Analyzing Proportions:
Fractional Response and Zero One Inflated Beta Models

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Sources:

Buis, M. L. (2020). Analysis of Proportions. In P. Atkinson, S. Delamont, A. Cernat, J.W. Sakshaug, & R.A. Williams (Eds.), SAGE Research Methods Foundations. https://dx.doi.org/10.4135/9781526421036852590

Jeffrey Wooldridge, 2011, "Fractional response models with endogenous explanatory variables and heterogeneity", http://www.stata.com/meeting/chicago11/materials/chi11_wooldridge.pdf.

"Econometric Methods for Fractional Response Variables with an Application to 401 (K) Plan Participation Rates" Leslie E. Papke and Jeffrey M. Wooldridge, Journal of Applied Econometrics, Vol. 11, No. 6 (Nov. - Dec., 1996), pp. 619-632. https://onlinelibrary.wiley.com/doi/10.1002/(SICI)1099-1255(199611)11:6%3C619::AID-JAE418%3E3.0.CO;2-1.

"How do you fit a model when the dependent variable is a proportion?" http://www.stata.com/support/faqs/statistics/logit-transformation/.

"How does one do regression when the dependent variable is a proportion?" https://stats.oarc.ucla.edu/stata/faq/how-does-one-do-regression-when-the-dependent-variable-is-a-proportion/

"Stata Tip 63: Modeling Proportions" Kit Baum, The Stata Journal, Volume 8 Number 2: pp. 299-303 http://www.stata-journal.com/article.html?article=st0147

"Stata command for fractional logit with endogenous regressor?" https://www.statalist.org/forums/forum/general-stata-discussion/general/1410304-stata-command-for-fractional-logit-with-endogenous-regressor#post1410304

NOTE: Material in handout is current as of April 3, 2022. Since `fracglm` and `fracivp` are still in beta form, there may be changes in the future. (But it won't surprise me if they remain beta forever!)

In many cases, the dependent variable of interest is a proportion, i.e. its values range between 0 and 1. Wooldridge (1996, 2011) gives the example of the proportion of employees that participate in a company's pension plan. Baum (2008) gives as examples the share of consumers' spending on food, the fraction of the vote for a candidate, or the fraction of days when air pollution is above acceptable levels in a city. This handout will discuss a few different ways for analyzing such dependent variables: fractional response models (both heteroskedastic and non-heteroskedastic), zero one-inflated beta models, and fractional ivprobit models.

Fractional Response Models. As Wooldridge notes, many Stata commands (`logit`, `probit`, `hetprob`) could analyze DVs that are proportions, but they impose the data constraint that the dependent variable must be coded as either 0 or 1, i.e. you can't have a proportion as the dependent variable even though the same formulas and estimation techniques would be appropriate with a proportion. Wooldridge offers his own short programs that relax this limitation; but a more flexible solution is offered by Richard Williams' user-written routine, `fracglm`, currently in (perpetual) beta testing. `fracglm` is adapted from `oglm`, and is probably easier to use than `oglm` when the dependent variable is a dichotomy (rather than an ordinal variable with 3 or more categories.)

To get `fracglm`, from within Stata type

```
net install fracglm, from(https://www3.nd.edu/~rwilliam/stata)
```

This is usually the best way to install. Files are placed in the right locations, and `adoupdate` will capture any updates.

That doesn't always work though. If it doesn't work for you, try pointing your browser to

https://www3.nd.edu/~rwilliam/stata/fracglmbeta.zip

Download the file (it may download automatically), unzip it, and follow the directions for installing that are in the Readme.txt file.

