Aircraft to Ground Profiling: Turbulence Measurements and Optical System Performance Modeling

Matthew Kalensky*1, Yakov Diskin†2, Matthew Whiteley‡2, Mitchell Grose2, Eric Jumper§1, Kevin Jackovitz2, Richard Drye2, Stanislav Gordeyev1, Aaron Archibald1, Brandon Hampshire2, Monte Owens2, Eric Smith2, and Eric Magee2

1University of Notre Dame, Notre Dame, IN, USA, 46556
2MZA Associates Corporation, Dayton, OH, 45430
3Air Force Institute of Technology, 2950 Hobson Way, Wright-Patterson AFB, OH 45433

A series of flight experiments have demonstrated a novel approach to measuring path resolved optical turbulence quantities, such as $C_n^2$, along an air-to-ground slant path. This manuscript describes the data acquisition experiments that involved two laser beams propagating between an orbiting airborne platform and a stationary ground terminal over an 8km slant path. Ground-based and in-flight measurements were collected simultaneously and $C_n^2$ profiles were computed using the Difference of Differential-Tilt Variance (DDTV) technique. This manuscript describes the DDTV technique which enables the path resolved measurement of turbulence strength resulting in $C_n^2$ profiles. The resulting turbulence profiles reveal what is believed to be the aero-optical contamination from the aircraft boundary layer within the statistics closest to the aircraft. Therefore, the contamination of the aero-optical environment can be quantified with respect to the rest of the atmospheric propagation path. Lastly, this manuscript presents analyses that compare the measured atmospheric turbulence profiles to state-of-the-art atmospheric models. The analyses extend beyond $C_n^2$ comparisons and show the measurement-versus-modeling comparison in terms of key directed energy system propagation parameters such as Greenwood frequency, coherence diameter, Rytov number, isoplanatic angle, Tyler frequency, open-loop jitter and open-loop Strehl ratio. The slant path turbulence is analyzed in the context of air-to-ground and ground-to-air directed energy systems.

Nomenclature

\[
\begin{align*}
\sigma_a & = \text{atmospheric tilt covariances} \\
\alpha & = \text{scaling factor based on atmospheric coherence lengths} \\
b & = \text{source separation}
\end{align*}
\]

*Corresponding Author, Graduate Researcher, mkalensk@nd.edu
†Research Engineer, Yakov.Diskin@mza.com
‡Vice President and Senior Scientist, Matthew.Whiteley@mza.com
§Aero-Optics Group Director, Roth-Gibson Professor, Department of Aerospace and Mechanical Engineering, EJumper@nd.edu
\[ C_f = \text{skin friction coefficient} \]
\[ C_n^2 = \text{refractive-index structure constant of turbulence, measures turbulence strength} \]
\[ d = \text{tilts} \]
\[ D = \text{aperture diameter} \]
\[ \Delta x = \text{subaperture separation} \]
\[ \delta = \text{boundary layer thickness} \]
\[ \Gamma = \text{gamma function} \]
\[ \kappa = \text{wavenumber} \]
\[ K_{GD} = \text{Gladstone Dale constant} \]
\[ L = \text{propagation path length} \]
\[ \lambda = \text{wavelength} \]
\[ M = \text{Mach number} \]
\[ \text{OPD}_{RMS} = \text{root mean square of Optical Path Difference} \]
\[ r_0 = \text{spherical-wave coherence diameter} \]
\[ \rho_\infty = \text{freestream density} \]
\[ \sigma_X = \text{Rytov parameter} \]
\[ \sigma_\delta = \text{DDTV parameter} \]
\[ \sigma_{HO}^2 = \text{tilt removed phase variance} \]
\[ \theta_0 = \text{isoplanatic angle} \]
\[ W = \text{weighting normalization constants} \]
\[ w = \text{weighting functions} \]
\[ \xi = \text{normalized path position} \]
\[ z = \text{location along the propagation path} \]

**I. Introduction**

Recently, there has been interest in the use of directed energy and other optical systems mounted on a flight vehicle [1-5] for a variety of applications. The fluctuations in the refractive index, or optical turbulence, in the atmospheric medium affects the performance of optical systems when a beam is propagated through the atmosphere. There are many system and engagement parameters that could influence an optical system’s sensitivity to turbulence. Atmospheric turbulence conditions fluctuate continuously with changes in weather, terrain, and altitude. Thus, for long propagation geometries, especially slant path geometries, the turbulence at one end of the path is often drastically different than at the other end. Atmospheric turbulence obstructs the spatial coherence of a laser beam as it is transmitted
through the atmosphere, and for airborne directed energy system applications, can drastically hinder the degree that the laser can be focused on a target [6]. However, if the aberrations imposed on the laser are measurable, adaptive optics technology can compensate for the distortions and alleviate the undesirable effects. A thorough understanding of the turbulence encountered along a path is necessary for adaptive optics system designers [7].

