**MidTerm Exam Math 10-270, March 5, 2014. Name**

*Show your work and place your final answer into the box provided.*

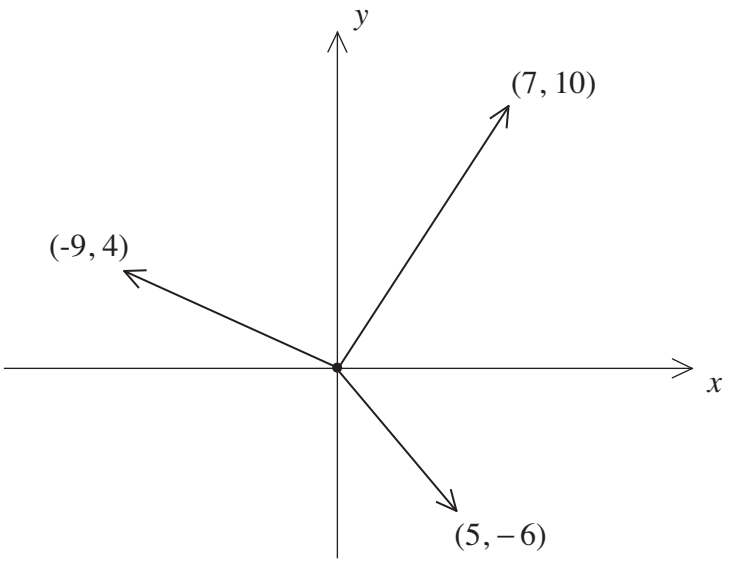
**1.** (15 pts) Make a careful drawing of a semicircular Roman arch that has 9 identical voussoirs. Assuming that each voussoir weights 350 pounds, compute the outward horizontal $H$ thrust generated by the top three voussoirs in each direction. (So it suffices to consider just one direction.) Can an accurate version of this drawing be executed with straightedge and compass?

$H =$

**2.** (10 pts) In the arch considered above, fuse the voussoirs together three at a time to form an arch that has 3 identical voussoirs. Compute the outward thrust that the keystone (it consists of the three top voussoirs of the previous arch) of this arch generates. (Again, one direction is sufficient.) Explain by addressing the structural difference between the two arches, why the horizontal thrust is less in the fused situation.

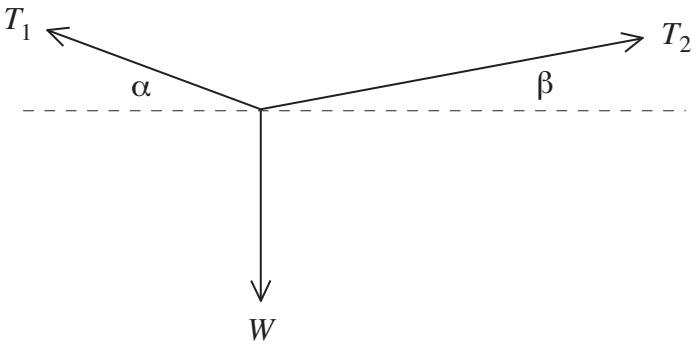
$H =$

**3.** (15 pts) The vectors in the $xy$-plane below represent forces. Compute their resultant $R$.




$R =$

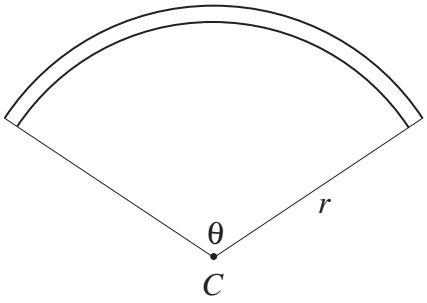
**4.** (15 pts) The diagram below depicts a cable that is firmly attached on both sides and a weight $W$ that is suspended from it. The angles of the two parts of the cable with the horizontal are $\alpha$ and $\beta$ and $T_1$ and $T_2$ are the respective tensions. Determine equations that express both $T_1$ and $T_2$ in terms of $W$, $\alpha$, and $\beta$. Compute $T_1$ and $T_2$ for $W = 450$ pounds, $\alpha = 25°$ and $\beta = 15°$. (Insert these answers into the boxes.)



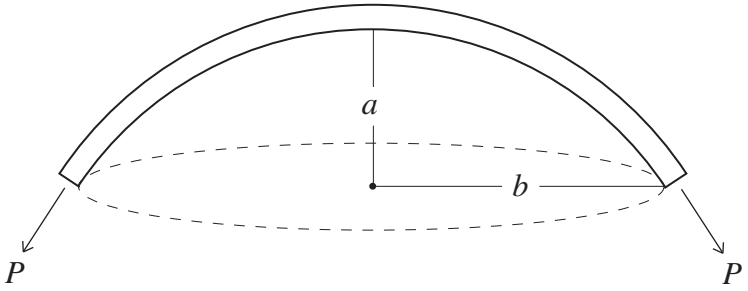

$T_1 =$

$T_2 =$

Figures (a) and (b) depict the essential structure of the original dome of the Hagia Sophia. Figure (a) shows the circular cross-section of its shell and its radius $r$, and Figure (b) marks the circular base of the dome with its radius $b = 47$ feet and the distance $a = 13$ feet from the center of the base to the top of the inside of the shell. Like the current dome, the original dome had 40 ribs.



(a)



(b)

**5.** (10 pts) Using the data supplied above, determine $r$ and $\theta$ for the original dome of the Hagia Sophia (both approximately).

| $r =$ | $\theta =$ |
|---|---|

Assume that the shell of the original dome was $2\frac{1}{2}$ feet thick and that it weighed 110 pounds per cubic foot (both the same as the current dome), and that 21,100 cubic feet is an estimate of its volume.

**6.** (10 pts) Estimate the total weight of the original shell. Then draw a force diagram depicting the downward slanting push $P$ of a typical rib (as shown in Figure (b)), its horizontal component $H$, and its vertical component. Use your force diagram to derive an estimate for $H$.

| $H =$ |
|---|

**7.** (15 pts) The number line and the Cartesian coordinate system are two of the most basic and important constructions in mathematics. Describe in specific terms the roadblock that the Greeks encountered that prevented them from making these discoveries.

**Formulas (some relevant, some not):** $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$ $c^2 = a^2+b^2-2ab\cos\theta$ $H_0 = \frac{W}{2}\cdot\frac{1}{\tan\frac{\alpha}{2}}$, $H_1 = W\cdot\frac{1}{\tan\frac{3\alpha}{2}}$, $H_2 = W\cdot\frac{1}{\tan\frac{5\alpha}{2}}$, $P_0 = \frac{W}{2}\cdot\frac{1}{\sin\frac{\alpha}{2}}$, $P_1 = W\cdot\frac{1}{\sin\frac{3\alpha}{2}}$, $P_2 = W\cdot\frac{1}{\sin\frac{5\alpha}{2}}$, $\sin\alpha = \frac{L/2}{P}$, $\tan\alpha = \frac{L/2}{H}$, $L \approx 2w\sqrt{d^2+h^2}$, $H \approx wd\sqrt{1+\frac{d^2}{h^2}}$ $y = mx+b$ $(x-h)^2+(y-k)^2 = r^2$