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THE STRAIGHT DOPE®

On "Let's Make a Deal," you pick Door #1. Monty opens Door #2--no prize. Do you stay with Door #1 or switch to #3?

02-Nov-1990

Dear Cecil:

I was perversely flipping through the Parade section of my Sunday newspaper when I stumbled upon Marilyn vos Savant's "Ask Marilyn" column. Even more perversely, I read it. It wasn't a total loss, though, because it appears she made another mistake, even worse than the one you pointed out in a very entertaining column a few months ago. Here's the question:

Suppose you're on a game show and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

ANSWER: Yes; you should switch. The first door has a one-third chance of winning, but the second door has a two-thirds chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door No. 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?

Correct me if I'm wrong, Cecil, but aren't the odds equal for the remaining doors--one in two? --Michael Grice, Madison, Wisconsin

P.S.: If the questions she answers are any indication of the intellect of the general population, this country is in a lot of trouble.

Dear Michael:

This is getting ridiculous. You're perfectly correct. If there are three doors your chances of picking the right one are one in three. Knock one out of contention and the chances of either of the remaining doors being the right one are equal--one in two. This business about a million doors is a bit of pretzel logic that maybe only somebody with the world's highest IQ (according to *Guinness*, anyway) can properly appreciate. *Parade*'s editors really ought to read the copy before they put it in the magazine.

WHOOPS

Dear Cecil:

I certainly won't be the only one to catch your latest error, but the perverse joy I get pointing it out offsets the small chance this letter has of being printed. I refer to your answer to Michael Grice's question about the game show conundrum--one prize, three doors, you pick Door #1, the host opens Door #3 to reveal no prize. Should you switch to the remaining door or stick with your original choice? You agreed with Grice that the odds of winning are equal for both--one in two. Wrong! The easiest route to the truth is to notice that resolving never to switch is equivalent to not having the option to switch, in which case, I'm sure you'll agree, the odds of winning remain one in three. Switching, therefore, has a two-thirds chance at the prize.

Your mistake was not realizing that opening Door #3 tells you more about Door #2 than about the door you originally picked. The reason for this is subtle. The host, in picking Door #3, does not choose from the full set of doors but rather from the subset of doors you did not pick. Each subset's probability of winning does not change but the probability for a particular door in the second subset does. If you don't get it find a friend who looks like Monty Hall and play 20 rounds. It will soon become obvious which strategy wins most often. --Robert E. Johanson, Chicago

Cecil replies:

Hmm. I'll admit I wasn't paying much attention when I wrote that column, assumed this was another instance of carelessness on Marilyn vos Savant's part, and fell into a sucker's trap. But now that I've had a chance to study the matter, it's apparent there is a subtlety that you eluded you as well.

First, though, I feel obliged to eat a bit of crow. The "common sense" answer, the one I gave, is that if you've got two doors and one prize, the chances of picking the right door are 50-50. Given certain key assumptions, which we'll discuss below, this is wrong.

Why? A different example will make it clear. Suppose our task is to pick the ace of spades from a deck of cards. We select one card. The chance we got the right one is 1 in 52. Now the dealer takes the remaining 51 cards, looks at them, and turns over 50, none of which is the ace of spades. One card remains. Should you pick it? Of course. Why? Because (1) the chances were 51 in 52 that the ace was in the dealer's stack, and (2) the dealer then *systematically eliminated all (or most) of the wrong choices.* The chances are overwhelming--51 out of 52, in fact--that the single remaining card is the ace of spades.

Which brings me to the subtlety I mentioned earlier. Your analysis of the game show question is correct, Bobo, only if we make several assumptions: (1) Monty Hall knows which door conceals the prize; (2) he only opens doors that do NOT conceal the prize; and (3) *he always opens a door.* Assumptions #1 and #2 are reasonable. #3 is not.

Monty Hall is not stupid. He knows, empirically at least, that if he *always* opens one of the doors without a prize behind it, the odds greatly favor contestants who switch to the remaining door. He also knows the contestants (or at least the highly vocal studio audience) will tumble to this eventually. To make the game more interesting, therefore, a reasonable strategy for him would be to open a door *only when the contestant has guessed right in the first place.* In that case the contestant would be a fool to change his pick.

But that's absurd, you say. If Monty only opened a door when you'd chosen correctly in the first place, no one would ever switch. Exactly--so it's likely Monty adds one last twist. Most of the time he only opens a door when you've chosen correctly--*but not always*. In other words, he tries to bluff the contestants, then counterbluff them.

This strategy changes the odds dramatically. In fact, it can be shown that if Monty always opens a door when the contestant is right and half the time when he's wrong--a perfectly rational approach--over the long haul the odds of the prize being behind Door #1 versus Door #2 are 50-50.

The lesson here is that probability isn't the cut-and-dried science you might assume from high school math class. Instead it involves a lot of educated guesses about human behavior. I'll admit I jumped to an unwarranted conclusion on this one. Don't be too sure you haven't done the same.

