Math 10270 : Final
May 12, 2017

Name:

_____

Calculators are allowed, together with an $8\frac{1}{2}'' \times 11''$ sheet of paper
with notes in your own handwriting.

Sign the pledge. "On my honor, I have neither given nor received
unauthorized aid on this Exam":

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1. (10 points.) Explain how to bisect a given angle using only a straightedge and compass.

Describe each stage carefully and illustrate the steps with sketches.

2. (10 points.) (i) A Gothic arch has a span of 50ft. Find the height of the arch at its highest point.

(ii) Repeat part (i) for a Roman arch of span 50 ft.

(iii) Repeat part (i) for a Gothic acute fifth with span 50 ft.

3. The distance from the focus F of a parabola to its directrix is 2 units. Suppose the parabola is cut by a line parallel to the directrix and 5 units away. Use the result of Archimedes to determine the area of the parabolic section.

4. (10 points.) What shape in the plane is described by the equation $(x-1)^2 + 2(y-2)^2 = 2$? Sketch the graph and label the major and minor axes and their lengths. Sketch also the line $y = x + 3$. Find the points of intersection with the graph just drawn.

5. (10 points.) Talk about some features distinguishing classical and Renaissance architecture.

6. (10 points.) (i) Find parametric equations for the line in space between the points $(1, 2, 3)$ and $(-1, 1, 2)$.

(ii) Find the points where the line intersects the sphere $x^2 + y^2 + z^2 = 10$.

7. (10 points.) (i) Find the intersection point of the two lines in the plane given by $y = 2x + 1$ and $y = 7 - x$.

(ii) Suppose that an artist's eye is at the point $(0, -2, 5)$ and a canvas lies in the xz-plane. Find the location of the image on the canvas of the intersection point on the xy-plane found in part (i).

(iii) Sketch the images of the two lines from part (i) on the canvas. Label their intersection point and limiting points on the horizon.

8. (10 points.) (i) What is hinging failure in an arch? Draw a sketch.

(ii) Give a brief statement of the Safe Theorem.

(iii) What is the relation of this to the dome of St. Peter's?

9. (10 points.) Consider the hanging truss shown.

(i) Assuming the truss to be symmetric and in equilibrium find the vertical components of the forces in each beam.

(ii) Suppose that the angle θ_1 is 30 degrees. Find the horizontal components of the forces and the angle θ_2.

10. (10 points.) A dome is formed by rotating about the z-axis the region in the xz-plane bounded by the curves

$$x^2 + z^2 = 2500; \qquad x^2 + z^2 = 2025; \qquad z = 0; \qquad x = 5.$$

(i) Find the volume of the dome.

(ii) If the dome is made of concrete of uniform density $150\text{lb}/\text{ft}^3$, find the weight of the dome.

Assume that x and z give lengths in feet.