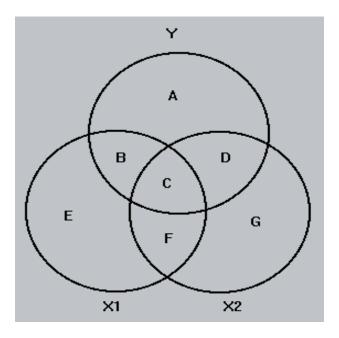
Supplemental notes on Semipartial Correlations

This discussion borrows heavily from <u>Applied Multiple Regression/Correlation Analysis</u> <u>for the Behavioral Sciences</u>, by Jacob and Patricia Cohen (1975 edition; there is also an updated 2003 edition now).

When I presented the following diagram, I pointed out that this was just one example of the many ways that the Xs and Ys could be interrelated:



I also presented the output for this specific empirical example:

Correlations									
		INCOME	EDUC	JOBEXP					
Pearson Correlation	INCOME	1.000	.846	.268					
	EDUC	.846	1.000	107					
	JOBEXP	.268	107	1.000					

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.919 ^a	.845	.827	4.07431

a. Predictors: (Constant), JOBEXP, EDUC

	Coefficients ^a													
			Unstandardized Coefficients		Standardi zed Coefficien ts		95% Confidence Interval for B		Correlations			Collinearity Statistics		
Ν	Nodel		В	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound	Zero-order	Partial	Part	Tolerance	VIF
1		(Constant)	-7.097	3.626		-1.957	.067	-14.748	.554					
		EDUC	1.933	.210	.884	9.209	.000	1.490	2.376	.846	.913	.879	.989	1.012
		JOBEXP	.649	.172	.362	3.772	.002	.286	1.013	.268	.675	.360	.989	1.012

a. Dependent Variable: INCOME

However, in February 2004, Eugene Paik, a graduate student at UNLV, emailed me and pointed out that the diagram didn't seem to correspond to the empirical example! (Later, Catherine Liu, graduate student at Notre Dame, made the same observation.) Paik's argument was as follows:

Given

- Y is the dependent variable.
- X1 and X2 are independent variables.
- $\mathbf{R}_{\mathbf{v}12}^2 = \mathbf{B} + \mathbf{C} + \mathbf{D}$

•
$$\mathbf{r}_{\mathbf{y}(1.2)}^2 = \mathbf{B}$$

r $_{y(2.1)}^2 = D$

Then

- $\mathbf{r}_{\mathbf{y}(1.2)}^2 + \mathbf{r}_{\mathbf{y}(2.1)}^2 \leq \mathbf{R}_{\mathbf{y}12}^2$ for all cases.
- In other words, $\mathtt{C} \geq 0$
- In other words, the sum of the squares of all semi-partial correlations cannot exceed R².

Because

- In term of the variance that each semi-partial correlation accounts for in Y, they are mutually exclusive by definition.
- In other words, the areas corresponding to semi-partial correlations do not overlap in the Ballantine diagram.

However

- In the example provided, this does not hold.
- The sum of squared semi-partial (part) correlations from the SPFF example is 0.879² + 0.360², which is around 0.9022.
- But R² (0.845) is smaller!!!
- If you calculate the area C, you get around -0.0591. That's <u>negative</u> 0.0591.

What does it mean for the sum of squared semi-partial correlations to exceed R²? What does it mean for the area of C to be negative?

Incidentally, I've seen even more extreme examples where C is largely negative (e.g., -0.41).

To summarize, the diagram implies that $R^2_{Y12} = B + C + D$, $sr^2_1 = B$, $sr^2_2 = D$, so

$$R^{2}_{Y12} \ge sr^{2}_{1} + sr^{2}_{2}$$
 because $B + C + D \ge B + D$.

However, in the actual example, $R^{2}_{Y12} = .845$, $sr^{2}_{1} = .773$, $sr^{2}_{2} = .1296$, hence $R^{2}_{Y12} < sr^{2}_{1} + sr^{2}_{2}$ because .845 < .902.

