MATH 13187: University Seminar Due Monday, March 27, 2017

Write a paper (targetted length is 5 to 7 pages) about topics from chapters 3 and 4 of the Codebook, or issues raised by the movie "The Imitation Game". I have listed a number of possible topics below, and will also suggest some more mathematical projects.

1. A number of events from the movie "The Imitation Game" never happened. For example, on page 186 of the Codebook, Singh says that the cryptanalysts were not told how their decipherings were used, which contradicts the scene near the end of the movie where Turing argues against revealing to the Navy that a convoy was targetted, as well as its aftermath. Collect further events in the movie that never happened, or involve mischaracterizations. Discuss the impact of including these events in the movie, and the broader issue of whether a historically-based movie should be historically accurate. One may argue that a movie is a work of art, and should be judged on that standard. Something else that may be of interest is that the book on which the movie is loosely based is called "Alan Turing: The Enigma". In what sense is Turing an enigma?

2. Write a film review of "The Imitation Game", and explain its strengths and weaknesses. You could also read other people's reviews and explain which reviews you find most reasonable.

3. Write a film review of some other movie related to the theme of artificial intelligence.

4. The literature I have read about cryptology in World War II is mostly based on allied successes. Look up whether there were any major cryptanalytic successes on the side of the Germans, Italians, or Japanese, and whether these had any impact on the war.

5. Alan Turing was arrested for homosexual behavior, which is no longer grounds for arrest in the Britain (or the United States). Look up the history of these laws in a few countries, and try to determine why they were instituted and if they have been repealed, why they were repealed. You could also study the way popular attitudes towards homosexuality have changed over time in various countries, the history of hormonal therapy, or the perspective (both official and tacit) towards homosexuality in the Catholic church.

6. When Joan Clarke (Keira Knightley) arrives for the crossword puzzle exam in the movie, she is treated dismissively because she is a women. Look up how women are treated in math and science, and look up how many women professors there are in math and/or science at elite universities, such as Notre Dame.

7. Read about artifical intelligence, which is a field Turing played a big role in starting. Explain what it is, and how it has developed since Turing's time.

8. Turing is often described as one of the inventors of the modern computer, but never actually built a machine used substantially by others. Write a short history of the development of the computer.

9. Turing is credited with formalizing the notion of an algorithm. Explain what an algorithm is, and discuss the impact of algorithms on modern society.

10. Turing is portrayed in "The Imitation Game" as very literal, and unaware of what other people thought. Is this accurate? Look at ways mathematicians and scientists are portrayed in popular culture (e.g., "Big Bang Theory" TV show, the movie "A Beautiful Mind").

11. The movie is titled "The Imitation Game", which is a game Turing imagined (sometimes called the Turing test), where a questioner poses questions to an entity, and tries to deduce from the answers whether the entity is a person or a machine. The questioner cannot see or talk to the entity, and the entity is allowed to lie to deceive the questioner. If you look up the wikipedia entry under "Turing test", you can find more information on this subject.

12. In the book "Alan Turing: The Enigma", Andrew Hodges discusses the notion of "Imitation Game" in a number of different contexts. One is the Turing test from the previous question. Another is the situation of someone who feels out of place in a society, and feels they have to imitate others in order to fit in. Hodges discusses this in the context of Turing's homosexuality, and the sense in which he may have felt compelled to imitate others to fit in. The same kind of argument can be applied to spies. Yet a third context arises for a cryptanalyst trying to decode an encoded message, who in a sense must get inside the head of the encoder and try to understand their thought process. This can be viewed as a kind of imitation.

13. In the movie, Turing is presented as working alone at the start, and the leading figure at Bletchley park. Look up the actual history of Bletchley park and the way work was divided, and the individuals who played an important role. The book "The Secrets of Station X" by Michael Smith is a good source, and you could read a chapter or two for this paper. The last chapter discusses what happened to some of the leaders at Bletchley park after the war.

14. Discuss the impact of math and/or science on the progress of World War II, in comparison to previous wars. I did a google search for "The impact of science on World War II", and a number of entries came up.

15. Nazi racial laws led to a number of influential German mathematicians and scientists losing their jobs or fleeing Germany and going to the United States or Britain. Document this trend and discuss its impact on World War II. The article "Science Under the Nazis" by Jacob Steinberg and Manfred Laubichler from Vol. 287, issue 5460, March 17 2000 of Science may be useful.

16. Turing's most significant mathematical achievement was to resolve what is called the "Entscheidungsproblem" or "Decision Problem". This was part of a series of negative results, that included work of Kurt Godel, who showed that in any axiomatic mathematical system, some assertions can neither be proved true or false (Turing showed that some quantities cannot be computed in a finite amount of time). Look up Godel's work and Turing's work and explain the context and the impact. The article "What is Godel's Theorem" from Scientific American in 1999 may be useful.