THE LAST YOU'LL EVER HAVE TO READ ABOUT THIS

Dear Cecil:

To beat the dead horse of Monty Hall's game-show problem: Marilyn was wrong, and you were right the first time ... --Eric Dynamic, Berkeley, California

Dear Cecil:

You really blew it. As any fool can plainly see, when the game-show host opens a door you did not pick and then gives you a chance to change your pick, he is starting a new game. It makes no difference whether you stay or switch, the odds are 50-50. --Emerson Kamarose, San Jose, California

Dear Cecil:

Suppose our task is to pick the ace of spades from a deck of cards. We select one card. The chance we got the ace of spades is 1 in 52. Now the dealer takes the remaining 51 cards. At this point his odds are 51 in 52. If he turns over 1 card which is not the ace of spades our odds are now 1 in 51, his are 50 in 51. After 50 wrong cards our odds are 1 in 2, his are 1 in 2. The idea that his odds remain 51 in 52 as more and more cards in his hand prove wrong is just plain crazy. --John Ratnaswamy, Chicago; similarly from Greg, Madison, Wisconsin; Stuart Silverman, Chicago; Frank Mirack, Arlington, Virginia; Dave Franklin, Boston; many others

Cecil replies:

Give it up, gang. It was bad enough that I screwed this up. But you guys have had the benefit of my miraculously lucid explanation of the correct answer! Since you won't listen to reason, all I can tell you is to play the game and see what happens. One writer says he played his buddy using the faulty logic in my first column and got skunked out of the price of dinner. Several other doubters wrote computer programs that, to their surprise, showed they were wrong and Marilyn vos Savant was right.

A friend of mine did suggest another way of thinking about the problem that may help clarify things. Suppose we have the three doors again, one concealing the prize. You pick door #1. Now you're offered this choice: open door #1, or open door #2 *and* door #3. In the latter case you keep the prize if it's behind either door. You'd rather have a two-in-three shot at the prize than one-in-three, wouldn't you? If you think about it, the original problem offers you basically the same choice. Monty is saying in effect: you can keep your one door or you can have the other two doors, one of which (a non-prize door) I'll open for you. Still don't get it? Then at least have the sense to keep quiet about it.

Other correspondents have passed along some interesting variations on the problem. Here's a couple from Jordan Drachman of Stanford, California:

- There is a card in a hat. It is either the ace of spades or the king of spades, with equal probability. You take another identical ace of spades and throw it into the hat. You then choose a card at random from the hat. You see it is an ace. What are the odds the original card in the hat was an ace? (Answer: 2/3.)
- There is a family with two children. You have been told this family has a daughter. What are the odds they also have a son, assuming the biological odds of having a male or female child are equal? (Answer: 2/3.)

Finally, this one from a friend. Suppose we have a lottery with 10,000 "scratch-off-the-dot" tickets. The prize: a car. Ten thousand people buy the tickets, including you. 9,998 scratch off the dots on their tickets and find the message YOU LOSE. Should you offer big money to the remaining ticketholder to exchange tickets with you? (Answer: hey, after all this drill, *you* figure it out.)

SO I LIED--*THIS* IS THE LAST YOU'LL EVER HAVE TO READ ABOUT THIS (I HOPE)

Dear Cecil:

The answers to the logic questions submitted by Jordan Drachman were illogical. In the first problem he says there is an equal chance the card placed in a hat is either an ace of spades or a king of spades. An ace of spades is then added. Now a card is drawn from the hat--an ace of spades. Drachman asks what the odds are that the original card was an ace. Drawing a card does not affect the odds for the original card. They remain 1 in 2 that it was an ace, not 2 in 3 as stated.

In the second problem we are told a couple has two children, one of them a girl. Drachman then asks what the odds are the other child is a boy, assuming the biological odds of having a male or female child are equal. His answer: 2 in 3. How can the gender of one child affect the gender of another? It can't. The answer is 1 in 2. --Adam Martin and Anna Davlantes, Evanston, Illinois

Dear Cecil:

In a recent column you asked, "Suppose we have a lottery with 10,000 `scratch off the dot' tickets. The prize: a car. Ten thousand people buy the tickets, including you. 9,998 scratch off the dots on their tickets and find the message `YOU LOSE.' Should you offer big money to the remaining ticketholder to exchange tickets with you?"

If you think the answer is "yes," you are wrong. If you think the answer is "no," then you are intentionally misleading your readers ... --Jim Balter, Los Angeles

Do you think I could possibly screw this up twice in a row? Of course I could. But not this time. Cecil is well aware the answer to the lottery question is "no"--if there are only two tickets left, they have equal odds of being the winner. The difference between this and the Monty Hall question is that we're assuming Monty *knows* where the prize is, and uses that information to select a non-prize door to open; whereas in the lottery example the fact that the first 9,998 tickets are losers is a matter of chance. I put the question at the end of a line of dissimilar questions as a goof--not very sporting, but old habits die hard.

The answers given to Jordan Drachman's questions--2 in 3 in both cases--were correct. The odds of the original card being an ace were 1 in 2 before it was placed in the hat. We are now trying to determine what card was actually chosen based on subsequent events. Here are the possibilities:

(1) The original card in the hat was an ace. You threw in an ace and then picked the *original* ace.

(2) The original card in the hat was an ace. You threw in an ace and then picked the *second* ace.

(3) The original card was a king; you threw in an ace. You then picked the ace.

In 2 of 3 cases, the original card was an ace. QED.

The second question is much the same. The possible gender combinations for two children are:

(1) Child A is female and Child B is male.

(2) Child A is female and Child B is female.

(3) Child A is male and Child B is female.

(4) Child A is male and Child B is male.

We know one child is female, eliminating choice #4. In 2 of the remaining 3 cases, the female child's sibling is male. QED.

Granted the question is subtle. Consider: we are to be visited by the two kids just described, at least one of which is a girl. It's a matter of chance who arrives first. The first child enters--a girl. The second knocks. What are the odds it's a boy? Answer: 1 in 2. Paradoxical but true. (Thanks to Len Ragozin of New York City.)

Cecil is happy to say he has heard from the originator of the Monty Hall question, Steve Selvin, a UCal-Berkeley prof (cf *American Statistician*, February 1975). Cecil is happy because he can now track Steve down and have him assassinated, as he richly deserves for all the grief he has caused. Hey, just kidding, doc. But next time you have a brainstorm, do us a favor and keep it to yourself.

--CECIL ADAMS

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