Paik is right; while I think the diagram is very useful and works for many situations, it doesn't accurately describe the specific example I am using. Here is the response I sent to Paik:

The example I use (which may be a good reason for not using it!) is an example of what Cohen and Cohen call Cooperative Suppression. Note that

- Educ and Jobexp are negatively correlated with each other (which makes sense; get more of one, you tend to get less of the other)
- Nonetheless, both have positive correlations and effects on income (which again makes sense; the more education and job experience you have, the more you can expect to make)
- As a result, the semipartial (part) correlations are actually larger than the zero-order correlations are.

In the attached excerpt from the 1975 edition of their book (which I think explains this more clearly than the 2003 edition does) they show how this can lead to the sort of situation you describe, i.e. the sum of the squared semipartials is greater than the R^2. (Their whole chapter is worth reading if you can get a copy of it.)

When cooperative suppression is present, the ballantine presentation breaks down a bit, because, as you say, you can't draw a negative area! Indeed, in their discussion, Cohen and Cohen present several diagrams, but they don't present one for cooperative suppression.

The diagram presented is fine when, say, X1, X2 and Y are all positively correlated; but that isn't the case in the current empirical example.

We will discuss the idea of suppression further during Stats II. Meanwhile, the attached page from Cohen and Cohen (1975) briefly discusses the idea.