In this work, turbulence profiling experiments are conducted which seek to measure turbulence strength along a path from a stationary ground station to an orbiting aircraft station. The flight experiments and corresponding analyses described in this manuscript have three major contributions to the atmospheric sciences and directed energy communities:

- A novel method to take air-to-ground path-resolved turbulence profiling measurements is presented.
- A unique filtering technique is employed which allows atmospheric and aero-optical induced effects to be decoupled.
- The measurement results are compared against state-of-the-art atmospheric turbulence models. The measured and modeled turbulence profiles are analyzed with respect to how they impact directed energy system performance.

The manuscript is organized as follows. The path resolved turbulence profiling algorithm used is described in Section II. The experimental setup and data collection process is described in Section III. The recorded turbulence measurements are affected by the aero-optical boundary layer of the aircraft and the anticipated magnitude of the aero-optical contamination in the turbulence profiles is discussed in Section IV. In Section V the measured turbulence profiles are compared to state-of-the-art atmospheric models. This section also describes the unique filtering technique that enables the aero-optical contamination to be isolated and removed from the rest of the atmospheric turbulence analysis. In Section VI the modeling-versus-measurement analysis examines how differences in turbulence profiles impact directed energy system performance parameters. The results are presented in the context of air-to-ground and ground-to-air directed energy systems propagating along the same slant path. The key findings are summarized and concluded in Section VII.

II. Difference of Differential Tilt Variances (DDTV) Profiling

$C_n^2$ is referred to as the atmospheric refractive index structure parameter and is a measure of optical turbulence strength [6,8]. Measuring $C_n^2$ at various positions along a path is typically referred to as “turbulence profiling” [9]. Many researchers have investigated the measurement of atmospheric optical turbulence characteristics using a variety of approaches [10–21]. The turbulence profiler used in this work, referred to as a Path-Resolved Optical Profiling System (PROPS), computes path resolved measurements by utilizing multiple point sources (red and blue) on one end of the path and multiple observing subapertures in a Shack Hartmann wavefront sensor (SHWFS) on the other. Ray paths from two subapertures can cross and reveal information about a particular part of the propagation path. The profiler uses a cooperative source and wavefront sensor to measure the $C_n^2$ profile by calculating subaperture tilt variations. The
profiling technique computes the DDTV to make path resolved optical turbulence measurements.

Generally, measuring optical turbulence along a path is challenging, especially if the path altitude above ground level is not constant. Rytov theory is the primary basis for turbulence approximations resulting from optical wave propagation along a path [6,8,9]. The Rytov parameter, $\sigma_{\chi}^2$, can be related to parameters such as $C_n^2$ by use of Eq. 1.

$$\sigma_{\chi}^2 = 0.5631 \left( \frac{2\pi}{\lambda} \right)^{7/6} \int_0^L C_n^2(z) \left[ z \left( 1 - \frac{z}{L} \right) \right]^{5/6} dz$$

(1)

Here, $\lambda$ is the wavelength, $z$ is the position along the path and $L$ is the full path length. Equation 1 is valid in propagation scenarios of weak turbulence ($\sigma_{\chi}^2 \leq 0.3$). In instances of strong turbulence or for long propagation path lengths, scintillation saturates and the Rytov parameter is no longer useful in generating meaningful turbulence approximations [22]. With the amendable nature of Rytov theory for calculating integrated optical turbulence strength parameters, we seek to extend the breadth of situations in which this theoretical construct can be applied. Rather than use irradiance-based quantities, the work described here utilizes the DDTV, first outlined and presented in Ref. [9], to produce meaningful statistics for computing turbulence parameters. The DDTV measurements are proportional to the Rytov parameter and can be used to estimate the value of the integral expression in Eq. 1[9]. This method relies on phase data, avoiding the issue of saturation. The DDTV technique uses differential statistics and thus also avoids contamination from undesirable gimbal motion and additive noise [9,23].