The following is adapted from the help for `fracglm`:

> fracglm estimates Fractional Response Generalized Linear Models (e.g. Fractional Probit, Fractional Logit) with or without heteroskedasticity. Fractional response variables range in value between 0 and 1. An example of a fractional response variable would be the percentage of employees covered by an employer's pension plan.
>
> fracglm also works with binary 0/1 dependent variables. fracglm supports multiple link functions, including logit (the default), probit, complementary log-log, log-log, log and cauchit. When these models include equations for heteroskedasticity they are also known as heterogeneous choice/ location-scale / heteroskedastic regression models.
>
> fracglm fills gaps left by other Stata commands. Commands like logit, probit and hetprob do not allow for fractional response variables. glm can estimate some fractional response models but does not allow an equation for heteroskedasticity.
>
> Several special cases of generalized linear models can also be estimated by fracglm, including the binomial generalized linear models of logit, probit and cloglog (which also assume homoskedasticity), hetprob, as well as similar models that are not otherwise estimated by Stata. This makes fracglm particularly useful for testing whether constraints on a model (e.g. homoskedastic errors) are justified, or for determining whether one link function is more appropriate for the data than are others.

In addition, Stata 14 introduced the `fracreg` command. It isn't quite as flexible as `fracglm` (e.g. it doesn't support as many link functions) but if you have Stata 14 it may be fine for your needs.

Example. Papke and Wooldridge (1996) give an example of participation rates in employer 401(k) pension plans. "Pension plan administrators are required to file Form 5500 annually with the Internal Revenue Service, describing participation and contribution behavior for each plan offered. Papke (1995) uses the plan level data to study, among other things, the relationship between the participation rate and various plan characteristics, including the rate at which a firm matches employee contributions."

In Wooldridge's (2011) version of this example, data are from 4,075 companies in 1987. The key variables used in this analysis are:

```
. use https://www3.nd.edu/~rwilliam/statafiles/401kpart, clear
. codebook prate mrate ltotemp age sole, compact

Variable    Obs Unique      Mean       Min       Max  Label
-------------------------------------------------------------------------------------
prate      4075   2597   .840607  .0036364         1  partic/employ
mrate      4075   3521   .463519         0         2  plan match rate, per $
ltotemp    4075   2147   6.97439   4.65396  13.00142  log(totemp)
age        4075     50  8.186503         1        71  age of the plan
sole       4075      2  .3693252         0         1  =1 if only pension plan
-------------------------------------------------------------------------------------------------
```

Wooldridge (2011) gives an example of a fractional probit model with heteroskedasticity. He recommends using robust standard errors (otherwise the standard errors are too large; you can confirm this by rerunning the following example with `vce(oim)`; you will see dramatic differences in the test statistics and standard errors.) He wrote his own program for this but `fracglm` can easily reproduce his results.

```
. fracglm prate mrate ltotemp age i.sole, het(mrate ltotemp age i.sole) link(p)

Heteroskedastic Fractional Probit Regression      Number of obs   =       4075
                                                  Wald chi2(4)    =     152.29
                                                  Prob > chi2     =     0.0000
Log pseudolikelihood = -1674.5212                 Pseudo R2       =     0.0632

------------------------------------------------------------------------------
             |               Robust
       prate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
prate        |
       mrate |   1.384694    .223861     6.19   0.000     .9459349    1.823454
     ltotemp |  -.1495096    .013966   -10.71   0.000    -.1768825   -.1221367
         age |   .0670722   .0100639     6.66   0.000     .0473474     .086797
      1.sole |    -.11827   .0932336    -1.27   0.205    -.3010046    .0644645
       _cons |   1.679377   .1058994    15.86   0.000     1.471818    1.886936
-------------+----------------------------------------------------------------
lnsigma      |
       mrate |    .240357   .0537812     4.47   0.000     .1349479    .3457662
     ltotemp |   .0375185   .0144217     2.60   0.009     .0092525    .0657845
         age |   .0171714   .0027289     6.29   0.000     .0118229    .0225199
      1.sole |  -.1627546   .0631069    -2.58   0.010    -.2864417   -.0390674
------------------------------------------------------------------------------
```

[NOTE: `vce(robust)` is the default for both `fracglm` and `fracreg`. If you are using `fracglm` with a binary dependent variable, you may wish to specify `vce(oim)` instead.]

