of the original dome was $2\frac{1}{2}$ feet thick (the same as for the rebuilt dome), one can derive the estimate of 21,100 cubic feet for its volume. With the assumption that the original dome was made of brick and mortar weighing 110 pounds per cubic foot (as in the rebuilt dome) this implies the estimate of 2.32 million pounds for the weight of the original dome.

5. (10 pts) Draw a force diagram depicting the downward slanting push P of a typical rib (as shown in the figure), its horizontal component H (this is the horizontal thrust that each rib generates), and its vertical component L (this is the weight of the dome per rib). Use the information in Problem 4 to determine the relevant angles of the force diagram.

(10 pts) Use this force diagram to derive an estimate for H . [Your answer for H should be more than 3 times greater than the estimate $H = 27,000$ pounds derived in the text for the horizontal thrust (per rib) of the rebuilt dome.] Describe the implications of this much greater horizontal thrust, both in terms of the requirements of the base on the original dome and the problem of hoop stress.

6. (20 pts) Explain in detail the geometry of the inner surface of the inner shell of the dome of Florence. Your explanation should include and be based on carefully drawn diagrams in both the coordinate plane and coordinate three space. [Since the geometry, rather than the dimensions, is the focus of the question, assume that the radius of the circle in which the octagon of the drum is inscribed is 1 unit.]

$$\text{Formulas: } H_0 = \frac{W}{2} \cdot \frac{1}{\tan \frac{\alpha}{2}}, \quad H_1 = W \cdot \frac{1}{\tan \frac{3\alpha}{2}}, \quad H_2 = W \cdot \frac{1}{\tan \frac{5\alpha}{2}}, \quad P_0 = \frac{W}{2} \cdot \frac{1}{\sin \frac{\alpha}{2}}, \quad P_1 = W \cdot \frac{1}{\sin \frac{3\alpha}{2}}, \\ P_2 = W \cdot \frac{1}{\sin \frac{5\alpha}{2}}, \quad \sin \alpha = \frac{L/2}{P}, \quad \tan \alpha = \frac{L/2}{H}, \quad L \approx 2w\sqrt{d^2 + h^2}, \quad H \approx wd\sqrt{1 + \frac{d^2}{h^2}}.$$