# Contingency Pricing for Information Goods and Services Under Industry-Wide Performance Standard

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ABSTRACT: This paper demonstrates that quality-contingent pricing is a useful mechanism for mitigating the negative effects of quality uncertainty in e-commerce and information technology services. Under contingency pricing of an information good or service, the firm pre-announces a rebate for poor performance. Consumers determine performance probabilities using publicly available historical performance data, and the firm may have additional private information with respect to its future probability distribution. Examining the monopoly case, we explicate the critical role of private information and differences in belief between the firm and market in the choice of pricing scheme. Contingent pricing is useful when the market underestimates the firm's performance; then it is optimal for the firm to offer a full-price rebate for misperformance, with a correspondingly higher price for meeting the performance stan-

*Journal of Management Information Systems* / Fall 2003, Vol. 20, No. 2, pp. 115–138.

0742–1222 / 2003 $9.50 + 0.00.

dard. We study the competitive value of contingency pricing in a duopoly setting where the firms differ in their probabilities of meeting the performance standard, but are identical in other respects. Contingency pricing is a dominant strategy for a firm when the market underestimates the firm's performance. Whereas both firms would earn equal profits if they were constrained to standard pricing, the superior firm earns greater profits under contingency pricing by setting lower expected prices. We show that contingency pricing is efficient as well, and consumer surplus increases because more consumers buy from the superior firm.

KEY WORDS AND PHRASES: contingency pricing, information goods, information-technology outsourcing, money-back guarantees, pricing of IT services, quality uncertainty.

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MANY INFORMATION TECHNOLOGY (IT) intensive commercial settings are characterized by quality uncertainty arising out of unobservability of quality or due to stochasticity in manufacturing or delivery processes. For digital goods, consumers must often make a purchase decision without inspecting the good. In e-commerce, consumers do not observe product quality until delivery and use (even for physical goods). Consumers interact with new firms, and vendors deal with wider ranges of consumers, creating greater uncertainty about the exchange. The IT sector has seen a large number of firms entering the market and an explosion of new products and services, further increasing occurrence of quality uncertainty. For many IT services, consumers require end-to-end services and quality levels, even though in many cases firms control only a part of the service chain. For instance, in online trading, firms may guarantee overall trade execution times although they do not control market-clearing and communication functions. Hence, there is inherent uncertainty in the quality of the delivered product. Quality uncertainty also arises in IT outsourcing since the outsourcing firm cedes control to the service provider. The tremendous growth in application outsourcing [18], data storage outsourcing [15], computer security outsourcing [17], and offshore software development [12] makes it vital to understand and address the effects of quality uncertainty. Although quality uncertainty has been studied in the economics and marketing literatures (see, e.g., [5, 6]), ours is one of the first papers to analyze quality uncertainty for information goods and services, and to propose a pricing mechanism to mitigate its effects.

How does quality uncertainty affect commerce—for instance, the existence of markets or competition between firms? There are obvious negative effects. Firms must take quality uncertainty into account in setting prices, thereby putting downward pressure on product prices. On the other hand, consumers also factor in quality uncertainty in determining their valuations for products, and often may balk from purchasing. Quality uncertainty deters trade, especially when firms and consumers differ in their expectations about product quality. It is common for firms to advertise performance promises, but these promises have little meaning when they are either vague, not enforceable, or do not specify a penalty when promised performance is not delivered.

We examine the use of quality-contingent prices as a mechanism to mitigate the effects of quality uncertainty. In contingency pricing, the firm announces quality–price pairs for various levels of quality instead of a single price. Often, the quality threshold emerges from industry-wide performance standards, such as a 55 ms latency in high-speed private networks. Any of the possible realizations of quality level may be achieved, and the consumer pays depending upon the quality of the product or service received. Contingency pricing works well when quality is objectively verifiable and is unaffected by use, thus avoiding moral hazard. The framework assumes the availability of performance information—how well the firms are likely to deliver on quality. Therefore, contingency pricing is especially well-suited to IT services because the information infrastructure enables easy quantification, capture, verification, and dissemination of quality and performance information. The emergence of businesses such as eTesting Labs (www.etestinglabs.com) that specialize in collecting and reporting performance data for various products and firms in the IT sector exemplifies this trend.

Examples of contingent pricing structures in IT goods and services include download speeds, ISP uptime guarantees (MSN), trade execution times (AmeriTrade), and ASP service level guarantees (see, e.g., [7]). For a specific example of the use of contingency pricing as well as the availability of performance information under quality uncertainty, consider the market for carrying Internet data traffic. In order to mitigate quality concerns, a leading Internet backbone provider, Sprint (www.sprintbiz.com/business/extranet/service.html), announced a 100 percent end-to-end availability guarantee, with the following contingencies and rebates: outages of an hour or less lead to a three-day, pro-rated monthly port; longer outages lead to a day's credit for each hour the outage persists. Customers can view performance statistics on the firm's Web site, and are offered a suite of tools to monitor and validate network performance and service levels. Similar contingent price offerings are becoming common for network latency (Genuity guarantees a 55 ms round-trip latency and 0.5 percent packet loss) and managed hosting services (99.5 percent Web server availability guarantees from Sprint).

Other mechanisms for dealing with quality uncertainty include money-back guarantees (see Mann and Wissink [9] and Moorthy and Srinivasan [13]), warranties (Lutz and Padmanabhan [8] and Matthews and Moore [11]), and limited time trials. Although contingency pricing shares some features with these mechanisms, it differs significantly from each. In money-back guarantees, there is no notion of quality contingency. Consumers may return the product for any reason; hence, transaction costs for the firm and consumer play a significant role, whereas with contingency pricing, products need not be returned; a simple monetary transaction completes the transaction. In the case of software goods, many firms offer 30-day trials to consumers to provide them a chance to observe quality, but this mechanism is essentially a money-back guarantee with a 30-day fuse. Warranties may be bundled with the product, or sold unbundled as extended warranties, often by a different firm. They involve the return and repair of the product to original specification, upon which the warranty may restart again, thereby involving the analysis of repeated gamble. Contingent

pricing does not restore the product to original quality. IT goods and services differ from traditional goods in providing the ability to monitor the quality of the good or service as it is delivered and the ability to act on this performance information. For many IT-intensive goods and services, where the product delivery and consumption are simultaneous, these mechanisms do not apply, whereas contingent prices work well.

Bazerman and Gillespie [2] write about the potential value of contingency pricing under quality uncertainty, and note the lack of managerial understanding regarding how to design optimal contingent pricing schemes. Bhargava and Sundaresan [3] motivate contingency pricing and analyze its applicability and implementability in modern IT settings, using a monopoly profit maximization model. Biyalogorsky and Gerstner [4] study contingency pricing under demand uncertainty, and show that a firm may benefit by offering a low-type customer a contingency sale at a lower price, where the sale becomes invalid if the firm receives a high-type customer in a specified time frame.