Wooldridge (2011) notes that a simple Wald test can be used to determine whether the coefficients in the heteroskedasticity equation are significantly different from zero. (I believe this is better than a likelihood ratio test because LR tests are problematic when using robust standard errors).

```
. test [lnsigma]

 ( 1)  [lnsigma]mrate = 0
 ( 2)  [lnsigma]ltotemp = 0
 ( 3)  [lnsigma]age = 0
 ( 4)  [lnsigma]0b.sole = 0
 ( 5)  [lnsigma]1.sole = 0
       Constraint 4 dropped

           chi2(  4) =  109.26
         Prob > chi2 =    0.0000
```

You interpret these results pretty much the same way you would interpret the results from a hetprobit model. A higher match rate, an older fund, and having fewer employees all increase participation rates.

If you want to make results more tangible, you can use methods like we have used before. For example,

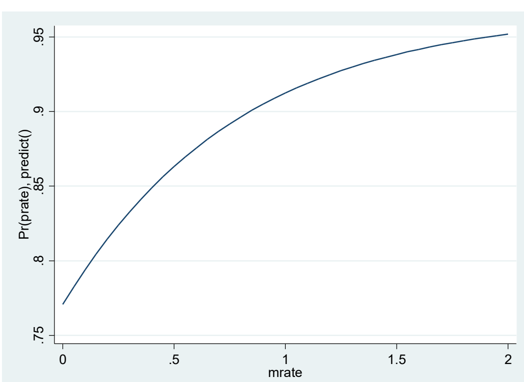
```
. margins, dydx(*)

Average marginal effects                          Number of obs   =       4075
Model VCE    : Robust

Expression   : Pr(prate), predict()
dy/dx w.r.t. : mrate ltotemp age 1.sole

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |    .159632   .0114214    13.98   0.000     .1372466    .1820175
     ltotemp |  -.0306262   .0020032   -15.29   0.000    -.0345524      -.0267
         age |   .0065659   .0006323    10.38   0.000     .0053266    .0078052
      1.sole |    .016089   .0061307     2.62   0.009     .0040729     .028105
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.

. mcp mrate, at1(0 (.05) 2)
```



Wooldridge (2011) also says you “should do a comparison of average partial effects [aka average marginal effects] between ordinary fractional probit and heteroskedastic fractional probit.” Non-heteroskedastic models can also be estimated with `fracglm`:

```
. fracglm prate mrate ltotemp age i.sole, link(p)

Fractional Probit Regression                     Number of obs   =       4075
                                                 Wald chi2(4)    =     695.89
                                                 Prob > chi2     =     0.0000
Log pseudolikelihood = -1681.9607                Pseudo R2       =     0.0591

------------------------------------------------------------------------------
             |               Robust
       prate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |   .5955726     .038756    15.37   0.000     .5196123    .6715329
     ltotemp |  -.1172851    .0080003   -14.66   0.000    -.1329655   -.1016048
         age |   .0180259    .0014218    12.68   0.000     .0152392    .0208126
      1.sole |   .0944158    .0271696     3.48   0.001     .0411645    .1476672
       _cons |   1.428854    .0593694    24.07   0.000     1.312493    1.545216
------------------------------------------------------------------------------

. margins, dydx(*)

Average marginal effects                         Number of obs   =       4075
Model VCE    : Robust

Expression   : Pr(prate), predict()
dy/dx w.r.t. : mrate ltotemp age 1.sole

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |   .1362769    .0088064    15.47   0.000     .1190167    .1535372
     ltotemp |  -.0268368    .0018454   -14.54   0.000    -.0304537   -.0232199
         age |   .0041246    .0003277    12.59   0.000     .0034824    .0047669
      1.sole |   .0213349    .0060421     3.53   0.000     .0094927    .0331771
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Based on the earlier Wald test we would prefer the heteroskedastic model. You can also see that there are some modest differences in the Average Marginal Effects estimated by the two models.

Using Stata 14's `fracreg` instead, the heteroskedastic and non-heteroskedastic models are estimated by

