

Economics 33530: Assignment 5 Key  
(due Friday, November 30)

Professor Jensen

Fall 2007

1. Assume the annual increment of a pine forest as a function of its age  $t$  is

$$MP = 1000 + 400t - 30t^2$$

and its mean annual increment is

$$AP = 1000 + 200t - 10t^2.$$

Determine the age  $A^*$  at which the flow of wood that can be harvested from this forest over time is maximized.

**This occurs at the age  $t$  where annual increment = mean annual increment, or  $1000+400t-30t^2 = 1000+200t-10t^2$ , which reduces to  $200t-20t^2=0$ . The roots of this are 0 and 10, so  $A^* = 10$ .**

2. Assume the value of a pine forest as a function its age  $t$  is given by

$$V = 400 + 1000t - 10t^2$$

so that its rate of change over time is

$$\Delta V/\Delta t = 1000 - 20t.$$

Also assume the interest rate is  $r=0.1$  and the opportunity cost of the land is  $OCL = 1585$ .

Determine the optimal rotation  $R^*$  that maximizes the net benefits of harvesting this forest.

**This occurs at the age where the benefits of waiting to harvest = the costs of waiting, or  $\Delta V/\Delta t = rV + OCL$ , which in this case is  $1000 - 20t = (.1)(400+1000t - 10t^2) + 1585$ . This simplifies to  $t^2 - 120t - 625=0$ , which has roots 125 and  $-5$ , so  $R^*=125$ .**