## Framework for Contingency Pricing

WE TREAT PRODUCT QUALITY $q$ as stochastic and in some bounded continuous interval as given by a probability distribution function that specifies the probability that quality is lower than $q$. Consumers have historical information about the firm's performance, typically using data that are metered and disseminated by third-party firms, regulatory agencies, or the seller. The IT sector has a unique ability, due to the digital nature of many goods and services, to collect and disseminate such information at a very low cost. For example, eTesting Labs provides performance information on dozens of IT products and firms, the FCC provides data on outages at Internet traffic carriers, and Sprint provides detailed performance data on its Web site. We note that this public historical information need neither be entirely accurate nor a perfect predictor of performance in the next period.

We focus on the case where the firm sets prices around just one quality threshold $\hat{q}$ so that there are two quality intervals, *standard* quality ($q_{High}$) and *inferior* quality ($q_{Low}$). The main results generalize to multipart contracts with multiple quality levels. Often, the quality threshold emerges as an industry-wide performance standard, for example, a 55 ms latency in data networks. Let $p$ represent the probability that performance falls below $\hat{q}$ (known to the firm), and let $\mu$ represent the consumers' belief that performance falls below $\hat{q}$ (we call this the public performance probability). The firm announces contingent prices $(R - r, R)$, where $R$ is the price for the standard quality, and $r$ is the rebate to the consumer if an inferior quality is realized. The final price paid by the buyer is contingent on the quality level realized.

The firm has constant expected marginal cost $\hat{C}$, which incorporates the production costs, costs due to customer service calls, error handling, or other processes required in dealing with inferior quality. The firm's objectives are to determine when to offer contingency pricing and to choose the optimal price and rebate. This design appears similar to the quality differentiation problem (see, e.g., [10, 14]), where the

firm also offers price–quality combinations. The crucial difference with the work on quality differentiation is that the literature has explored the case where the firm is able—with *certainty*—to offer each quality level on the menu: the intent is to let consumers *self-select* into segments. Contingency pricing, on the other hand, focuses on quality uncertainty: the multiple qualities are only possible realizations for product quality. The combination of the stochastic quality levels and associated prices (along with associated probabilities) defines a single choice item offered to all users—users do not have the option to select a particular quality.

Our framework assumes that the firm possesses private information with regard to the process that generates the quality distribution. Such information—about resource allocations, continuous improvements to technology, planned maintenance and breakdowns, process changes, and recent events—allows it to measure the true probabilities of meeting performance levels. Hence, we assume that the firm's information, the probability $p$, reflects the true state of the world and it may differ from the publicly available performance information $\mu$.

To model consumer choice under quality uncertainty, let the index $v$ represent consumer types, and let $U(v)$ represent consumer type $v$'s expected valuation for the product, where

$$U(v) = (1-\mu)U\left(v, q_{High}\right) + \mu U\left(v, q_{Low}\right).$$

Consumers are heterogeneous in their valuation for the product and have different sensitivities to quality. Consumers are aware of the public performance probability $\mu$, and make decisions as expected value maximizers. All consumers see the same expected price

$$\bar{R} = R(1-\mu) + (R-r)\mu = R - \mu r$$

and the firm gets an expected revenue per sale of

$$\hat{R} = R(1-p) + (R-r)p = R - pr.$$

We order the consumer types $v$ so that $U(v)$ is increasing in $v$.

## Contingency Pricing by Monopoly

WE BEGIN OUR ANALYSIS OF CONTINGENCY PRICING by examining a monopolistic firm that offers a single product. This enables us to isolate the impact of contingency pricing and serves as a starting point to develop a model of contingency pricing under competition (see the fourth section). The insights from the monopoly model are directly applicable when a single firm dominates the market. The monopoly results are also applicable when multiple firms have considerable market power with respect to groups of customers (who may have high switching costs or otherwise are locked-in to the respective firm).

For ease of exposition, assume that consumer types are uniformly distributed along the support [0,1] (the results easily generalize to other distributions). A type $v$ customer has expected surplus $U(v) - \bar{R}$ on buying the product with expected price $\bar{R}$. Setting $U(v) - \bar{R}$ yields the marginal customer $v_m$ who is indifferent toward buying the product, thus $U(v_m) - \bar{R}$. Hence, the fraction of buyers is $1 - v_m$. The demand function depends only on the market price expectation $\bar{R}$ rather than the actual prices $(R - r, R)$. Rewriting $v_m$ as $U^{-1}(\bar{R})$, the demand function is $D(\bar{R}) = 1 - U^{-1}(\bar{R})$. We make the usual assumption that the demand function satisfies nondecreasing price elasticity, that is, that the proportional fall in demand caused by a proportional change in expected price weakly increases with price. Formally, we assume that the term $(-D'(\bar{R}))/(D(\bar{R}))\,\bar{R}$ increases in $\bar{R}$.

## Role of Private Information

The firm's expected profit under quality uncertainty and contingency pricing is

$$\pi = \left(\hat{R} - \hat{C}\right) D\left(\bar{R}\right),$$

where $\hat{R}$ is the firm's revenue expectation per sale. By definition, $\hat{R} = R - pr$, so that $\hat{R} = \bar{R} + r(\mu - p)$. Notice that the standard pricing scheme with a single price and no rebate is just a special case of the contingency pricing scheme with $r = 0$. A second special case of interest is the *full rebate* solution $r = R$ where the firm offers a full-price rebate for inferior quality. The optimal pricing scheme is

$$\left(R^*, r^*\right) = \underset{(R,r)}{\arg\max}\, \pi = \underset{(R,r)}{\arg\max} \left(\bar{R} + r\left(\mu - p\right) - \hat{C}\right) D\left(\bar{R}\right).$$

Solving for optimal prices, we find that:

> *Proposition 1: It is never optimal for the firm to offer less than a full-price rebate ($r < R$) except when the market has perfect information about the performance probability at quality level $\hat{q}$ (i.e., $\mu = p$). When public information is perfect, a convex combination of prices is optimal, including a full-rebate contingent price contract and a standard single price contract, indicating that contingency pricing offers no additional value in this case.*

See Appendix A for the proof. This proposition formalizes the intuition that when there is any benefit from offering a certain rebate $r$, then a higher rebate (and a smaller increase in the price $R$) will confer an even greater benefit (note that a unit increase in $R$ requires a greater increase in rebate because the probability of rebate is less than one). If $\bar{R}^*$ is the expected price that induces the optimal number of consumers to buy, the firm may achieve $\bar{R}^*$ by a number of price–rebate combinations $(\bar{R} + r\mu, r)$. Since the firm's profits increase with the offered rebate (and corresponding higher price), it will offer the maximum possible rebate, equal to price. When the market

underestimates the firm's performance, the firm is able to play this game. However, when public information is perfect, the firm has no advantage in playing this game. Under perfect information, the optimal expected price can be achieved with infinite combinations of price and rebate, including the zero-rebate standard pricing scheme. All of these combinations yield the same market share and the same margin, hence identical profit.

## Optimal Contingency Pricing Under Private Information

Proposition 1 indicates that the optimal pricing scheme under private information is either a standard single price $R^s$ (no penalty for performance failure) or a contingency pricing scheme with price $R^c$ and a full-price rebate for inferior performance. We compute and compare the optimal prices and profits under each case, and determine conditions under which each pricing scheme is optimal.