```
. use https://www3.nd.edu/~rwilliam/statafiles/401kpart, clear
. fracreg probit prate mrate ltotemp age i.sole, het(mrate ltotemp age i.sole) nolog

Fractional heteroskedastic probit regression    Number of obs     =      4,075
                                                Wald chi2(5)      =     152.30
                                                Prob > chi2       =     0.0000
Log pseudolikelihood = -1674.5212               Pseudo R2         =     0.0088

------------------------------------------------------------------------------
             |               Robust
       prate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
prate        |
       mrate |   1.384675    .2238372     6.19   0.000     .9459625    1.823388
     ltotemp |  -.1495096    .0139658   -10.71   0.000    -.1768822   -.1221371
         age |   .0670714    .0100629     6.67   0.000     .0473485    .0867943
      1.sole |  -.1182733    .0932298    -1.27   0.205    -.3010003    .0644537
       _cons |   1.679372    .1058965    15.86   0.000     1.471819    1.886926
-------------+----------------------------------------------------------------
lnsigma      |
       mrate |   .2403557    .0537805     4.47   0.000     .1349478    .3457635
     ltotemp |   .0375172      .01442     2.60   0.009     .0092545    .0657799
         age |   .0171714    .0027289     6.29   0.000     .0118229    .0225199
      1.sole |  -.1627574     .063104    -2.58   0.010    -.2864389   -.0390759
------------------------------------------------------------------------------
```

```
. fracreg probit prate mrate ltotemp age i.sole, nolog

Fractional probit regression                    Number of obs     =      4,075
                                                Wald chi2(5)      =     695.89
                                                Prob > chi2       =     0.0000
Log pseudolikelihood = -1681.9607               Pseudo R2         =     0.0591

------------------------------------------------------------------------------
             |               Robust
       prate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |   .5955726    .038756    15.37   0.000     .5196123    .6715329
     ltotemp |  -.1172851   .0080003   -14.66   0.000    -.1329655   -.1016048
         age |   .0180259   .0014218    12.68   0.000     .0152392    .0208126
      1.sole |   .0944158   .0271696     3.48   0.001     .0411645    .1476672
       _cons |   1.428854   .0593694    24.07   0.000     1.312493    1.545216
------------------------------------------------------------------------------
```

Other Comments on Fractional Response Models:

1. Other than the fact that the heteroskedastic model fits better in this case, what is the rationale for it? I asked Jeffrey Wooldridge about this and he emailed me the following:

> I think of it [the heteroskedastic model] mainly as a simple way to get a more flexible functional form. But this can also be derived from a model where, say, y(i) is the fraction of successes out of n(i) Bernoulli trials, where each binary outcome, say w(i,j), follows a heteroskedastic probit. Then E(y(i)|x(i)) would have the form estimated by your Stata command.
>
> Or, we could start with an omitted variable formulation: E[y(i)|x(i),c(i)] = PHI[x(i)*b + c(i)], where the omitted variable c(i) is distributed as Normal with mean zero and variance h(x(i)). As an approximation, we might use an exponential function for 1 + h(x(i)), and then that gives the model, too.

2. As noted in Wooldridge (2011) and in the Stata FAQ cited above, the `glm` command can also be used to estimate non-heteroskedastic models. Specify family(binomial) and either link(p) or link(l). These are the same results that `fracglm` gave for the non-heteroskedastic model.