The semipartial correlations are $x_1 = \frac{29 - 24(-30)}{\sqrt{1 - 30^2}} = \frac{29 + 0720}{9539} = 379$, $x_2 = \frac{24 - 29(-30)}{\sqrt{1 - 30^2}} = \frac{24 + 0870}{9539} = 379$, $x_2 = \frac{24 - 29(-30)}{\sqrt{1 - 30^2}} = \frac{24 + 0870}{9539} = 343$, both larger than their respective validity coefficients. R_{7-12}^2 is found by Eq. (3.3.1) to be $R_{7-12}^2 = \frac{24 - 20(-30)}{1 - (-30)^2} = 2016$, $R_{7-12}^2 = \frac{29^2 + 24^2 - 2(29)(24)(-30)}{1 - (-30)^2} = 2016$, and we note that $r_{7}^2 r_1 + r_{7}^2 < R_{7-12}^2 < xr_1^2 + xr_2^2$ $R_{7-13}^2 = \frac{29^2 + 24^2 - 2(29)(24)(-30)}{1 - (-30)^2} = 2016$, Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a larger proportion of "Computing the fixe fract dramating enhancing under conditions of "cooperative" suppression, and each variable accounts for a larger proportion of "Computing the fixe find enths rist does along." Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a larger proportion of "cooperative" suppression, and each other than it does along." Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each other than it does along." The agam and excored is we find that $R_1 = 398$ and $R_2 = 3597$, both of which retain the gipm and excored is the finite science studies. The detailed presentation here is in the interest of enabling the researcher to encodities $\sqrt{R_1}$ coefficient which falls outside the limits defined by r_{Y1} and 0 signab the presence of suppression. If the X' in question has a zero (in praction, wery small) correlation with Y, the situation is one of classical appression. If its R_1 R_1 the opposite sign from its r_{Y1} and is of the same sign, cooperative suppression is indicated. r_{Y1}
$\mathbf{y}_1 = \frac{29 - 24(30)}{\sqrt{130^2}} = \frac{29 + .0720}{9539} = 379,$ $\mathbf{y}_2 = \frac{2420(30)}{\sqrt{130^2}} = \frac{24 + .0870}{9539} = 343,$ both larger than their respective validity coefficients. $R_{2^{-1},1}^2$ is found by Eq. (3.3.1) to be $R_{2^{-1},12}^2 = \frac{2420(30)}{1 - (30)^2} = 2016,$ $R_{2^{-1},12} = \frac{29^2 + .24^2 - 2(.29)(.24)(30)}{1 - ((30)^2)} = 2016,$ and we note that $R_{2^{-1},12}^2 = \frac{29^2 + .24^2 - 2(.29)(.24)(30)}{1 - ((30)^2)} = 2016,$ (Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a larger proportion of the Y variance in the presence of the other than it does alone. Computing the β we find that $\beta_1 = .398$ and $\beta_2 = .359$, both of which retain the sign and exceed in magnitude their respective r_{Y_1} . 29 and 24. Defined by r_{Y_1} and to set under the interest of enabling the researcher to recognize them when they do occur, and for their value as quasiparadotical, wery small) correlation here is in the interest of enabling the researcher to recognize them when they do occur, and for their value as quasiparadotical, wery small) correlation with Y , the situation is one of classical supression. If its β_1 B °r' opposite sign from its r_{Y_1} it is serving as a net suppression. If its β_1 B °r' opposite sign from its r_{Y_1} it is serving as a net suppression. If its β_1 B °r' opposite sign from its r_{Y_1} and is of the same sign, cooperative suppression is indicated.
$\mathbf{x}_{2} = \frac{24 - 29(-30)}{\sqrt{1 - 30^{2}}} = \frac{24 + .0870}{9339} = ^{2}333,$ both larger than their respective validity coefficients. R_{7-12}^{2} is found by Eq. (3.3.1) to be $R_{7-12}^{2} = \frac{29^{2} + 24^{2} - 2(29)(24)(-30)}{1 - (-30)^{2}} = 2016,$ and we note that $R_{7-12}^{2} = \frac{29^{2} + 24^{2} - 2(29)(24)(-30)}{1 - (-30)^{2}} = 2016,$ Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a larger proportion of the Y variance in the presence of the other than it does alone. Computing the fave find that $\beta_{1} = 398$ and $\beta_{2} = 359$, both of which retain the gipn and exceed in magnitude their respective γ_{T1} , 29 and 24 . Find cooperative—are not frequently found in behavioral science studies. The sign and ecceed in magnitude their respective γ_{T1} , γ_{2} and 24 . For suppression -classical, net, and cooperative are not frequently found in behavioral science studies. The sign and ecceed in magnitude their respective γ_{T1} , γ_{2} and 24 . For the presence of suppression. If the X_{1} in question has a zero (in practice, wery small) correlation with Y_{1} the situation is one of classical suppression. If its β_{1} is serving as a net suppression is indicated. We exceed its γ_{T1} and is of the same sign, cooperative suppression is indicated.