*Lemma 1: The optimal single price $R^s$ is the unique solution to the equation*

$$\frac{-D(\bar{R})}{\bar{R}\cdot D'(\bar{R})} = \frac{\bar{R}-\hat{C}}{\bar{R}}. \tag{2}$$

*For contingency pricing, the optimal price is $R^c = (\bar{R}^c/(1-m))$, where the market price expectation $\bar{R}^c$ is the unique solution to the equation*

$$\frac{-D(\bar{R})}{\bar{R}\cdot D'(\bar{R})} = \frac{\bar{R}-\left(\frac{1-\mu}{1-p}\right)\hat{C}}{\bar{R}} = \frac{\hat{R}^c-\hat{C}}{\hat{R}^c}. \tag{3}$$

See Appendix A for the proof. For the standard pricing scheme, the optimal price represents the usual condition that the inverse elasticity of demand equals the firm's market power (margin relative to price). The optimal contingent price follows a similar structure, except that inverse elasticity of demand now equals the firm's *effective* market power. Figure 1 illustrates the optimal solutions under the standard and contingency pricing schemes for the case where $\mu > p$ (i.e., the market underestimates the firm's ability to deliver quality level $\hat{q}$, $(1-\mu)/(1-p) < 1$).

It is clear from the two expressions (and Figure 1) that when $(1-\mu)/(1-p) < 1$, consumers see a lower expected price under the optimal contingency pricing scheme than the optimal standard price $(\bar{R}^c < R^s)$, thus the firm expands its market coverage with contingency pricing. Further, since the inverse elasticity of demand is downward sloping, the firm's effective market power is greater when it employs contingency pricing. The optimal profits for the two cases are $\pi^s = D(R^s)(R^s - \hat{C})$ and $\pi^c = D(\bar{R})(\hat{R}^c - \hat{C})$, respectively.

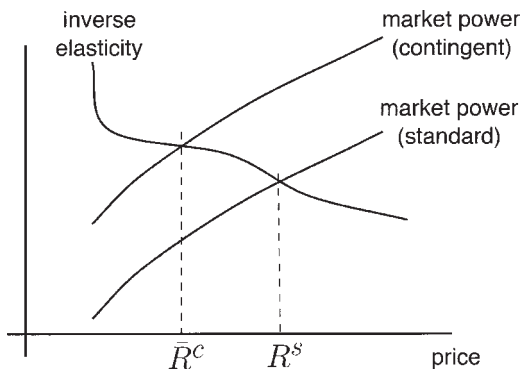


*Figure 1.* Optimal Prices Under Standard Pricing and Full-Rebate Contingency Pricing when the Market Underestimates Firm Performance

> *Proposition 2: The firm will apply contingency pricing if and only if $\mu > p$, that is, when the market underestimates the firm's performance. When $\mu > p$, the firm offers a contingent contract with price $R^c$ (and a full-price rebate); when $\mu < p$ the firm should offer a standard single-price contract with price $R^s$.*

We provide the complete proof in Appendix A but the main idea is straightforward. We design a contingent price contract that has a price expectation equal to the optimal standard price—hence it yields the same market share—but generates higher expected revenue per sale and profit (because $\mu > p$).

To understand this result, let us interpret a contingent contract as a lottery. When the market overestimates the firm's inability to deliver promised quality—that is, consumers overestimate the probability of receiving a rebate—a contingent contract represents a lottery biased in favor of the firm. Since the bias increases with the size of the rebate, the firm offers the maximum (full-price) rebate. Conversely, when the firm realizes that its performance will be poorer than what the market believes, a contingent contract represents a lottery biased against the firm. Hence, the firm chooses not to employ this mechanism. It is interesting to note that the literature on money-back guarantees [9, 13] often assumes that the price will be fully refunded when consumers return the product. Here we formally show that a "full-rebate" policy is indeed optimal under market overestimation. Propositions 1 and 2 also highlight a rather simple *all-or-nothing* contingency pricing strategy for managers when they have good reason to believe that they can beat market expectations. The result also mirrors the spirit of the full-insurance conditions in insurance markets under similar conditions.

## Illustration

We illustrate the effect and design of contingency pricing with a linear form for consumer valuations. Specifically, consumers discern only two quality levels—standard and inferior. Let $U(v,q_{High}) = v$ and $U(v,q_{Low}) = v - (\alpha + \delta v)$, where $(\alpha + \delta v)$ is consumer

type $v$'s disutility for inferior quality $q_{Low}$ so that disutility is proportionally increasing with valuation. For instance, if a network is down for an hour, the loss is greater to the consumer who uses the computer network for higher-value activities. The variables $\hat{C}$, $\mu$, and $p$ represent expected marginal costs, and public and private performance probabilities, respectively, as before.

We investigate the existence of the market by inspecting participation constraints for the consumer as well as the firm. For the market to exist under contingency pricing, there must exist an expected price $\bar{R}$ such that both the firm and the highest value consumer ($v = 1$) earn a positive surplus.

$$\hat{R} - \hat{C} > 0$$
$$\left(1 - \mu\left(\alpha + \delta\right)\right) - \hat{R} > 0.$$

Combining these two equations, and using the condition that $\hat{R} = \bar{R} + r(\mu - p)$, we get the constraint

$$\hat{C} - \left(1 - \mu\left(\alpha + \delta\right)\right) < r\left(\mu - p\right), \tag{4}$$

which is looser than the corresponding existence constraint under standard pricing (which requires $r = 0$) when $\mu - p > 0$. Hence, contingency pricing expands the likelihood of market existence.

The shape of the feasible regions for both types of contracts depends on the relationship between the disutility and the firm's expected cost $\hat{C}$, as illustrated in Figure 2. The feasibility region is the region below the respective market participation constraints. Standard pricing is feasible only for certain combinations of $p$ and $\mu$ (the shaded region), whereas contingency pricing can yield a feasible solution for additional combinations (the region with the x's).

The relative outcomes for the optimal standard and optimal contingency pricing schemes are given in Table 1. It can be seen that when $\mu > p$, contingency pricing yields a lower expected price and increases market coverage. The increase in profit due to contingency pricing is

$$\frac{\mu - p}{4\left(1 - \delta\mu\right)}\left(\frac{\left(1 - \left(\alpha + \delta\right)\mu\right)^2}{1 - \mu} - \frac{\hat{C}}{1 - p}\right),$$

which is positive (when $\mu > p$) as long as the market participation constraint is satisfied. Moreover, contingency pricing improves the ex post total consumer surplus in the amount

$$\frac{\left(\mu - p\right)^2 \hat{C}^2}{8\left(1 - p\right)^2\left(1 - \delta\mu\right)},$$

demonstrating that contingency pricing improves efficiency, raising both consumer surplus and firm profits.

