Math 60210, Basic Algebra, Study questions for Exam 1, Fall 2018
Exam 1 is Fri, September 28

1. Let $\sigma = (1\ 2) \in S_n$, and let C_σ be the conjugacy class of σ.

(A) Compute $|C_\sigma|$.

(B) Compute the subgroup $C_{S_n}(\sigma)$, i.e., determine the elements of S_n which centralize σ.

2. Let G be a nonabelian group with center $Z(G)$. Show that there exists an abelian subgroup H of G such that $Z(G) \subset H$ but $Z(G) \neq H$.

3. Let p be an odd prime. Find the conjugacy classes in the dihedral group D_{2p}. Verify the class equation.

4. Find the conjugacy classes in D_8. Verify the class equation.

5. Let $G = S_4$ and let

$$V = \{e, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$$

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(i) Prove that V is a normal subgroup of G.

(ii) Let $H = \{\sigma \in S_4 : \sigma(4) = 4\}$ (in class we observed that $H \cong S_3$). Define $\sigma : H \to \mathrm{Aut}(V)$ by $\sigma(\tau) = \sigma\tau\sigma^{-1}$ for $\sigma \in H$, $\tau \in V$. Prove that $V \ltimes_\sigma H \cong S_4$.

6. Let $G = D_{12}$, the symmetry group of a regular hexagon T_6, with vertices $x_0, x_1, \ldots, x_5$, where $x_j = (\cos(2\pi j/6), \sin(2\pi j/6))$, for $j = 0, 1, 2, 3, 4, 5$. Let L_0 be the line from x_0 to x_3, let L_1 be the line from x_1 to x_4, and let L_2 be the line from x_2 to x_5. For $g \in G$ and $i = 0, 1, 2$, we let $g \cdot L_i = \{g(v) : v \in L_i\}$.

(i) Prove that $g \cdot L_i$ is one of L_0, L_1, L_2.

(ii) Let $Y = \{L_0, L_1, L_2\}$. Prove that $(g, L_i) \mapsto g \cdot L_i$ is a G-action on Y.

(iii) Prove the kernel of this action is r_π, rotation by an angle π, and that $D_4/\{e, r_\pi\} \cong S_3$.