```
. glm prate mrate ltotemp age i.sole, vce(robust) link(p) family(binomial) nolog
note: prate has noninteger values

Generalized linear models                          No. of obs      =      4075
Optimization     : ML                              Residual df     =      4070
                                                   Scale parameter =         1
Deviance         =  885.9205448                    (1/df) Deviance =  .2176709
Pearson          =  896.7484978                    (1/df) Pearson  =  .2203313

Variance function: V(u) = u*(1-u/1)                [Binomial]
Link function    : g(u) = invnorm(u)               [Probit]

                                                   AIC             =  .6352659
Log pseudolikelihood = -1289.354251                BIC             = -32946.47
```

```
------------------------------------------------------------------------------
             |               Robust
       prate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |   .5955726    .038756    15.37   0.000     .5196123    .6715329
     ltotemp |  -.1172851   .0080003   -14.66   0.000    -.1329655   -.1016048
         age |   .0180259   .0014218    12.68   0.000     .0152392    .0208126
      1.sole |   .0944158   .0271696     3.48   0.001     .0411645    .1476672
       _cons |   1.428854   .0593694    24.07   0.000     1.312493    1.545216
------------------------------------------------------------------------------
```

Zero Inflated Beta Models. The Stata FAQ (http://www.stata.com/support/faqs/statistics/logit-transformation/) warns that other types of models may be advisable depending on why the 0s or 1s exist. From the FAQ (it talks about a logit transformation but the same is true for probit):

> A traditional solution to this problem [the dependent variable is a proportion] is to perform a logit transformation on the data. Suppose that your dependent variable is called y and your independent variables are called X. Then, one assumes that the model that describes y is
>
> ```
> y = invlogit(XB)
> ```
>
> If one then performs the logit transformation, the result is
>
> ```
> ln( y / (1 - y) ) = XB
> ```
>
> We have now mapped the original variable, which was bounded by 0 and 1, to the real line. One can now fit this model using OLS or WLS, for example by using **regress**. Of course, one cannot perform the transformation on observations where the dependent variable is zero or one; the result will be a missing value, and that observation would subsequently be dropped from the estimation sample.
>
> A better alternative is to estimate using **glm** with **family(binomial)**, **link(logit)**, and **robust**; this is the method proposed by Papke and Wooldridge (1996).
>
> In either case, there may well be a substantive issue of interpretation. Let us focus on interpreting zeros: the same kind of issue may well arise for ones. Suppose the y variable is proportion of days workers spend off sick. There are two extreme possibilities. The first extreme is that all observed zeros are in effect sampling zeros: each worker has some nonzero probability of being off sick, and it is merely that some workers were not, in fact, off sick in our sample period. Here, we would often want to include the observed zeros in our analysis and the **glm** route is attractive. The second extreme is that some or possibly all observed zeros must be considered as structural zeros: these workers will not ever report sick, because of robust health and exemplary dedication. These are extremes, and intermediate cases are also common. In practice, it is often helpful to look at the frequency distribution: a marked spike at zero or one may well raise doubt about a single model fitted to all data.
>
> A second example might be data on trading links between countries. Suppose the y variable is proportion of imports from a certain country. Here a zero might be structural if two countries never trade, say on political or cultural grounds. A model that fits over both the zeros and the nonzeros might not be advisable, so that a different kind of model should be considered.

Baum (2008) elaborates on the problem of structural zeros and 1s. He notes "the managers of a city that spends none of its resources on preschool enrichment programs have made a discrete choice. A hospital with zero heart transplants may be a facility whose managers have chosen not to offer certain advanced services. In this context, the glm approach, while properly handling both zeros and ones, does not allow for an alternative model of behavior generating the limit values."

He suggests alternatives such as the "zero-inflated beta" model, which allows for zero values (but not unit values) in the proportion and for separate variables influencing the zero and nonzero values (i.e. something similar to the zero-inflated or hurdle models that you have for count data). A one-inflated beta model allows for separate variables influencing the one and non-one values

Both the zero and one inflated beta models can be estimated via Maarten Buis's `zoib` program, available from SSC. The help file for `zoib` says

> zoib fits by maximum likelihood a zero one inflated beta distribution to a distribution of a variable depvar. depvar ranges between 0 and 1: for example, it may be a proportion. It will estimate the probabilities of having the value 0 and/or 1 as separate processes. The logic is that we can often think of proportions of 0 or 1 as being qualitatively different and generated through a different process as the other proportions.

Here is how we can apply the one-inflated beta model to the current data. In these data, no company has a value of zero, but about a third of the cases have a value of 1, so we use the `oneinflate` option to model the 1s separately.