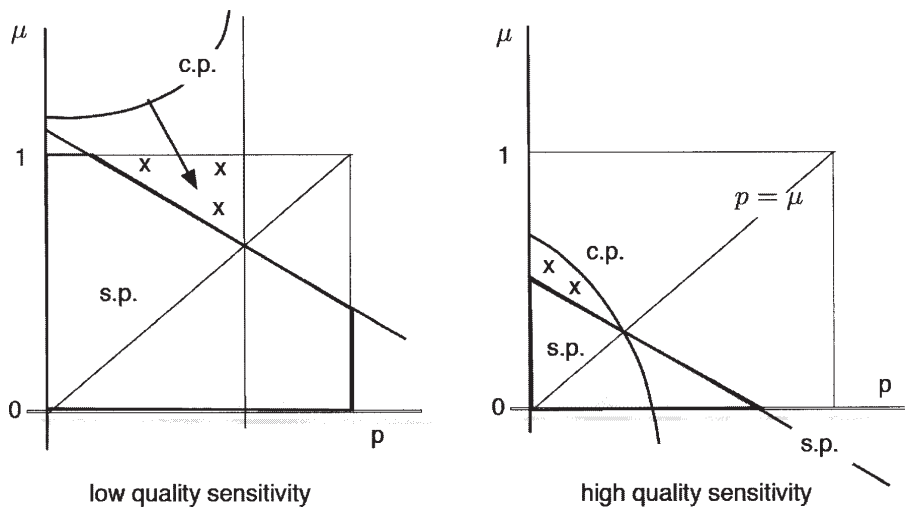


*Figure 2.* Feasible Regions for Single Price and Contingent Price Contracts when Disutility from Inferior Quality is Low (left) and High (right). *Notes:* The region marked with the x's indicates the combinations of $p$ and $\mu$ values under which contingency pricing expands the likelihood of market existence.

Table 1. Optimal Price and Market Coverage Under Standard and Contingency Pricing

| | Standard pricing | Contingency pricing |
|---|---|---|
| Expected price | $\frac{1}{2}\left(1+\hat{C}-(\alpha+\delta)\mu\right)$ | $\frac{1}{2}\left(1+\hat{C}\frac{1-\mu}{1-p}-(\alpha+\delta)\mu\right)$ |
| Market coverage | $\frac{1}{2}-\frac{1}{2(1-\delta\mu)}\left(\hat{C}+\alpha\mu\right)$ | $\frac{1}{2}-\frac{1}{2(1-\delta\mu)}\left(\hat{C}\frac{1-\mu}{1-p}+\alpha\mu\right)$ |

In the region where contingent pricing is preferred, it is interesting to examine how the fraction of buyers, firm's margin, and profit change as the public probability $\mu$ changes. We state the results below, please see Appendix A for the proof.

> *Proposition 3: When customers are highly quality sensitive ($\alpha + \delta > 1$), then the firm's optimal price (and rebate) $R^c$, expected revenue per sale $R^c(1 - p)$, and margin decrease with an increase in $\mu$, the market's estimate of firm mis-performance. When quality sensitivity is low (i.e., $\alpha + \delta < 1$), these outcomes increase with an increase in $\mu$. In both cases, however, the firm sets prices such that the consumers see a lower expected price for higher $\mu$.*

Consumer underestimation of the firm's performance ($\mu > p$) introduces a trade-off for the firm: it can exploit the difference in belief via contingency pricing, but it must

also overcome the lower valuations by offering a lower expected price. A change in $\mu$ changes this trade-off. When customers are less sensitive to quality, an increase in $\mu$ increases the positive effect from contingency pricing more than the increase in the negative effect from lower valuations. When customers are highly sensitive to quality, the negative consequence of lower price dominates the benefit from contingency pricing.

## Contingency Pricing Under Duopolistic Competition

HAVING DEMONSTRATED THE VALUE OF contingency pricing for a monopoly firm, in this section, we discuss contingency pricing in a duopolistic setting since it allows us to incorporate the strategic interactions that characterize competition while maintaining the tractability of the analytical formulation.

Suppose there are two firms of low- and high-quality $L$ and $H$, respectively. The higher-quality firm has a lower probability of failing to meet the level of specified performance, thus $p_H \leq p_L$. The firms are identical in other respects such as consumer preferences, marginal costs, and market estimation of their performance $\mu$. This framework applies to a scenario where two firms are incorporating a new technology into an established product. One firm—the high-quality firm—has a superior adaptation of the technology. However, the market estimates reflect the historical performance of the established product (common to both firms), hence consumers are unable to distinguish between the two firms. As a specific example, consider competition between backbone Internet service providers where latency is one of the measures of quality, the industry standard today is 55 ms round-trip delay, and most firms have similar technologies; however, different adaptations of a new algorithm or faster switching hardware can cause a short-term difference in performance levels between competing firms. The market's best estimate may be identical for all firms, due to lack of distinguishing information.

As before, consumers have heterogeneous valuations, high-value consumers have a greater disutility for quality failure, and the pricing schemes are designed around an exogenously specified industry quality threshold $\hat{q}$. Each firm $i$ possesses private information on its own probability of failure $p_i$. Each firm chooses its pricing scheme with price $R_i$ and rebate $r_i$ to maximize its profits. As before, we write $\bar{R}_i$ and $\hat{R}_i$ to denote the consumers' and firm's price expectations. Consumers' purchase decision is based on both expected prices and additional attributes of each firm with respect to their preferences. To make the problem interesting, we assume that the two firms cover the market. We seek to investigate optimal pricing strategy (whether to offer contingency pricing and how to set prices) for each firm and the impact of contingency pricing on market shares, profits, and social welfare.

In order to isolate the effects of contingency pricing, we develop a model of competition in which the firms differ in their true performance distribution, but are otherwise equally competitive with respect to costs and consumer preferences. Specifically, the firms would split the market equally if they were to offer the same expected price

$\bar{R}$. A parsimonious way to formalize this is to consider the firms to engage in Bertrand competition under a given demand model

$$D_i = \beta_1 - \beta_2 \bar{R}_i + \beta_2 \bar{R}_j. \tag{5}$$

In order to illustrate this sort of competition under quality uncertainty, we extend the Salop model of spatial competition between firms [16]. In Salop's circular city model, consumers are distributed uniformly along a circle that represents some product attribute for which consumers have heterogeneous preferences. Firms locate along the circle, all locations being equally advantageous, a priori. Salop's model enables one to examine price competition between firms with differentiated products. The quality uncertainty in our framework raises the need to model multiple quality levels, hence we superimpose a quality dimension on top of the two-dimensional circular space.

Imagine that consumers are distributed uniformly along a circle (here, of radius $1/\pi$). The two firms are positioned diametrically opposite in the attribute space, which follows the equilibrium of maximal differentiation. For either firm, customers have a valuation $v$ for the standard quality product, but incur a misfit cost $xt$ proportional to their distance $x$ along the arc from the firm's product, yielding a net valuation $v - xt$. We assume an additional misfit cost for inferior quality, given by $a + d(v - xt)$, so that the loss is monotonically increasing in proximity to the product, hence higher-value customers suffer a greater loss in valuation on receiving inferior quality. Here, $a$ may be interpreted as a constant loss for all consumers, and $d$ is the percentage loss in net valuation for customer at distance $x$. Thus, the net valuation of the inferior quality is $(v - xt) - (a + d(v - xt))$. To depict the valuations for standard and inferior quality, we represent the distance $x$ on a line segment of length 1 (the distance between the two firms) as shown in Figure 3.

