

Marginal Effects for Continuous Variables

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References: Long 1997, Long and Freese 2003 & 2006 & 2014, Cameron & Trivedi's "Microeconomics Using Stata" Revised Edition, 2010

Overview. Marginal effects are computed differently for discrete (i.e. categorical) and continuous variables. This handout will explain the difference between the two.

With binary independent variables, marginal effects measure *discrete change*, i.e. how do predicted probabilities change as the binary independent variable changes from 0 to 1?

Marginal effects for continuous variables measure the *instantaneous rate of change* (defined shortly). They are popular in some disciplines (e.g. Economics) because they often provide a good approximation to the amount of change in Y that will be produced by a 1-unit change in X_k . But then again, they often do not.

Example. We will show Marginal Effects at the Means (MEMS) for both the discrete and continuous independent variables in the following example.

```
. use http://www3.nd.edu/~rwilliam/statafiles/glm-logit.dta, clear
. logit grade gpa tuce i.psi, nolog
```

```
Logistic regression                               Number of obs   =           32
                                                    LR chi2(3)      =           15.40
                                                    Prob > chi2     =           0.0015
Log likelihood = -12.889633                       Pseudo R2      =           0.3740
```

grade	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
gpa	2.826113	1.262941	2.24	0.025	.3507938 5.301432
tuce	.0951577	.1415542	0.67	0.501	-.1822835 .3725988
1.psi	2.378688	1.064564	2.23	0.025	.29218 4.465195
_cons	-13.02135	4.931325	-2.64	0.008	-22.68657 -3.35613

```
. margins, dydx(*) atmeans
```

```
Conditional marginal effects                       Number of obs   =           32
Model VCE    : OIM

Expression   : Pr(grade), predict()
dy/dx w.r.t. : gpa tuce 1.psi
at           : gpa
              : tuce
              : 0.psi
              : 1.psi
              : = 3.117188 (mean)
              : = 21.9375 (mean)
              : = .5625 (mean)
              : = .4375 (mean)
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]
gpa	.5338589	.237038	2.25	0.024	.069273 .9984447
tuce	.0179755	.0262369	0.69	0.493	-.0334479 .0693989
1.psi	.4564984	.1810537	2.52	0.012	.1016397 .8113571

Note: dy/dx for factor levels is the discrete change from the base level

Discrete Change for Categorical Variables. Categorical variables, such as psi, can only take on two values, 0 and 1. It wouldn't make much sense to compute how $P(Y=1)$ would change if, say, psi changed from 0 to .6, because that cannot happen. The MEM for categorical variables therefore shows how $P(Y=1)$ changes as the categorical variable changes from 0 to 1, holding all other variables at their means. That is, for a categorical variable X_k

$$\text{Marginal Effect } X_k = \Pr(Y = 1|X, X_k = 1) - \Pr(y=1|X, X_k = 0)$$

In the current case, the MEM for psi of .456 tells us that, for two hypothetical individuals with average values on gpa (3.12) and tuce (21.94), the predicted probability of success is .456 greater for the individual in psi than for one who is in a traditional classroom. To confirm, we can easily compute the predicted probabilities for those hypothetical individuals, and then compute the difference between the two.

```
. margins psi, atmeans
```

```
Adjusted predictions          Number of obs   =          32
Model VCE      : OIM

Expression      : Pr(grade), predict()
at              : gpa          =      3.117188 (mean)
                : tuce         =      21.9375 (mean)
                : 0.psi        =       .5625 (mean)
                : 1.psi        =       .4375 (mean)
```

		Delta-method				[95% Conf. Interval]	
	Margin	Std. Err.	z	P> z			
psi							
0	.1067571	.0800945	1.33	0.183	-.0502252	.2637393	
1	.5632555	.1632966	3.45	0.001	.2432001	.8833109	