```
. zoib prate mrate ltotemp age i.sole, oneinflate( mrate ltotemp age i.sole)

Iteration 0:   log likelihood = -1350.3099
Iteration 1:   log likelihood = -881.01326
Iteration 2:   log likelihood =  -860.4238
Iteration 3:   log likelihood = -860.34541
Iteration 4:   log likelihood = -860.34541

ML fit of oib                                     Number of obs   =       4075
                                                  Wald chi2(4)    =     438.05
Log likelihood = -860.34541                       Prob > chi2     =     0.0000

------------------------------------------------------------------------------
       prate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
proportion   |
       mrate |   .7549614   .0570378    13.24   0.000     .6431693    .8667535
     ltotemp |  -.1138555   .0111967   -10.17   0.000    -.1358006   -.0919104
         age |   .0236683   .0022057    10.73   0.000     .0193452    .0279915
      1.sole |   .0035928   .0397814     0.09   0.928    -.0743772    .0815629
       _cons |   1.507286   .0870497    17.32   0.000     1.336672    1.677901
-------------+----------------------------------------------------------------
oneinflate   |
       mrate |   .9482935   .0859375    11.03   0.000     .7798591    1.116728
     ltotemp |  -.2918532    .027768   -10.51   0.000    -.3462776   -.2374288
         age |   .0190046   .0038664     4.92   0.000     .0114266    .0265826
      1.sole |   .6041419   .0762853     7.92   0.000     .4546255    .7536583
       _cons |   .4242109   .2017944     2.10   0.036     .0287012    .8197205
-------------+----------------------------------------------------------------
ln_phi       |
       _cons |   1.621576   .0262855    61.69   0.000     1.570057    1.673094
------------------------------------------------------------------------------
```

The one-inflate equation shows that companies with higher match rates, fewer total employees, older plans, and that have only one pension plan available are more likely to have 100% participation in their plans. When participation is not 100%, these same variables (except sole) also are associated with higher participation rates.

Other Models & Programs. I am not familiar with most of these, but the help for `zoib` suggests that some of these programs may also sometimes be helpful when modeling proportions:

`betafit` fits by maximum likelihood a two-parameter beta distribution to a distribution of a variable depvar. depvar ranges between 0 and 1: for example, it may be a proportion.

`dirifit` fits by maximum likelihood a Dirichlet distribution to a set of variables depvarlist. Each variable in depvarlist ranges between 0 and 1 and all variables in depvarlist must, for each observation, add up to 1: for example, they may be proportions.

`fmlogit` fits by quasi maximum likelihood a fractional multinomial logit model. Each variable in depvarlist ranges between 0 and 1 and all variables in depvarlist must, for each observation, add up to 1: for example, they may be proportions. It is a multivariate generalization of the fractional logit model proposed by Papke and Wooldridge (1996).

For the latter two programs, the help files give as examples models where the dependent variables are the proportions of a municipality's budget that are spent on governing, public safety, education, recreation, social work, and urban planning. Independent variables include whether or not there are any left-wing parties in city government. If I understand the models correctly, the coefficients tell you how the independent variables increase or decrease the proportion of spending in each area. For example, the results show that when there is no left wing party in city government, less of the city budget tends to get spent on education.

Fractional ivprobit commands. `fracivp` is a beta program adapted from Stata 12's `ivprobit` program. It relaxes the assumption that the dependent variable be coded 0/1 and allows it to be a proportion instead. `fracivp` estimates Fractional Response Probit models with continuous endogenous regressors. This is a use at your own risk program; it seems to work ok but I haven't fully tested it yet. `cmp` (discussed next) may be a better (or at least more proven) choice. Comments are welcome. To get `fracivp`, from within Stata type