The DDTV method utilizes an arbitrary number of sources and subapertures separated by a propagation range. The sources themselves are physically separated by a distance, $b$, considered small relative to the propagation path, $L$. On the side receiving the light from the sources is a subaperture array where each subaperture is separated from the others by varying distances, $\Delta x$. The light from the sources is initially undisturbed. As the light propagates through the atmosphere, the small spatially and temporally dependent fluctuations in the atmosphere’s refractive index perturb the light causing its departure from planarity. Using a SHWFS, each subaperture focuses the light it receives to a point creating an array of discrete illuminated spots, or centroids. The incoming light is filtered to observe the light received from each source. The $x$ and $y$ deviations, or local tilts of these centroids away from their time resolved expected spatial locations is an indication of the local deviation from planarity of the incoming light. The geometry of the source/aperture arrangement enables the existence of locations along the path where light emitted from the sources, cross paths with the light received through different subaperture pairs. At these locations, there is a commonality in how the light is affected by the surrounding physical environment. Figure[1] illustrates the source/aperture schematic. By analyzing different combinations of subaperture separations, the crossing of the beams is moved to different parts of the propagation path. For widely separated subapertures and fixed sources, the crossing point is far from the receiving aperture as seen in the top illustration in Fig. [1] For minimally separated (side by side) subapertures, the crossing point is very close to the SHWFS as shown in the bottom illustration in Fig. [1] In this diagram, a yellow circle highlights the
location where the red (R) and blue (BL) light cross paths for a particular subaperture pair combination. Equation 2 describes how the DDTV approach is applied from acquired tilt data.

\[
\sigma_\delta^2 = \langle (d_{R1} - d_{BL2})^2 \rangle - \langle (d_{R2} - d_{BL1})^2 \rangle \quad (2)
\]

In this equation, \(\sigma_\delta^2\) is the DDTV measurement computed from \(d\) which represents the local tilts for different subaperture/source combinations. R and BL indicate the light source, red and blue, respectively. ‘1’ and ‘2’ represent the subapertures receiving the light source. Each local tilt measurement is a combination of atmospheric tilt as well as noise and platform motion components. After expanding, simplifying, and ignoring the negligible components of Eq. 2, only the difference in atmospheric tilt covariances, \(a\), remains \([9, 23]\). This result is shown in Eq. 3.

\[
\sigma_\delta^2 = 2(\langle a_{R1}a_{BL2} \rangle - \langle a_{R2}a_{BL1} \rangle) \quad (3)
\]

For two subapertures separated by a physical distance, \(\Delta x\), the covariance of Zernike \(\chi\)-tilt coefficients can be calculated using Eq. 4 \([24, 26]\). The covariance for a specific \(C_n^2(z)\) is a function of the normalized aperture and source separation. In Eq. 4 \(D\) is the subaperture diameter, \(\lambda\) is the wavelength, \(L\) is the full path length, and \(\xi\) is normalized position along the path defined by \(z/L\). The \(W(\xi)\) term represents a weighting function applied to the expression. Equation 4 can
easily be substituted into the result determined from the DDTV geometry, and with simplification, yields Eq. 5. This result produces an expression for the DDTV parameter with \( C_n^2 \) embedded in this weighted integral equation. Here, \( W_0c \) and \( W_0 \) are normalization constants and \( w_c(\xi) \) and \( w(\xi) \) are weighting functions for the cross path and non-cross path geometries, respectively. If the known quantities in Eq. 5 are gathered into a quantity \( \langle a_1, a_2 \rangle \) as seen in Eq. 6, then Eq. 5 can be rearranged to form a linear system presented in Eq. 7. Here, \( \rho \) is a matrix of weighting functions for \( m \) number of DDTV measurements by \( n \) number of propagation path partitions. Using the pseudo matrix inverse, \( C_n^2 \) can be calculated for partitions along the path. A more thorough explanation of the procedure and theoretical construct behind DDTV can be found in Refs. [9, 23].