both larger than their respective validity coefficients. R_{Y-12}^{2} is found by Eq. (3.3.1) to be $R_{Y-12}^{2} = \frac{29^{2} + 24^{2} - 2(.29)(.24)(30)}{1 - (30)^{2}} = .2016$, $R_{Y-12}^{2} = \frac{29^{2} + .24^{2} - 2(.29)(.24)(30)}{1 - (30)^{2}} = .2016$, and we note that $r_{Y1}^{2} + r_{Y2}^{2} < R_{Y-12}^{2} < sr_{T}^{2} + sr_{T}^{2}$ 0.841 + .0576 < .2016 < .1440 + .1175. (Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a <i>larger</i> proportion of the <i>Y</i> variance in the presence of the other than it does alone. Computing the fs we find that $\beta_{1} = .398$ and $\beta_{2} = .359$, both of which retain the sign and exceed in magnitude their respective r_{Y1} , .29 and .24. Finally, it is important to note that all three kinds of suppression-classical, net, and cooperative-are not frequently found in behavioral science studee.) The detailed presentation here is in the interest of enabling the researcher to recognize them when they do occur, and for their value as quasiparadoxical curiosities $\Delta \beta_{i}$ coefficient which falls outside the limits defined by r_{Y1} and 0 signals the presence of suppression. If the X_{i} in question has a zero (in practice, wery small) correlation with Y , the situation is one of classical suppression. If its β_{i} B 'of opposite sign from its r_{Y1} , it is serving as a net suppression. If its β_{i} b 'of opposite sign, cooperative suppression is indicated.
$R_{T,12}^2 = \frac{.20^2 + .24^2 - 2(.29)(.24)(30)}{1 - (30)^3} = .2016,$ and we note that $r_{T,1}^2 + r_{T,2}^2 < R_{T,12}^2 < x_T^2 + x_T^2$ $r_{T,1175}^2 = .0016 < .11440 + .1175.$ (Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a larger proportion of the <i>Y</i> variance in the presence of the other than it does alone] Computing the fix we find that $R_1 = .398$ and $R_2 = .359$, both of which retain the gian and exceed in magnitude their respective r_{T_1} , 29 and .24. Finally, it is important to note that all three kinds of suppression-classical, net, and cooperative—are not frequently found in behavioral science studies) The detailed presentation here is in the interest of enabling the researcher to recognize them when they do occur, and for their value as quasiparadoxical curiosities Δ_{R_1} coefficient which falls outside the limits defined by r_{T_1} and 0 signals the presence of suppression. If the X_1 in question has a zero (in practice, wery small) correlation with Y_1 , the situation is one of classical suppression. If its R_1 is opposite sign from its r_{T_1} , it is serving as a net suppression. If its R_1 is opposite sign from its r_{T_1} , it is serving as a net suppression. If its R_1 exceeds its r_{T_1} and is of the same sign, cooperative suppression is indicated.
and we note that $r_{Y_1}^2 + r_{Y_2}^2 < r_{Y_{-12}}^2 < sr_{1}^2 + sr_{2}^2$ $r_{Y_1}^2 + r_{Y_2}^2 < r_{Y_{-12}}^2 < sr_{1}^2 + sr_{2}^2$ (2000 the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a larger proportion of the Y variance in the presence of the other than it does alone. Computing the fix we find that $\beta_1 = .398$ and $\beta_2 = .359$, both of which retain the gau and exceed in magnitude their respective r_{Y_1} , 29 and 24. Finally, it is important to note that all three kinds of suppression-classical, net, and cooperative-are not frequently found in behavioral science studies) The mode magnitude their respective r_{Y_1} , 29 and 24. Finally, it is important to note that all three kinds of suppression-classical, net, and cooperative-are not frequently found in behavioral science studies) The sum when they do occur, and for their value as quasiparadoxical curiosities $\Delta_i \beta_i$ coefficient which falls outside the limits defined by r_{Y_1} and 0 signals the presence of suppression. If the X_i in question has a zro (in practice, wery small) correlation with Y_i the situation is one of classical appression. If its β_i is by for opposite sign from its r_{Y_1} , it is serving as a net suppression is indicated.
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TANK PARTY

3 MRC: TWO OR MORE IVs

8

3.4.6 Cooperative Suppression: $r_{12} < 0$

positively with Y correlate negatively with each other (or, equivalently, the reverse), 72, being negative, involves a portion of the variance in the IVs all of which it melevant to Y; thus, when each variable is partialled from the other, all indices of relationship with Y are enhanced. researcher (although rarely attained) than one in which IVs which correlate In his never ending search for high R3, no circumstance is more attractive to the

For example, imagine that a Director of Personnel is establishing a procedure for selecting sales personnel from among job applicants. A sample of current salesmen is drawn, and each person is rated for overall success in sales performance. These ratings constitute the "criterion" or dependent variable, Y. A series of interviews leads the researcher to suspect that two major components of sales success are social aggressiveness and habits and skills with regard to record keeping. Measures of these two characteristics are devised and administered to the tample of salesmen. The correlation between the measure of social aggressiveness (X_1) and sales success (Y) is found to be .29, the correlation between record keeping (X_2) and Y is .24, and $r_{12} = -.30$, indicating an overall tendency for those high on social aggressiveness to be relatively low on record keeping. į