```
. display .5632555 - .1067571
.4564984
```

For categorical variables with more than two possible values, e.g. religion, the marginal effects show you the difference in the predicted probabilities for cases in one category relative to the reference category. So, for example, if relig was coded 1 = Catholic, 2 = Protestant, 3 = Jewish, 4 = other, the marginal effect for Protestant would show you how much more (or less) likely Protestants were to succeed than were Catholics, the marginal effect for Jewish would show you how much more (or less) likely Jews were to succeed than were Catholics, etc.

Keep in mind that these are the marginal effects when all other variables equal their means (hence the term MEMs); the marginal effects will differ at other values of the X_s .

Instantaneous rates of change for continuous variables. What does the MEM for gpa of .534 mean? It would be nice if we could say that a one unit increase in gpa will produce a .534 increase in the probability of success for an otherwise “average” individual. Sometimes statements like that will be (almost) true, but other times they won't. For example, if an

“average” individual (average meaning $\text{gpa} = 3.12$, $\text{tuces} = 21.94$, $\text{psi} = .4375$) saw a one point increase in their gpa, here is how their predicted probability of success would change:

```
. margins, at(gpa = (3.117188 4.117188)) atmeans
```

```
Adjusted predictions          Number of obs   =           32
Model VCE      : OIM
```

```
Expression      : Pr(grade), predict()
```

```
1._at          : gpa          =    3.117188
                 tuces         =    21.9375 (mean)
                 0.psi         =     .5625 (mean)
                 1.psi         =     .4375 (mean)
```

```
2._at          : gpa          =    4.117188
                 tuces         =    21.9375 (mean)
                 0.psi         =     .5625 (mean)
                 1.psi         =     .4375 (mean)
```

	Margin	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
_at						
1	.2528205	.1052961	2.40	0.016	.046444	.459197
2	.8510027	.1530519	5.56	0.000	.5510265	1.150979

```
. display .8510027 - .2528205
.5981822
```

Note that (a) the predicted increase of .598 is actually more than the MEM for gpa of .534, and (b) in reality, gpa couldn't go up 1 point for a person with an average gpa of 3.117.

MEMs for continuous variables measure the *instantaneous rate of change*, which may or may not be close to the effect on $P(Y=1)$ of a one unit increase in X_k . The appendices explain the concept in detail. What the MEM more or less tells you is that, if, say, X_k increased by some very small amount (e.g. .001), then $P(Y=1)$ would increase by about $.001 * .534 = .000534$, e.g.

```
. margins, at(gpa = (3.117188 3.118188)) atmeans noatlegend
```

```
Adjusted predictions          Number of obs   =           32
Model VCE      : OIM
```

```
Expression      : Pr(grade), predict()
```

	Margin	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
_at						
1	.2528205	.1052961	2.40	0.016	.046444	.459197
2	.2533547	.1053672	2.40	0.016	.0468388	.4598706

```
. display .2533547 - .2528205
.0005342
```

Put another way, for a continuous variable X_k ,

$$\text{Marginal Effect of } X_k = \lim_{\Delta \rightarrow 0} [\Pr(Y = 1|X, X_k + \Delta) - \Pr(y=1|X, X_k)] / \Delta]$$

The appendices show how to get an exact solution for this.

There is no guarantee that a bigger increase in X_k , e.g. 1, would produce an increase of $1 * .534 = .534$. This is because the relationship between X_k and $P(Y = 1)$ is nonlinear. When X_k is measured in small units, e.g. income in dollars, the effect of a 1 unit increase in X_k may match up well with the MEM for X_k . But, when X_k is measured in larger units (e.g. income in millions of dollars) the MEM may or may not provide a very good approximation of the effect of a one unit increase in X_k . That is probably one reason why instantaneous rates of change for continuous variables receive relatively little attention, at least in Sociology. More common are approaches which focus on discrete changes, which we will discuss later.

Conclusion. Marginal effects can be an informative means for summarizing how change in a response is related to change in a covariate. For categorical variables, the effects of discrete changes are computed, i.e., the marginal effects for categorical variables show how $P(Y = 1)$ is predicted to change as X_k changes from 0 to 1 holding all other X s equal. This can be quite useful, informative, and easy to understand.

For continuous independent variables, the marginal effect measures the instantaneous rate of change. If the instantaneous rate of change is similar to the change in $P(Y=1)$ as X_k increases by one, this too can be quite useful and intuitive. However, there is no guarantee that this will be the case; it will depend, in part, on how X_k is scaled.

Subsequent handouts will show how the analysis of discrete changes in continuous variables can make their effects more intelligible.

Appendix A: AMEs for continuous variables, computed manually

Calculus can be used to compute marginal effects, but Cameron and Trivedi (Microeconometrics using Stata, Revised Edition, 2010, section 10.6.10, pp. 352 – 354) show that they can also be computed manually. The procedure is as follows:

Compute the predicted values using the observed values of the variables. We will call this prediction1.

Change one of the continuous independent variables by a very small amount. Cameron and Trivedi suggest using the standard deviation of the variable divided by 1000. We will refer to this as delta (Δ).

Compute the new predicted values for each case. Call this prediction2.

For each case, compute

$$xme = \frac{\text{prediction2} - \text{prediction1}}{\Delta}$$

Compute the mean value of xme. This is the AME for the variable in question.

Here is an example:

