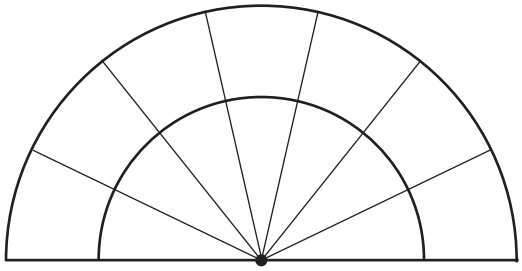
Final Exam. Math 10-270. May 11, 2012. Name

Note: *calculators may not be used to solve calculus problems.*

Problems 1 and 2 deal with a semi-circular Roman arch that has 7 identical voussoirs as shown in the figure. The inner and outer radii of the arch are 6 and 8 units long,



respectively. Each voussoir weighs 175 pounds. The voussoirs are the only load on the arch.

1. Of the numbers listed below which is the best estimate of the horizontal thrust that the top three voussoirs of the arch (the keystone is one of them) generate in each direction?

   A. 219 pounds B. 383 pounds C. 1206 pounds D. 603 pounds * E. 688 pounds

2. As to the possibility of constructing the above diagram of the arch with a straight edge and compass, which one of the following is correct? (You are given a segment of length 1.)

   A. The entire diagram can be constructed

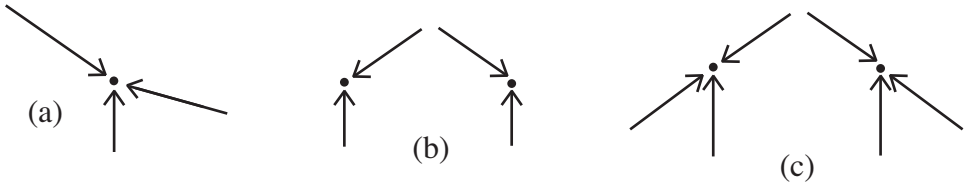
   B. The diagram cannot be constructed because angles cannot be trisected

   C. The two semicircles can be constructed, but the voussoirs cannot *

   D. Neither the semicircles nor the voussoirs can be constructed

   E. Not enough information is supplied to answer the question

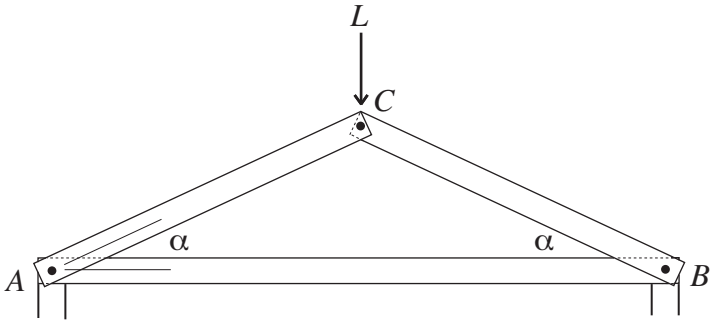
Problems 3 and 4 deal with vectors and forces. All vectors in the diagrams (a), (b), and (c) below represent forces. All forces acting at the indicated points are shown, and all are relevant to the structure that is represented. In each case, the units are chosen so that the magnitude of the force is equal to the length of the vector.



3. Which of the following answers provide correct information about the structural scheme of a stable Gothic cathedral:

   A. only (a) * B. only (b) C. only (c) D. both (a) and (b) E. both (a) and (c)

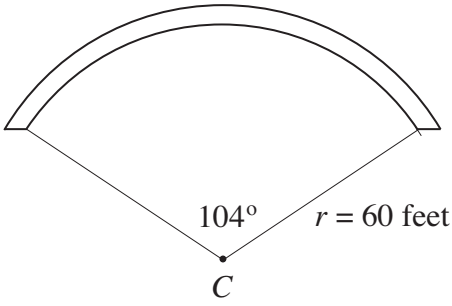
Consider the simple truss shown below. Assume that $L = 10{,}000$ pounds and that $\alpha = 25°$.

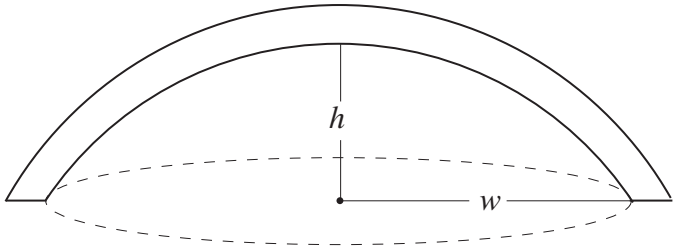


4. The tie beam pulls at the point A with a force (in pounds) of approximately

   A. 17,556 B. 11,833 C. 10,723 * D. 23,662 E. 21,445

Problems 5 and 6 deal with the shell of a dome. The figures below depict the vertical cross section of a shell of a dome much like that of the Hagia Sophia above its gallery of 40 windows. Between each pair of windows is a load bearing rib. The inner and outer boundaries of the shell lie on concentric spheres. In the figure on the left, C is the common center of the two spheres, $r = 60$ feet is the radius of the inner sphere, and $\theta = 104°$ is the angle that determines the extent of the shell. The horizontal circular base of the shell along with its center are





shown in the figure on the right. The circular base has radius $w = 47$ feet and its distance from the top of the inside of the shell is $h = 23$ feet. It is known that the shell is 2 feet thick.