Combining the valuations for standard and inferior quality, under a mis-performance estimate $\mu$, the consumer at distance $x$ from firm $i$ has an expected surplus $(v - (a + dv)\mu) - t(1 - \mu d)x - \bar{R}_i$ on purchasing from firm $i$. The consumer who gets equal surplus from both firms is indifferent between them; others consumers on either side of this indifferent consumer prefer the respective firm closer to them. This generates the demand functions $D_H$ and $D_L$ (as a function of expected prices), where

$$D_i = D_i\left(\bar{R}_i, \bar{R}_j\right) = \frac{1}{2} - \frac{\bar{R}_i}{2t\left(1-\mu d\right)} + \frac{\bar{R}_j}{2t\left(1-\mu d\right)},$$

which is mathematically identical to Equation (5) with suitable transformation of parameters. Firm $i$'s profit function is $\pi_i = (\hat{R}_i - \hat{C})\, D_i$. It is easy to see that the firms capture equal market share if they offer identical expected prices; otherwise, the higher-priced firm captures a smaller share of the market. Furthermore, we note that were the two firms both constrained to use only standard pricing, they would be equally competitive and capture equal market share and profits, even though firm $H$ is, unknown to consumers, superior.

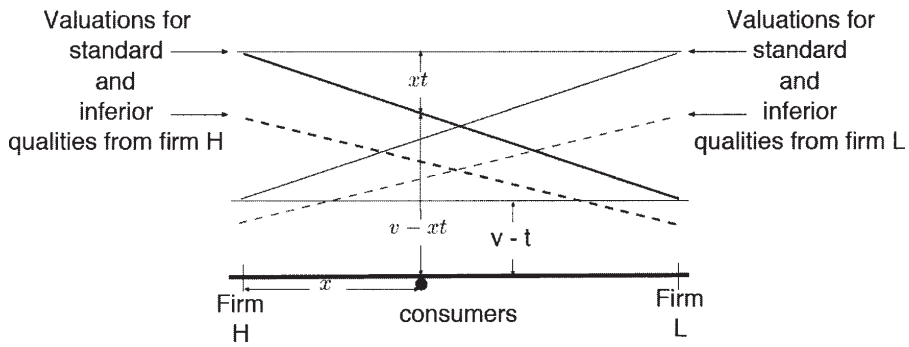


*Figure 3.* Duopoly Competition Under Quality Uncertainty

## Standard or Contingency Pricing?

Let $\bar{R}_j$ denote the expected price of firm *j*. Firm *i*'s pricing scheme is a best response to $\bar{R}_j$, regardless of whether firm *j* employs standard or contingency pricing. This follows easily from the nature of the demand function, since consumer reaction (and firm *i*'s profit function) is on the competitor's expected price.

Let $\bar{R}_j$ denote any standard price chosen by firm *i* given $\bar{R}_j$, and let $\pi_i$ be the corresponding profit. Consider an alternate contingent contract with a contingent price $\bar{R}_i/(1-\mu)$ and a full rebate. This offer generates the same expected price $\bar{R}_i$ but yields a higher expected revenue per sale $\bar{R}_i((1-p_i)/(1-\mu))$ when $\mu > p_i$. Hence, the firm earns greater profits. Thus,

> *Proposition 4: Contingency pricing is a dominant strategy for the firm when the market underestimates the firm's performance.*

From a managerial perspective, we see that each firm chooses its pricing strategy based on its own performance, and then determines actual prices as a standard best-response to competitor's prices. Each firm chooses contingency pricing when the market underestimates its own performance, and standard pricing when the market overestimates performance. It might seem surprising that the firm's choice of pricing strategy does not depend on the pricing scheme chosen by the rival firm. The pricing scheme employed by firm *i* depends on whether contingency or standard pricing yields a better profit. Within each scheme, the optimal expected price of firm *i* depends on expected price (to consumers) of the rival firm *j*, which itself depends on whether firm *j* employs contingency or standard pricing. It appears, therefore, that firm *i*'s choice of pricing scheme should depend on firm *j*'s pricing scheme—why, then, does it not? The reason is that regardless of the expected price offered by firm *j*, and regardless of whether the expected price results from a single or contingent prices, firm *i* can construct a better response using contingency pricing than if it employs standard pricing (as shown above). Another way to understand the independence between the competing firms' choices of pricing scheme is that the profit of each firm depends on its profit margin and the demand it can capture. The demand is affected

by competitive pressure reflected in the two firms' expected prices. However, given an expected price, each firm has the flexibility to determine its true profit margin by adjusting the price and rebate. When the market underestimates a firm's performance, it can always (as in the monopoly case) improve its profit margin by offering a rebate for mis-performance.

## Price Response Functions

Firm $i$'s prices are a best response to firm $j$'s expected price $\bar{R}_j$, but its pricing strategy depends only on the market's under- or overestimation of its own performance. We first determine the best-response pricing functions in each case and then compare the respective optimal prices.

Consider how the demand function for a firm changes with respect to its own price. Let $D'$ denote the partial derivative of firm $i$'s demand $D_i$ with respect to its expected price $\bar{R}_i$, so that $D' = (-1/(2t(1 - \mu d))$. Examining first-order conditions (please see Appendix B), the optimal price–rebate policy lies on the boundary $r_i = 0$ or $r_i = R_i$, except when public information is perfect ($p = \mu$) in which case several combinations of price and rebate are optimal. Thus, *neither firm will offer less than full-price rebate* should it choose contingency pricing, analogous to the result of Proposition 1.

Therefore we study the optimal pricing of each firm under the two special cases of standard pricing and full-rebate contingency pricing. Let $\Omega_i^c(\bar{R}_j)$ be firm $i$'s best full-rebate contingency price response function to firm $j$'s expected price. Note that the function returns the expected price seen by consumers, given the full-rebate pricing scheme employed by the firm. Similarly, $\Omega_i^s(\bar{R}_j)$ is firm $i$'s best standard price response to $\bar{R}_j$.