```
. * Appendix A: Compute AMEs manually
. webuse nhanes2f, clear
. * For convenience, keep only nonmissing cases
. keep if !missing(diabetes, female, age)
(2 observations deleted)

. clonevar xage = age
. sum xage
```

Variable	Obs	Mean	Std. Dev.	Min	Max
xage	10335	47.56584	17.21752	20	74

```
. gen xdelta = r(sd)/1000
. logit diabetes i.female xage, nolog
```

```
Logistic regression                               Number of obs   =       10335
                                                    LR chi2(2)      =       345.87
                                                    Prob > chi2     =       0.0000
Log likelihood = -1826.1338                          Pseudo R2      =       0.0865
```

diabetes	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
1.female	.1549701	.0940619	1.65	0.099	-.0293878 .3393279
xage	.0588637	.0037282	15.79	0.000	.0515567 .0661708
_cons	-6.276732	.2349508	-26.72	0.000	-6.737227 -5.816237

```
. margins, dydx(xage)
```

```
Average marginal effects          Number of obs   =       10335  
Model VCE      : OIM
```

```
Expression      : Pr(diabetes), predict()  
dy/dx w.r.t.   : xage
```

```
-----+-----  
          |              Delta-method  
          |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]  
-----+-----  
    xage |   .0026152   .000188   13.91   0.000   .0022468   .0029836  
-----+-----
```

```
. predict xage1
```

```
(option pr assumed; Pr(diabetes))
```

```
. replace xage = xage + xdelta
```

```
xage was byte now float  
(10335 real changes made)
```

```
. predict xage2
```

```
(option pr assumed; Pr(diabetes))
```

```
. gen xme = (xage2 - xage1) / xdelta
```

```
. sum xme
```

```
-----+-----  
Variable |      Obs      Mean      Std. Dev.      Min      Max  
-----+-----  
    xme |    10335   .0026163   .0020192   .0003549   .0073454  
-----+-----
```