17. It is perfectly fine to write your essay about a subject unrelated to Turing and the movie. You can look for topics in chapters 3 and 4 of the Codebook, or you could also read Chapter 5 of the Codebook, which is about how people learned to decipher ancient languages. You could also look at the references in the Codebook on pages 397 to 400. The Al Kadi article on Arab cryptanalysis mentioned on page 398, the Barbara Tuchman book on the Zimmerman telegraph mentioned on page 399, and the Robert Harris novel discussed on page 399 could be interesting. It may also be interesting to research why certain societies that were once very advanced scientifically lost their scientific edge. This includes Arab societies, but also China. If you are interested in exploring a topic from one of your earlier essays further, that would also be fine.

18. Possible math projects. Note if you try to explain a proof given in a book, you should try to explain the proof in your own words. It is not valuable to just repeat the text from the book.

    (a) Explain how to prove that every positive integer can be factored uniquely as a product of prime numbers. You can read a proof near the start of the book "Elementary Number Theory" by William Stein, which is available for free download. Just google "William Stein Number theory", and click on "free legal pdf".

    (b) Look up the proof of the formula we use to compute $\phi(n)$ from the prime factorization of n, and explain how to prove that this formula is true. This is explained in chapter two of Stein's book, discussed in part (a). It uses the modular arithmetic that we will discuss after break, and this may be better as a final project/paper.

    (c) Some of the units on the course website are probably not optimally written. Rewrite a unit (or a part of a unit) in a way that you think would explain the material better. You will not hurt my feelings by doing this.

    (d) Look up the Fermat factorization method (see for example, Section 3.6, pp. 130-131) of the book "Elementary Number Theory" by Kenneth Rosen. Explain how it works, and illustrate through examples.

    (e) Try to complete option 3 from the second writing assignment, which is copied below.

    Option 3: For a and b positive integers, show that the set of combinations of a and b is exactly the set of multiples of $gcd(a, b)$ by completing the following

steps (note: this is an alternative approach to the assertion in Option 2, and mathematically deeper).

(I) We say a subset H of the integers is a *subgroup* if (i) 0 is in H, (ii) if x is in H, then $-x$ is in H, and (iii) if x and y are in H, then $x + y$ is in H. For a positive integer k or $k = 0$, let H_k be the set of all multiples of k. Show that H_k is a subgroup of the integers.

(II) Show that if H is any subgroup of the integers, then $H = H_k$ for some k. My hint for this is that it should be intuitively clear that any collection of positive integers has a least element, and if d is the least positive element in H, you want to show that $H = H_d$. For this, let c be in H, and write $c = q \cdot d + r$, as in ordinary division with $0 \leq r < d$. Show that r is in H, and conclude that $r = 0$ using the fact that d is the least positive element in H.

(III) Let $H_{a,b}$ be the collection of all combinations of a and b, so that in symbols, $H_{a,b} = \{x \cdot a + y \cdot b\}$ as x and y range over integers. Show that $H_{a,b}$ is a subgroup of the integers, and use step (II) above to show that $H_{a,b} = H_d$, where d is the least positive element in $H_{a,b}$. In the process, we have seen that $d = xa + yb$ for some x and y.

(IV) Now we would have completed our argument if we can show that d is the greatest common divisor of a and b. For this, it suffices to show two things. First, show that if c divides a and c divides b, then c also divides d (the end of (III) will help with this). Second, show that d divides a and d divides b. For this, write $a = q \cdot d + r$ with $0 \leq r < d$, and deduce that r is in $H_{a,b} = H_d$. Use the defining property of d to show that $r = 0$ so d divides a. Use similar reasoning to show that d divides b. Now explain why these two steps show that $d = gcd(a, b)$.

Note that the argument in option 3 turns the argument on option 2 on its head. In option 2, we start with $gcd(a, b)$, and we show that the multiples of $gcd(a, b)$ are exactly the combinations of a and b. In option 3, we really start with the least positive number d that is a combination of a and b, and show that this number d is actually $gcd(a, b)$. The approach in option 3 generalizes much better to other situations.

SOME REFERENCES:

The Codebook

"Alan Turing: The Enigma" by Andrew Hodges (source for the movie, but more than 600 pages).

"The Secrets of Station X" by Michael Smith.

"The Man Who Knew too Much: Alan Turing and the Invention of the Computer", by David Leavitt (this seemed to me like more of a popular version of the book by Hodges).