For the case of standard pricing, let $\bar{R}_i^s$ be firm $i$'s single price, and let $\pi_i^s$ be the corresponding profit. As shown in Appendix B, the first-order condition yields the best response function $\Omega_i^s(\bar{R}_j)$ for firm $i$'s price as

$$\Omega_i^s\left(\bar{R}_j\right) = \frac{\hat{C}}{2} + \frac{1}{2}\left(\bar{R}_j + (1-\delta\mu)t\right). \tag{6}$$

To examine the best contingency price response, let $R_i^c$ be the contingent price (and rebate) for firm $i$, so that the market expectation of the firm's price is $\bar{R}_i^c = R_i^c(1 - \mu)$ and its expected revenue per sale is $\bar{R}_i^c = R_i^c(1 - p_i)$. The first-order condition yields the price response function (see Appendix B)

$$\Omega_i^c\left(\bar{R}_j\right) = \frac{\hat{C}}{2}\frac{1-\mu}{1-p_i} + \frac{1}{2}\left(\bar{R}_j + (1-\delta\mu)t\right). \tag{7}$$

Since $\mu > p_i$, we can see by comparing the RHS of Equations (6) and (7) that

*Proposition 5: Firm i's contingent price response (to any expected price $\bar{R}_j$ of the competing firm) always yields a lower expected price to the consumer than*

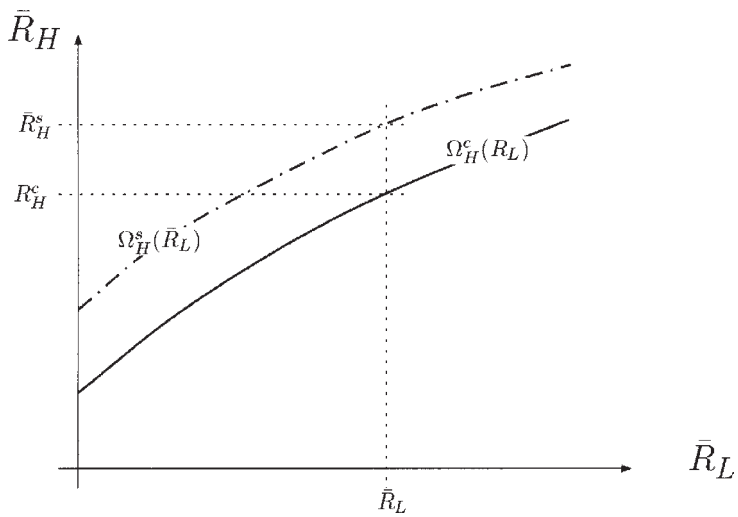


*Figure 4.* Price Response Functions Under Standard and Contingency Pricing

*its best response under single pricing, when the market underestimates firm i's performance. Formally, $\Omega_i^c < \Omega_i^s$ when $\mu > p_i$.*

Figure 4 provides a graphical illustration. This proposition generalizes the monopoly result that the optimal expected price is lower under contingency pricing when the market underestimates the firm's performance.

## Equilibrium Prices and Outcomes Under Market Underestimation

We focus now on the case where the market underestimates both firms' performance, ignoring the less interesting cases where the market overestimates performance (where the firm chooses standard pricing). Under this case, both firms will independently prefer to offer full-rebate contingency pricing. The equilibrium prices are obtained at the intersection of the two firms' best-response functions $Q_i^c(\bar{R}_j)$ given by the respective realizations of Equation (7). The unique optimal values of $\bar{R}_H^c$ and $\bar{R}_L^c$ are

$$\bar{R}_H^c = \frac{\hat{C}(1-\mu)}{3(1-p_L)} + \frac{2\hat{C}(1-\mu)}{3(1-p_H)} - (t-\mu d) \tag{8}$$

$$\bar{R}_L^c = \frac{\hat{C}(1-\mu)}{3(1-p_H)} + \frac{2\hat{C}(1-\mu)}{3(1-p_L)} - (t-\mu d). \tag{9}$$

Under duopoly, Proposition 4 extended the monopoly result and demonstrated that each firm benefits from contingency pricing when the market underestimates the firm's performance. But when the firms are heterogeneous, which one does better

under contingency pricing? Clearly, both firms cannot improve market coverage with contingency pricing, since this is now a zero-sum game. Will firm *H* gain market share at the expense of *L?* Or will it prefer to improve its margin instead? How do their prices compare? Does contingency pricing improve both firm's profits?

*Proposition 6: Under contingency pricing, the optimal expected price* $\bar{R}_H^c$ *of the higher quality firm is lower than the optimal expected price* $\bar{R}_L^c$ *of the lower quality firm. Hence firm H captures a greater market share under contingency pricing.*

Please see Appendix B for the proof. Note that Proposition 6 does not claim that firm *H* earns smaller expected revenue per sale; rather, it states that consumers see a lower expected price from firm *H*. Prior research demonstrates that higher prices can signal a higher-quality product (see, e.g., [1]) and finds positive price–quality correlation (see, e.g., [19]). Our result points to a rather counterintuitive result that the firm with better quality should offer a lower expected price to the consumer. Why does the higher-quality firm set a lower expected price? Each firm, in examining its optimal contingency prices under market underestimation, sees an advantage in luring customers with a lower expected price by offering a rebate since it is aware that the true expected cost of rebate is lower. Since firm *H* is superior in this respect, it is better able to deploy this tool and therefore captures (at the expense of firm *L*) a greater market share in equilibrium. Firm *H* chooses to set a lower expected price since this is the only mechanism it has to take advantage of its better performance, given that the consumers see no difference between the performances of the two firms. Thus, contingency pricing may be seen as a mechanism that allows the firm to benefit from superior quality, and its use is somewhat different from mechanisms that signal higher quality such as advertising or higher prices.

Despite offering the lower expected price, the superior firm *H* makes greater profit than firm *L* when $p_L > p_H$. This is quite intuitive since, for any expected price from firm *L*, firm *H* earns higher expected revenue per sale by offering identical price and rebate. We formally state the result below, please see Appendix B for proof.

*Proposition 7: The superior firm H makes greater profit than firm L when* $\mu > p_L > p_H$.

Contingency pricing confers competitive advantage to the superior firm. Whereas both firms benefit from the use of contingency pricing, the superior firm gains more. This contrasts with the benchmark case of standard pricing where both firms would earn equal profit, since the better performance would make no difference under standard pricing (when the market estimation of their performance is identical). Finally, the gain in consumer surplus when both firms optimally use contingency pricing, compared with if they were constrained to use standard pricing, is positive:

$$\Delta(\text{consumer surplus}) = \frac{\hat{C}}{2}\left(\frac{\mu - p_H}{1 - p_H} + \frac{\mu - p_L}{1 - p_L}\right) + \frac{\hat{C}^2 (p_L - p_H)^2 (1-\mu)^2}{36(1-\delta\mu)t(1-p_H)^2(1-p_L)^2},$$

again demonstrating that contingency pricing is not only profit-maximizing but also efficient—a better way to trade when there are differences in belief.

## Conclusion

MANY IT PRODUCTS AND SERVICES ARE CHARACTERIZED by quality uncertainty. We expect performance uncertainty to be an increasingly important issue in the IT sector due to the increases toward various forms of IT outsourcing—managed hosting, data center services, application, and business process outsourcing. Establishing product credibility under performance uncertainty is also crucial for new IT products and services, such as network troubleshooting software. In spite of the widespread occurrence of quality uncertainty in the IT sector, there is little analysis of this phenomenon in the information systems literature. Traditional mechanisms, such as warranties and money-back guarantees, are not suitable for IT goods when quality is realized only upon delivery or use. This paper proposes and develops a pricing mechanism to address and mitigate quality uncertainty for information goods and services. Contingency pricing is feasible in such environments because the IT infrastructure and the digital nature of these goods and services makes it possible to define quality dimensions that can be objectively measured, verified, and disseminated. We demonstrate that contingency pricing is an attractive mechanism for dealing with quality uncertainty for IT goods and services.