```
.  
end of do-file
```

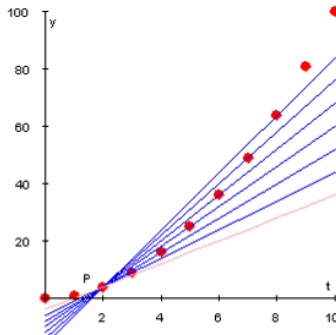
Appendix B: Technical Discussion of Marginal Effects (Optional)

In binary regression models, the marginal effect is the slope of the probability curve relating X_k to $\Pr(Y=1|X)$, holding all other variables constant. But what is the slope of a curve??? A little calculus review will help make this clearer.

Simple Explanation. Draw a graph of $F(X_k)$ against X_k , holding all the other X 's constant (e.g. at their means). Chose 2 points, $[X_k, F(X_k)]$ and $[X_k + \Delta, F(X_k + \Delta)]$. When Δ is very very small, the slope of the line connecting the two points will equal or almost equal the marginal effect of X_k .

More Detailed Explanation. Again, what is the slope of a curve? Intuitively, think of it this way. Draw a graph of $F(X)$ against X , e.g. $F(X) = X^2$. Chose specific values of X and $F(X)$, e.g. $[2, F(2)]$. Choose another point, e.g. $[8, F(8)]$. Draw a line connecting the points. This line has a slope. The slope is the *average rate of change*.

Now, choose another point that is closer to $[2, F(2)]$, e.g. $[7, F(7)]$. Draw a line connecting these points. This too will have a slope. Keep on choosing points that are closer to $[2, F(2)]$. The *instantaneous rate of change* is the limit of the slopes for the lines connecting $[X, F(X)]$ and $[X+\Delta, F(X + \Delta)]$ as Δ gets closer and closer to 0.



Source: <http://www.ugrad.math.ubc.ca/coursedoc/math100/notes/derivative/slope.html>

Last accessed January 24, 2017

See the above link for more information

Calculus is used to compute slopes (& marginal effects). For example, if $Y = X^2$, then the slope is $2X$. Hence, if $X = 2$, the slope is 4. The following table illustrates this. Note that, as Δ gets smaller and smaller, the slope gets closer and closer to 4.

$$F(X) = X^2 \quad X = 2 \quad F(2) = 4$$

δ	X+ δ	F(X+ δ)	Change in F(X)	Change in X	Slope
100	102	10404	10400	100	104
10	12	144	140	10	14
1	3	9	5	1	5
0.1	2.1	4.41	0.41	0.1	4.1
0.01	2.01	4.0401	0.0401	0.01	4.01
0.001	2.001	4.004001	0.004001	0.001	4.001
0.0001	2.0001	4.00040001	0.00040001	0.0001	4.0001

Marginal effects are also called instantaneous rates of change; you compute them for a variable while all other variables are held constant. The magnitude of the marginal effect depends on the values of the other variables and their coefficients. The *Marginal Effect at the Mean* (MEM) is popular (i.e. compute the marginal effects when all x's are at their mean) but many think that *Average Marginal Effects* (AMEs) are superior.

Logistic Regression. Again, calculus is used to compute the marginal effects. In the case of logistic regression, $F(X) = P(Y=1|X)$, and

$$\text{Marginal Effect for } X_k = P(Y=1 | X) * P(Y = 0|X) * b_k.$$

Returning to our earlier example,

```
. use http://www3.nd.edu/~rwilliam/statafiles/glm-logit.dta, clear
. logit grade gpa tuce psi, nolog
```

```
Logistic regression                               Number of obs   =           32
                                                    LR chi2(3)      =           15.40
                                                    Prob > chi2     =           0.0015
Log likelihood = -12.889633                       Pseudo R2       =           0.3740
```

grade	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
gpa	2.826113	1.262941	2.24	0.025	.3507938 5.301432
tuce	.0951577	.1415542	0.67	0.501	-.1822835 .3725988
psi	2.378688	1.064564	2.23	0.025	.29218 4.465195
_cons	-13.02135	4.931325	-2.64	0.008	-22.68657 -3.35613