5. If the shell weighs 2,400,000 pounds, then the best estimate for the horizontal thrust generated by each of the 40 ribs is:

   A. 77,000 pounds B. 47,000 pounds * C. 57,000 pounds D. 67,000 pounds
   E. due to insufficient evidence no estimate can be provided

6. Which of the expressions below provides the best estimate for the volume of the shell of the dome (with cross section provided by the figures).

   A. $\displaystyle\int_0^{47} \pi\sqrt{62^2 - x^2}\,dx - \int_0^{47} \pi\sqrt{60^2 - x^2}\,dx$ B. $\displaystyle\int_{23}^{62} \pi(62^2 - x^2)\,dx - \int_{23}^{60} \pi(60^2 - x^2)\,dx$

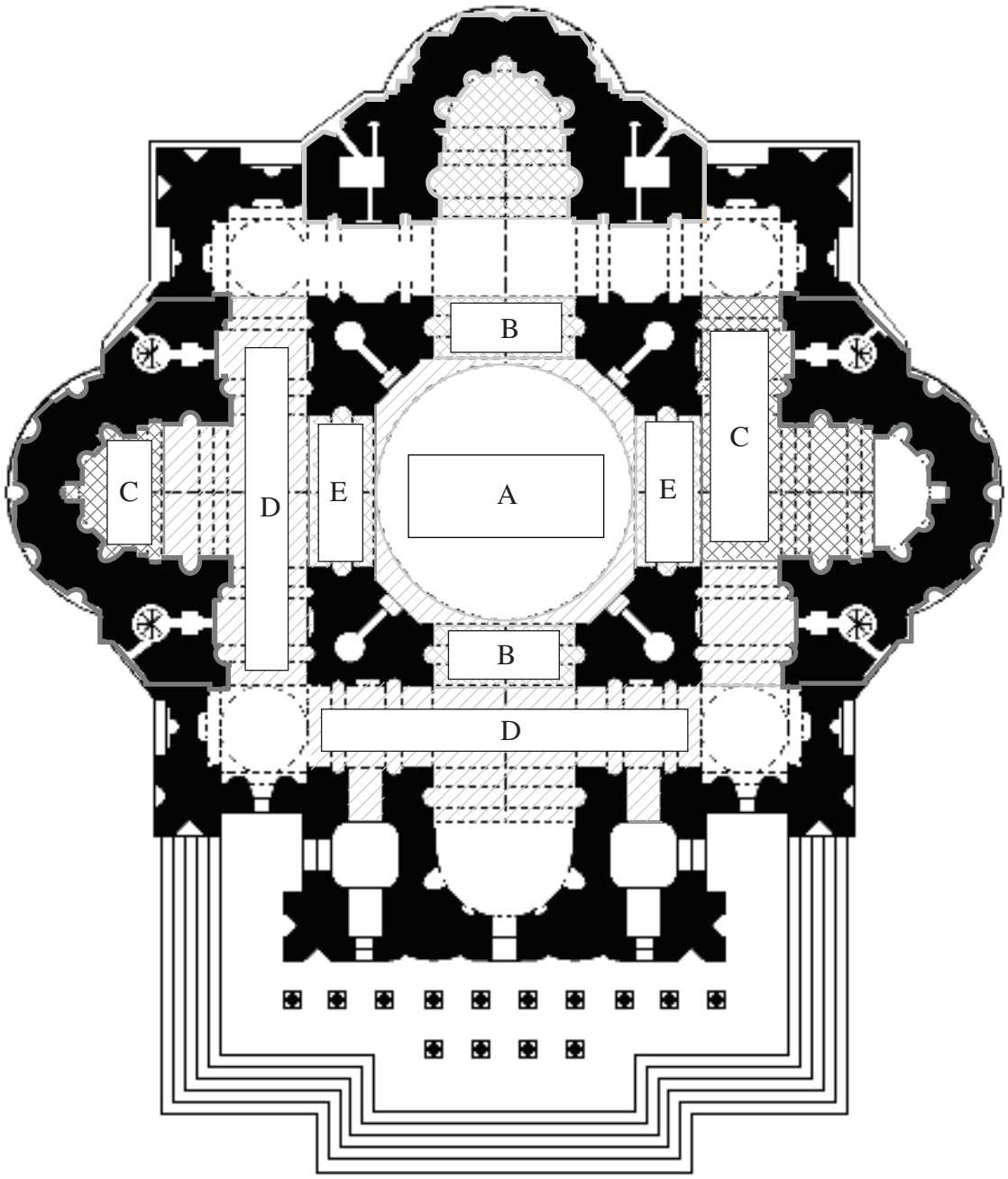
   C. $\displaystyle\int_{37}^{62} \pi(62^2 - x^2)\,dx - \int_{37}^{60} \pi(60^2 - x^2)\,dx$ * D. $\displaystyle\int_{47}^{62} \pi(62^2 - x^2)\,dx - \int_{47}^{60} \pi(60^2 - x^2)\,dx$