We have developed economic models to examine the optimal design of contingency pricing and establish that contingency pricing can expand commerce by mitigating the effects of quality uncertainty. Our analysis demonstrates the critical role of performance probabilities and private information in this framework. Contingency pricing is attractive when the firm has private information about performance (specifically when it expects to perform better than perceived by the market), both increasing profits and enabling market existence in situations where no single-price contract could provide positive social welfare. Interestingly, in this case, the firm offers a full-price rebate for inferior quality. Under duopolistic competition, each firm's optimal choice regarding the use of contingency pricing depends only on whether the market underestimates the firm's own performance. When the market underestimates both firms—the scenario examined in this article—then both employ contingency pricing. The superior firm, however, derives a competitive advantage, gaining market share and a greater increase in profits due to contingency pricing. It is better able to employ the contingency price mechanism, and its choice of rebate results in a lower-price expectation for consumers, thus presenting the superior firm with greater market share. Compared to the case of standard pricing, where both firms are equally competitive in the market, contingency pricing decreases competitive intensity, but improves consumer surplus, because the superior firm can signal its better performance thereby attracting more buyers.

These results provide actionable recommendations to managers, especially since the information infrastructure and information-content of the relevant quality metrics allows implementation of the contingency pricing framework. We find that contin-

gency pricing is relevant, applicable, and implementable in many IT-intensive business contexts such as electronic commerce, information goods, online transactional services, telecommunications, and IT infrastructure services. Private information has a crucial role in making contingency pricing attractive, and the value of contingency pricing increases when the firm is more confident, relative to the market, about its performance. The differences in public and private information can be very significant for the IT sector, because new technologies are deployed at a rapid pace and new firms are constantly emerging with new products. Since buyers are unfamiliar with these new offerings and new firms, contingency pricing can serve as an effective signaling mechanism for new entrants.

We have not investigated the case where the firm's information itself may be inaccurate, but consider it a useful avenue for future research. A related topic for future work concerns the detailed analysis of signaling that a firm may send out by offering different types of contracts. Similarly, while our analysis considers a one-shot game between the firm and the consumers, the outcome and choices may differ under repeated interaction. Consumers may learn to adjust their beliefs about firm performance based on the actual outcomes in prior interactions. However, such learning will have a limited effect on consumer behavior, since the firm's performance (in the next period) itself may change from one period to the next. Determining the net impact of such learning needs careful modeling and further research. Other topics for future research include the role of multipart contingent contracts, menu of contingent contracts (contingency pricing and price discrimination), and empirical studies of consumer response to contingency pricing.

*Acknowledgments:* The authors gratefully acknowledge the eBusiness Research Center and the Smeal College of Business Administration at Penn State University for supporting this research. They thank the editors and referees for the *JMIS* Special Section—their comments and suggestions have substantially strengthened the paper.

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# Appendix

## Detailed Derivations and Proofs: Monopoly

**Proof of Proposition 1.** The firm's optimization problem is to choose price and rebate ($R$, $r$) to maximize $\pi = D(\bar{R})(\hat{R} - \hat{C})$. Recall that $\hat{R} = R - rp$ and $\bar{R} = R = r\mu$. First-order conditions with respect to $R$ and $r$ yield

$$\frac{\partial \pi}{\partial R} = \left(\hat{R} - \hat{C}\right) D'\left(\bar{R}\right) + D\left(\bar{R}\right)$$

$$\frac{\partial \pi}{\partial r} = -\mu\left(\hat{R} - \hat{C}\right) D'\left(\bar{R}\right) - pD\left(\bar{R}\right).$$

When $p \neq \mu$, the stationary point yields a zero-profit solution (with $D = 0$ or $\hat{R} = \hat{C}$), hence the optimal price–rebate combination must lie at one of the boundaries. For the public information problem where $p = \mu$, the two first-order conditions reduce to a single relation

$$\left(\hat{R} - \hat{C}\right) D'\left(\bar{R}\right) + D\left(\bar{R}\right) = 0.$$

The above equation can be rewritten such that the optimal single price $\bar{R}$ satisfies the condition stated in Equation (2)

$$\frac{-D(R)}{R \cdot D'(R)} = \frac{R - \hat{C}}{R}.$$

This condition guarantees a unique solution for $\bar{R}$ under nondecreasing price elasticity of demand. However, as indicated above, this effective expected price can be achieved by many price–rebate combinations including the two extreme cases of $r = 0$ and $r = R$, and all convex combinations of these two solutions. ■

### Candidate Optima Under Private Information

**Proof of Lemma 1.** We examine the two candidate optima at the boundaries $r = 0$ and $r = R$.

*No rebate profit function.* Setting $r = 0$, the firm's problem is to choose $R$ to maximize $\pi^s = (R - \hat{C})\, D(R)$. The first-order condition is $(R - \hat{C})\, D'(R) + D(R) = 0$. Rewriting this equation, the optimal single price $R^s$ is the unique solution to the equation (also referred to as Equation (2) in the paper)

$$\frac{-D(R)}{R \cdot D'(R)} = \frac{R - \hat{C}}{R}. \tag{10}$$

We will show below that the profit function is concave in $R$ and hence this solution is optimal.

*Concavity of profit function under nondecreasing price elasticity.* The assumption of nondecreasing price elasticity of demand, formally, is $(\partial/\partial x)((-xD'(x))/(D(x))$, which, after computing and rearranging the terms, simplifies to

$$xD''(x) < \left[\frac{-xD'(x)}{D(x)}\right]\left(-D'(x)\right) - D'(x). \tag{11}$$

Substituting for the price elasticity term in the square brackets on the right hand side of Equation (11) from the first-order condition in Equation (10) and expressing in terms of $R$, we get

$$RD''(R) < \left(\frac{R}{R-\hat{C}}\right)\left(-D'(R)\right) - D'(R),$$

which can be rearranged as

$$\left(R-\hat{C}\right)D''(R) + 2D'(R) < \frac{\hat{C}}{R}D'(R). \tag{12}$$

Concavity of the profit function requires that the second derivative $(\partial^2\pi/\partial R^2) = (\hat{R}-\hat{C})\,D''(R) + 2D'(R)$ is negative. Since we know that $D'(R)$ is negative, Equation (12) guarantees that the second derivative is indeed negative.