```
. adjust gpa tuce psi, pr
```

```
-----
Dependent variable: grade      Equation: grade      Command: logit
Covariates set to mean: gpa = 3.1171875, tuce = 21.9375, psi = .4375
-----
```

All	pr
	.25282

Key: pr = Probability

. mfx

Marginal effects after logit
 y = Pr(grade) (predict)
 = .25282025

variable	dy/dx	Std. Err.	z	P> z	[95% C.I.]	X
gpa	.5338589	.23704	2.25	0.024	.069273	.998445		3.11719
tuce	.0179755	.02624	0.69	0.493	-.033448	.069399		21.9375
psi*	.4564984	.18105	2.52	0.012	.10164	.811357		.4375

(*) dy/dx is for discrete change of dummy variable from 0 to 1

Looking specifically at GPA – when all variables are at their means, $\text{pr}(Y=1|X) = .2528$, $\text{Pr}(Y=0|X) = .7472$, and $b_{\text{GPA}} = 2.826113$. The marginal effect at the mean for GPA is therefore

$$\text{Marginal Effect of GPA} = P(Y=1 |X) * P(Y = 0|X) * b_{\text{GPA}} = .2528 * .7472 * 2.826113 = .5339$$

The following table again shows you that, in logistic regression, as the distance between two points gets smaller and smaller, i.e. as Δ gets closer and closer to 0, the slope of the line connecting the points gets closer and closer to the marginal effect.

Logistic Regression

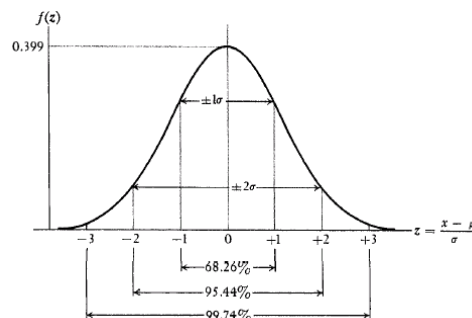
$F(X, \text{GPA}) = P(Y=1|X, \text{GPA})$ GPA=3.11719 Other X's at Mean $F(X, 3.11719) = .25282025107643$

δ	GPA+ δ	F(X,GPA+ δ)	Change in F(X,GPA)	Change in GPA	Slope
10	13.11719	1	0.747179749	10	0.0747179749
1	4.11719	0.851002558	0.598182307	1	0.5981823074
0.1	3.21719	0.309808293	0.056988042	0.1	0.5698804165
0.05	3.16719	0.280431679	0.027611428	0.05	0.5522285640
0.01	3.12719	0.258196038	0.005375787	0.01	0.5375787403
0.001	3.11819	0.253354486	0.000534235	0.001	0.5342350788
0.0001	3.11729	0.252873644	5.3393E-05	0.0001	0.5339297264

Probit. In probit, the marginal effect is

$$\text{Marginal Effect for } X_k = \Phi(XB) * b_k$$

where Φ is the probability density function for a standardized normal variable. For example, as this diagram shows, $\Phi(0) = .399$:



Example:

```
. use http://www3.nd.edu/~rwilliam/statafiles/glm-logit.dta, clear
. probit grade gpa tuce psi, nolog
```

```
Probit estimates                               Number of obs   =           32
                                                LR chi2(3)      =           15.55
                                                Prob > chi2     =           0.0014
Log likelihood = -12.818803                    Pseudo R2      =           0.3775
```

grade	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
gpa	1.62581	.6938818	2.34	0.019	.2658269	2.985794
tuce	.0517289	.0838901	0.62	0.537	-.1126927	.2161506
psi	1.426332	.595037	2.40	0.017	.2600814	2.592583
_cons	-7.45232	2.542467	-2.93	0.003	-12.43546	-2.469177

```
. mfx
```

```
Marginal effects after probit
y = Pr(grade) (predict)
= .26580809
```

variable	dy/dx	Std. Err.	z	P> z	[95% C.I.]		X
gpa	.5333471	.23246	2.29	0.022	.077726	.988968	3.11719
tuce	.0169697	.02712	0.63	0.531	-.036184	.070123	21.9375
psi*	.464426	.17028	2.73	0.006	.130682	.79817	.4375

(*) dy/dx is for discrete change of dummy variable from 0 to 1

```
. * marginal change for GPA. The invnorm function gives us the Z-score for the stated
. * prob of success. The normalden function gives us the pdf value for that Z-score.
. display invnorm(.2658)
-.62556546
. display normalden(invnorm(.2658))
.32804496
. display normalden(invnorm(.2658)) * 1.62581
.53333878
```

$$\text{Marginal Effect for GPA} = \Phi(XB) * b_k = .32804496 * 1.62581 = .5333$$

The following table again shows you that, in a probit model, as the distance between two points gets smaller and smaller, i.e. as Δ gets closer and closer to 0, the slope of the line connecting the points gets closer and closer to the marginal effect.

Probit
 $F(X, GPA) = P(Y=1|X, GPA)$ GPA=3.11719 Other X's at Mean $F(X, 3.11719) = .26580811$

δ	GPA+ δ	F(X,GPA+ δ)	Change in F(X,GPA)	Change in GPA	Slope
10	13.11719	1	0.73419189	10	0.0734191890
1	4.11719	0.841409951	0.575601841	1	0.5756018408
0.1	3.21719	0.321696605	0.055888495	0.1	0.5588849545
0.01	3.12719	0.27116854	0.00536043	0.01	0.5360430345
0.001	3.11819	0.266341711	0.000533601	0.001	0.5336010479
0.0001	3.11729	0.26586143	5.33203E-05	0.0001	0.5332029760

Using the margins command for MEMs & AMEs,