```
net install fracivp, from(https://www3.nd.edu/~rwilliam/stata)
```

That doesn't always work though. If it doesn't work for you, try pointing your browser to

https://www3.nd.edu/~rwilliam/stata/fracivpbeta.zip

Download the file (it may download automatically), unzip it, and follow the directions for installing that are in the Readme.txt file.

Another choice for fractional ivprobit (and lots of other things) is Dennis Roodman's `cmp` (Conditional mixed process estimator with multilevel random effects and coefficients) command (available from SSC). `cmp` is incredibly powerful. Among other things, it can estimate fractional ivprobit models. See https://www.statalist.org/forums/forum/general-stata-discussion/general/1410304-stata-command-for-fractional-logit-with-endogenous-regressor for a discussion. I'll give an example but read the thread and the `cmp` help file if you want to understand it better.

```
. use https://www3.nd.edu/~rwilliam/statafiles/401kpart, clear

. cmp setup
$cmp_out      = 0
$cmp_missing  = .
$cmp_cont     = 1
$cmp_left     = 2
$cmp_right    = 3
$cmp_probit   = 4
$cmp_oprobit  = 5
$cmp_mprobit  = 6
$cmp_int      = 7
$cmp_trunc    = 8  (deprecated)
$cmp_roprobit = 9
$cmp_frac     = 10

. cmp (prate = mrate ltotemp i.sole age) (age= mrate ltotemp i.sole agesq),
ind($cmp_frac $cmp_cont)

Note: fractional probit models imply vce(robust).

Fitting individual models as starting point for full model fit.
Note: For programming reasons, these initial estimates may deviate from your
specification.
      For exact fits of each equation alone, run cmp separately on each.

      Source |       SS           df       MS      Number of obs   =     4,075
-------------+----------------------------------   F(4, 4070)      =    216.54
       Model |  25.1282708         4  6.2820677    Prob > F        =    0.0000
    Residual |  118.073925     4,070  .029010793   R-squared       =    0.1755
-------------+----------------------------------   Adj R-squared   =    0.1747
       Total |  143.202196     4,074  .035150269   Root MSE        =    .17033

------------------------------------------------------------------------------
       prate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |   .1072729   .0066685    16.09   0.000     .0941991    .1203468
     ltotemp |  -.0281719   .0018764   -15.01   0.000    -.0318507   -.0244931
      1.sole |   .0177024   .0060337     2.93   0.003     .0058732    .0295317
         age |      .0037   .0002979    12.42   0.000     .0031159    .0042841
       _cons |   .9505378   .0143984    66.02   0.000     .9223091    .9787665
------------------------------------------------------------------------------
(4,075 real changes made)
```

```
      Source |       SS           df       MS      Number of obs   =     4,075
-------------+----------------------------------   F(4, 4070)      =   6899.10
       Model |  304240.033         4  76060.0083   Prob > F        =    0.0000
    Residual |  44870.2245     4,070  11.0246252   R-squared       =    0.8715
-------------+----------------------------------   Adj R-squared   =    0.8713
       Total |  349110.258     4,074  85.6922577   Root MSE        =    3.3203

------------------------------------------------------------------------------
         age |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |   1.215105   .1289421     9.42   0.000     .9623082    1.467902
     ltotemp |   .1388149   .0365146     3.80   0.000     .0672263    .2104035
      1.sole |   .2171782   .1176027     1.85   0.065    -.0133874    .4477438
       agesq |   .0262239    .000164   159.94   0.000     .0259024    .0265454
       _cons |   2.570793   .2813236     9.14   0.000     2.019244    3.122341
------------------------------------------------------------------------------

Fitting full model.