   E. $\displaystyle\int_0^{23} \pi(62^2 - x^2)\,dx - \int_0^{23} \pi(60^2 - x^2)\,dx$

7. In an xy-plane you are given the circle of radius 2 with center the origin and a line through $(0, 5)$ with slope m. The line goes through the circle if m satisfies the condition:

   A. $m > \frac{\sqrt{21}}{5}$ and $m < -\frac{\sqrt{21}}{5}$ but no other m B. $m > \frac{5}{4}$ and $m < -\frac{5}{4}$ but no other m
   C. $m > \frac{\sqrt{21}}{4}$ and $m < -\frac{\sqrt{21}}{4}$ but no other m D. $m > \frac{\sqrt{21}}{10}$ and $m < -\frac{\sqrt{21}}{10}$ but no other m
   E. $m > \frac{\sqrt{21}}{2}$ and $m < -\frac{\sqrt{21}}{2}$ but no other m *

Problems 8 and 9 deal with the plan below:



8. It is a plan for

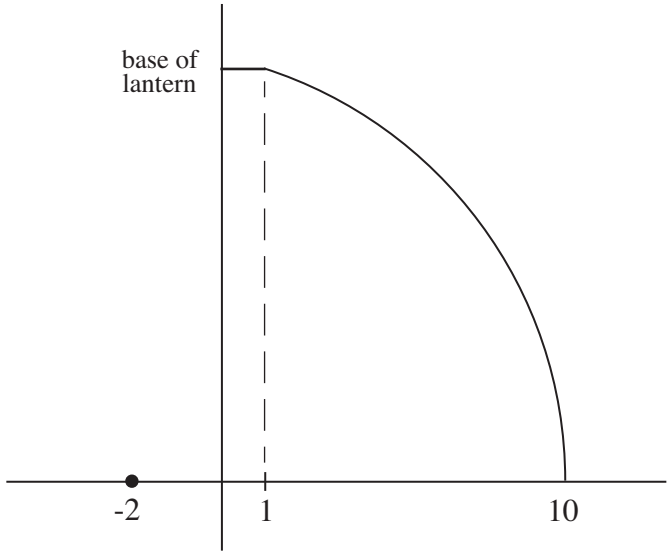
   A. the Milan cathedral B. the Santa Maria del Fiore C. the old St. Peter's Baslica
   D. San Marco in Venice E. the new St. Peter's Basilica *

Consider the following architects: 1. Alberti, 2. Bernini, 3. Bramante, 4. Brunelleschi, 5. della Porta, 6. Maderno, 7. Michelangelo, 8. Palladio, 9. Raphael, and 10. Sangallo.

9. Each of the boxes labelled A, B, C, D, and E of the plan refers to a section of the structure constructed by one of the architects on the list. Place into the listing below the number that identifies the architect responsible for building the corresponding section of the structure.

   A. B. C. D. E.

Problems 10 and 11 deal with the cross section of the interior of the shell of a dome like that of the Santa Maria del Fiore. The diagram in the figure below consists of an xy-axis system (the meter is the operative unit of length) and an arc of the circle of radius 12 centered at the point $x = -2$. The base of the lantern is also depicted. The x-axis is the springing line of the circular arc. The (vertical) distance from the springing line to the base of the lantern is the interior height h of the dome. The y-axis is the vertical central axis of the dome.



10. The height to span ratio of the interior of the dome is

    A. $\frac{\sqrt{139}}{20}$ B. $\frac{\sqrt{135}}{20}$ * C. $\frac{\sqrt{133}}{20}$ D. $\frac{\sqrt{131}}{20}$ E. $\frac{\sqrt{137}}{20}$

11. The integral that expresses the length of the circular arc of the cross section from the springing to the base of the lantern is:

    A. $\int_1^{10} \sqrt{1 + \frac{(x+2)^2}{140-x^2-4x}}\, dx$ * B. $\int_1^{10} \sqrt{1 + \frac{(x+1)^2}{140-x^2-4x}}\, dx$ C. $\int_1^{10} \pi(144 - (x+2)^2)\, dx$

    D. $\int_1^{10} \sqrt{1 + \frac{1}{140-x^2-4x}}\, dx$ E. $\int_1^{10} \sqrt{1 + \frac{2(x+1)^2}{140-x^2-4x}}\, dx$ F. $\int_1^{10} \sqrt{1 + \frac{\frac{1}{2}(x+1)^2}{140-x^2-4x}}\, dx$

12. The area under the graph of $y = \sqrt{x}$ and over the interval from $x = 0$ to $x = 4$ is:

    A. $\frac{14}{3}$ B. $\frac{17}{3}$ C. $\frac{16}{3}$ * D. $\frac{13}{3}$ E. $\frac{19}{3}$