```
. quietly probit grade gpa tuce l.psi, nolog
. margins, dydx(*) atmeans
```

```
Conditional marginal effects           Number of obs   =           32
Model VCE      : OIM
```

```
Expression      : Pr(grade), predict()
dy/dx w.r.t.    : gpa tuce l.psi
at              : gpa               =      3.117188 (mean)
                 tuce               =      21.9375 (mean)
                 0.psi               =       .5625 (mean)
                 l.psi               =       .4375 (mean)
```

	Delta-method				
	dy/dx	Std. Err.	z	P> z	[95% Conf. Interval]
gpa	.5333471	.2324641	2.29	0.022	.0777259 .9889683
tuce	.0169697	.0271198	0.63	0.531	-.0361841 .0701235
l.psi	.464426	.1702807	2.73	0.006	.1306819 .7981701

Note: dy/dx for factor levels is the discrete change from the base level.

```
. margins, dydx(*)
```

```
Average marginal effects           Number of obs   =           32
Model VCE      : OIM
```

```
Expression      : Pr(grade), predict()
dy/dx w.r.t.    : gpa tuce l.psi
```

	Delta-method				
	dy/dx	Std. Err.	z	P> z	[95% Conf. Interval]
gpa	.3607863	.1133816	3.18	0.001	.1385625 .5830102
tuce	.0114793	.0184095	0.62	0.533	-.0246027 .0475612
l.psi	.3737518	.1399913	2.67	0.008	.099374 .6481297

Note: dy/dx for factor levels is the discrete change from the base level.

As a sidelight, note that the marginal effects (both MEMs and AMEs) for probit are very similar to the marginal effects for logit. This is usually the case.

OLS. Here is what you get when you compute the marginal effects for OLS:

```
. use http://www3.nd.edu/~rwilliam/statafiles/glm-reg.dta, clear
. reg income educ jobexp black
```

Source	SS	df	MS	Number of obs =	500
Model	33206.4588	3	11068.8196	F(3, 496) =	787.14
Residual	6974.79047	496	14.0620776	Prob > F	= 0.0000
				R-squared	= 0.8264
				Adj R-squared	= 0.8254
Total	40181.2493	499	80.5235456	Root MSE	= 3.7499

income	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
educ	1.840407	.0467507	39.37	0.000	1.748553	1.932261
jobexp	.6514259	.0350604	18.58	0.000	.5825406	.7203111
black	-2.55136	.4736266	-5.39	0.000	-3.481921	-1.620798
_cons	-4.72676	.9236842	-5.12	0.000	-6.541576	-2.911943

```
. margins, dydx(*) atmeans
```

```
Conditional marginal effects      Number of obs =      500
Model VCE      : OLS
```

```
Expression      : Linear prediction, predict()
dy/dx w.r.t.    : educ jobexp black
at              : educ          =      13.16 (mean)
                  jobexp       =      13.52 (mean)
                  black         =       .2 (mean)
```

	Delta-method				[95% Conf. Interval]	
	dy/dx	Std. Err.	z	P> z		
educ	1.840407	.0467507	39.37	0.000	1.748777	1.932036
jobexp	.6514259	.0350604	18.58	0.000	.5827087	.7201431
black	-2.55136	.4736266	-5.39	0.000	-3.479651	-1.623069

```
. margins, dydx(*)
```

```
Average marginal effects      Number of obs =      500
Model VCE      : OLS
```

```
Expression      : Linear prediction, predict()
dy/dx w.r.t.    : educ jobexp black
```

	Delta-method				[95% Conf. Interval]	
	dy/dx	Std. Err.	z	P> z		
educ	1.840407	.0467507	39.37	0.000	1.748777	1.932036
jobexp	.6514259	.0350604	18.58	0.000	.5827087	.7201431
black	-2.55136	.4736266	-5.39	0.000	-3.479651	-1.623069

The marginal effects are the same as the slope coefficients. This is because relationships are linear in OLS regression and do not vary depending on the values of the other variables.