Iteration 0:   log pseudolikelihood =  -19544.64
Iteration 1:   log pseudolikelihood = -14607.297
Iteration 2:   log pseudolikelihood = -12676.462
Iteration 3:   log pseudolikelihood = -12400.374
Iteration 4:   log pseudolikelihood = -12351.718
Iteration 5:   log pseudolikelihood = -12351.342
Iteration 6:   log pseudolikelihood = -12351.342

Mixed-process regression                        Number of obs     =      4,075
                                                Wald chi2(8)      =    2241.87
Log pseudolikelihood = -12351.342               Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
prate        |
       mrate |   .6009217   .0386194    15.56   0.000     .5252291    .6766143
     ltotemp |  -.1160397   .0079939   -14.52   0.000    -.1317074    -.100372
      1.sole |   .0961139   .0271793     3.54   0.000     .0428435    .1493842
         age |   .0162197   .0014474    11.21   0.000     .0133828    .0190566
       _cons |   1.431081   .0592794    24.14   0.000     1.314896    1.547267
-------------+----------------------------------------------------------------
age          |
       mrate |   1.215105   .1705324     7.13   0.000     .8808678    1.549342
     ltotemp |   .1388149   .0452943     3.06   0.002     .0500397    .2275901
      1.sole |   .2171782    .117622     1.85   0.065    -.0133566     .447713
       agesq |   .0262239   .0010564    24.82   0.000     .0241534    .0282944
       _cons |   2.570793   .3393261     7.58   0.000     1.905726     3.23586
-------------+----------------------------------------------------------------
    /lnsig_2 |   1.199452   .0643687    18.63   0.000     1.073292    1.325612
/atanhrho_12 |   .0319316   .0113532     2.81   0.005     .0096797    .0541834
-------------+----------------------------------------------------------------
       sig_2 |   3.318297   .2135944                      2.924991    3.764489
      rho_12 |   .0319207   .0113416                      .0096794    .0541304
------------------------------------------------------------------------------
```

```
. * Test only the first equation, since that is what fracivp does
. test [prate]

 ( 1)  [prate]mrate = 0
 ( 2)  [prate]ltotemp = 0
 ( 3)  [prate]0b.sole = 0
 ( 4)  [prate]1.sole = 0
 ( 5)  [prate]age = 0
       Constraint 3 dropped

           chi2(  4) =  674.30
         Prob > chi2 =    0.0000

. fracivp prate mrate ltotemp i.sole  (age=agesq), vce(robust) nolog

Probit model with endogenous regressors         Number of obs     =      4,075
                                                Wald chi2(4)      =     674.30
Log pseudolikelihood = -12351.342               Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0162198   .0014474    11.21   0.000     .0133829    .0190567
       mrate |   .6009217   .0386194    15.56   0.000     .5252291    .6766143
     ltotemp |  -.1160397   .0079939   -14.52   0.000    -.1317074   -.1003721
      1.sole |   .0961139   .0271793     3.54   0.000     .0428435    .1493842
       _cons |   1.431082   .0592794    24.14   0.000     1.314896    1.547267
-------------+----------------------------------------------------------------
     /athrho |   .0319314   .0113532     2.81   0.005     .0096795    .0541832
    /lnsigma |   1.199452   .0643687    18.63   0.000     1.073292    1.325612
-------------+----------------------------------------------------------------
         rho |   .0319205   .0113416                      .0096792    .0541303
       sigma |   3.318298   .2135944                      2.924991    3.764489
------------------------------------------------------------------------------
Instrumented:  age
Instruments:   mrate ltotemp 1.sole agesq
------------------------------------------------------------------------------
Wald test of exogeneity (/athrho = 0): chi2(1) =     7.91 Prob > chi2 = 0.0049
```

Both `fracivp` and `cmp` produce identical results, which makes me feel good about `fracivp`. `fracivp` may have more (untested) post-estimation options that might make it a better choice in some cases.