*Full rebate profit function.* With $r = R$, the firm's problem is to choose $R$ to maximize $\pi^c = (R(1-p)-\hat{C})D(\hat{R})$. The first-order condition is

$$(1-\mu)\left(R(1-p)-\hat{C}\right)D'(\bar{R}) + (1-p)D(\bar{R}) = 0,$$

and rewriting this equation, the optimal single price $R^c$ is the unique solution to the equation (also referred to as Equation (3))

$$\frac{-D(\bar{R})}{\bar{R}\cdot D'(\bar{R})} = \left(\frac{\bar{R} - \frac{1-\mu}{1-p}\hat{C}}{\bar{R}}\right) = \left(\frac{\hat{R}^c - \hat{C}}{\hat{R}^c}\right). \tag{13}$$

Once again, we will show that the second derivative $(\partial^2\pi/\partial R^2) = (1-\mu)^2[\,(\hat{R}-\hat{C})\,D''(\bar{R}) + 2((1-p)/(1-\mu))D'(\bar{R})\,]$ is negative to verify the concavity of the profit function. As before, substituting the expression for the inverse elasticity of demand from the first-order condition 13 in Equation (11), expressing in terms of $R$ and rearranging terms, we get

$$\left(\hat{R}-\hat{C}\right)D''\left(\bar{R}\right)+2\frac{1-p}{1-\mu}D'\left(\bar{R}\right)<\frac{\hat{C}}{\bar{R}}D'\left(\bar{R}\right), \tag{14}$$

which implies the second-order condition required since $D'(\bar{R})$ is negative. Hence the optimal prices in the no-rebate and full-rebate cases are as given in Equations (2) and (3), respectively. ■

**Proof of Proposition 2.** We construct a full-rebate contingent price contract that outperforms the optimal standard price when $\mu > p$. Given the optimal standard price $R^s$ with profit $\pi^s$, define a contingency pricing scheme with price $R$ such that the consumers' price expectation is $R(1-\mu) = R^s$, that is, $R = (R^s/(1-\mu))$. Hence the firm gets the same market coverage as under the single price. However, its expected revenue per sale $\hat{R} = ((1-p)/(1-\mu))R^s$ is greater than $R^s$, hence the firm earns a higher profit with this contingent price contract. ■

**Proof of Proposition 3.** The marginal consumer is $v_m = (\bar{R} + \alpha\mu)/(1-\mu\delta)$, hence the firm's profit function is

$$\pi=\left(1-\frac{\bar{R}+\alpha\mu}{1-\mu\delta}\right)\left(\frac{1-p}{1-\mu}\bar{R}-\hat{C}\right).$$

The optimal contingency price $R^c$ is $(1/2)(1 + \hat{C}((1-\mu)/(1-p)) - (\alpha+\delta)\mu)$. We see that the derivative

$$\frac{\partial R^c}{\partial\mu}=\frac{1}{2}\frac{1-(\alpha+\delta)}{(1-\mu)^2}$$

is positive (or negative) if and only if $\alpha + \delta$ is less than (greater than) 1. Hence, the price (and rebate), expected revenue per sale and margin all decrease with $\mu$ when quality sensitivity is high, and increase with $\mu$ when quality sensitivity is low.

Similarly, we compute the derivative

$$\frac{\partial\bar{R}^c}{\partial\mu}=\frac{1}{2(1-p)}\left(\hat{C}+(\alpha+\delta)(1-p)\right),$$

which is always negative, hence the consumers' expected price decreases as $\mu$ increases. ■

## Detailed Derivations and Proofs: Duopoly

*Price response functions.* The derivatives with respect to a firm's announced price and rebates are $(\partial D_i)/(\partial R_i) = D'$ and $(\partial D_i)/(\partial r_i) = -\mu D'$. Computing first derivatives

for the profit function of firm $i$, with respect to its announced price and rebate, given an expected price $\bar{R}_j$ from firm $j$,

$$\frac{\partial \pi_i}{\partial R_i} = D'\left(\hat{R}_i - \hat{C}\right) + D_i$$

$$\frac{\partial \pi_i}{\partial r_i} = -\mu D'\left(\hat{R}_i - \hat{C}\right) - p_i D_i.$$

Comparing the two first-order conditions, we see that the optimal lies on the boundary $r_i = 0$ or $r_i = R_i$, except when public information is perfect ($p = \mu$), in which case several combinations of price and rebate are optimal.

*Price response functions under standard pricing.* Under standard pricing with $r_i = 0$, firm $i$'s problem is to choose $R_i$ to maximize $\pi_i = (R_i - \hat{C})\, D_i$. After substituting for the demand function, the first-order condition is

$$\frac{\partial \pi_i}{\partial R_i} = \frac{1}{2(1-\delta\mu)t}\left[\hat{C} - \left(2\bar{R}_i - R_j - (1-\delta\mu)t\right)\right] = 0$$

and it is clear that the second derivative is negative. Solving the first-order condition for $\bar{R}_i$, the optimal single best response price $\Omega_i^s$ is given by Equation (6):

$$\Omega_i^s\left(\bar{R}_j\right) = \frac{\hat{C}}{2} + \frac{1}{2}\left(\bar{R}_j + (1-\delta\mu)t\right).$$

*Price response functions under contingency pricing.* Under full rebate, firm $i$'s problem is to choose $R_i$ to maximize $\pi_i = (\hat{R}_i - \hat{C})\, D_i$. The first-order condition is

$$\frac{\partial \pi_i}{\partial R_i} = \frac{1}{2(1-\delta\mu)t}\left[\hat{C} - \frac{1-p_i}{1-\mu}\left(2\bar{R}_i - \bar{R}_j - (1-\delta\mu)t\right)\right] = 0$$

and again the second derivative is negative. Solving the first-order condition for $\bar{R}_i$, the optimal single best response price $\Omega_i^c$ is given by Equation (7)

$$\Omega_i^c\left(\bar{R}_j\right) = \frac{\hat{C}}{2}\frac{1-\mu}{1-p_i} + \frac{1}{2}\left(\bar{R}_j + (1-\delta\mu)t\right).$$

**Proof of Proposition 6.** We rewrite the optimal prices given by Equations (8) and (9)

$$\bar{R}_H^c = \frac{\hat{C}(1-\mu)}{3(1-p_L)} + \frac{2\hat{C}(1-\mu)}{3(1-p_H)} - (\mathrm{t} - \mu d)$$

$$\bar{R}_L^c = \frac{\hat{C}(1-\mu)}{3(1-p_H)} + \frac{2\hat{C}(1-\mu)}{3(1-p_L)} - (\mathrm{t} - \mu d).$$

Therefore,

$$\bar{R}_H^c - \bar{R}_L^c = \frac{\hat{C}(1-\mu)}{3}\left[\frac{1}{1-p_H} - \frac{1}{1-p_L}\right],$$

which is negative since $p_H < p_L$. ■

**Proof of Proposition 7.** Let $\bar{R}^c$ be *any* expected price of firm $L$ under a full-rebate contingent price solution. Then its expected revenue per sale is $\bar{R}^c((1-p_L)/(1-\mu))$. For firm $H$, construct a full-rebate solution with the same expected price $\bar{R}^c$. Hence, the two firms gain equal market share under these prices. However, firm $H$ has a greater expected revenue per sale $(\bar{R}^c((1-p_H)/(1-\mu)))$ since $1-p_H > 1-p_L$. Therefore, we see that, for any expected price $\bar{R}^c$ from firm $L$, there exists a *feasible* contingent price solution for firm $H$ such that $\pi_H > \pi_L$. Hence, firm $H$ can respond in a similar way to firm $L$'s *optimal* price $\bar{R}_L^c$ and earn greater profit by offering the same expected price. Firm $H$'s *optimal* price $\bar{R}_H^c$ must surely be at least as profitable as this feasible price, hence firm $H$'s optimal profits exceed that of firm $L$. ■