1. Assume two firms produce an identical product with market demand curve 

\[ q = D(p) = 2 - p, \]

where \( q \) is total quantity sold and \( p \) is price. Each firm i can produce at a constant average and marginal cost of 1 up to its production capacity \( x_i < 1/4 \); that is, its cost function is

\[ C_i(q_i) = q_i \quad \text{for} \quad q_i < x_i \]
\[ = \infty \quad \text{otherwise}. \]

a. Show that, under proportional rationing, \((p_1, p_2) = (p^*, p^*)\), where \( p^* = 2 - (x_1 + x_2) \), is a Bertrand equilibrium.

b. Determine the corresponding equilibrium profits.

2. Two firms produce differentiated products with inverse demand given by

\[ P_1 = 10 - 2q_1 - q_2 \]
\[ P_2 = 10 - 2q_2 - q_1 \]

where \( P_i \) and \( q_i \) are firm i’s price and output, and \( A \) is a positive constant. Firm i’s total cost function is

\[ C_i(q_i) = q_i. \]

a. Assume the firms compete in quantities. Determine the Nash equilibrium quantities, and the corresponding prices and profits.

b. Assume firms compete in prices instead. Determine the Nash equilibrium prices, and the corresponding quantities and profits.

c. Compare your answers to these questions, and explain any differences.