\[
\langle a_1, a_2 \rangle = 16\sqrt{3}\Gamma(8/3) \left(\frac{2\pi}{\lambda}\right)^2 D^{5/3} L \int_0^L C_n^2(z) W(\xi) d\xi
\]

\[
\sigma_\delta^2 = \frac{128\sqrt{3}\Gamma(8/3)}{\pi^2} \left(\frac{2\pi}{\lambda}\right)^2 D^{5/3} L \int_0^L C_n^2(z)(W_0c w_c(\xi) - W_0 w(\xi)) d\xi
\]

\[
m_m = \frac{\sigma_\delta^2}{\frac{128\sqrt{3}\Gamma(8/3)}{\pi^2} \left(\frac{2\pi}{\lambda}\right)^2 D^{5/3} LW}
\]

\[
\begin{bmatrix}
m_1 \\
\vdots \\
m_M
\end{bmatrix} = \begin{bmatrix}
p_{11} & \cdots & p_{1N} \\
\vdots & \ddots & \vdots \\
p_{M1} & \cdots & p_{MN}
\end{bmatrix} \begin{bmatrix}
C_{n1}^2 \\
\vdots \\
C_{nN}^2
\end{bmatrix}
\]

The curves shown in the left plot of Fig. 2 illustrate how different weights are applied along the propagation path based on the DDTV geometry. The solid blue curve represents the scenario in Fig. 1 where source \( R \) is received by aperture 1 and source \( B \) is received by aperture 2, namely separate path geometry. The red dashed line in the left plot of Fig. 2 represents when source \( R \) is received by aperture 2 and source \( B \) is received by aperture 1, otherwise referred to as the cross-path geometry. At the intersection of the red and blue light, the same turbulence environment is experienced. Therefore, the DDTV technique computes the difference between the separate path and the cross-path weighting functions to produce a new weighting function, shown by the green line in the left plot of Fig. 2 which maximizes the sensitivity to the specific portion of the path where the red and blue intersect. Different subaperture pairs change the location along the path where the sources intersect. Consequently, the point on the path where the weight is applied also changes. This method is applied to all source/subaperture combinations resulting in a set of DDTV weighting functions that resolve the path as seen in the right plot of Fig. 2. The plot shows the weighting function covering approximately 50% of the full propagation path closest to the subaperture array. Therefore, to obtain full path sensitivity a second profiler system is setup in the opposite configuration to have sources and a SHWFS on both ends of the path.
III. Experimental Setup and Data Acquisition Campaign

A. Data Acquisition

The 2019 turbulence profiling flight campaign [27, 28] in Grand Rapids, Michigan, aimed to collect air-to-ground path-resolved turbulence measurements for comparison and validation with state-of-the-art atmospheric turbulence models. This was the third campaign of its kind with the first two occurring in 2018. The unique source and subaperture geometries of the profiler yield differential local jitter measurements to compute path specific weighting functions resulting in a 30-bin profile (L/30 resolution). The ground station terminal was installed in an open field and the other terminal was installed in an Airborne Aero-Optics Laboratory - Beam Control (AAOL-BC) aircraft. During data collection, the aircraft continuously orbits around the ground station for approximately 3 hour flights with data collected on a per-orbit basis. The flight geometry is nominally a ground radius of 7.4km and an altitude 2.1km, yielding an approximately 8km slant path. Actual geometries during flight rely heavily on cloud altitude and wind conditions. Thus exact geometries were recorded with a Trimble GPS unit installed on the aircraft.

Automated tracking software on both ends of the propagation path is necessary for data acquisition. For the ground station, the system was able to maintain track approximately 80 percent of each orbit with both wide field of view and narrow field of view tracking engaged. Tracking conditions on the air side were more susceptible to environmental conditions such as changes in altitude and cloud cover. The air station was able to successfully acquire and maintain track for approximately 50 percent of each orbit. For more detail on the tracking and data collection procedures for these flight campaigns, see Refs [27, 28].
B. Ground Station Setup

Figure 3 shows images of the ground station profiler terminal from the front and back. The terminal consists of a 0.2032m Meade telescope on a Meade LX200 gimbal mount. The sources, red and blue LEDs (ground station used LEDs instead of lasers), are attached to the side, and the color SHWFS is on the back of the telescope for data collection. Additional hardware components were added to the terminal for aircraft tracking. The added components included a Wide Field of View (WFOV) camera for aircraft acquisition, a Narrow Field of View (NFOV) camera for aircraft fine-tracking, and a track beacon utilized by the air station for ease of tracking the ground station. The profiler terminal utilizes the rails on the Meade telescope to maintain parallel optical axes between the telescope and LED source assembly. This experimental setup required modifications to the standard PROPS hardware. The customized terminal, depicted in Fig. 3, shows additional mounting hardware designed to keep the LED source assembly in a vertical configuration. The customized ground station terminal also includes a modified beacon transmitting at 950nm. The new beacon is detectable by the short-wave infrared track sensors on the aircraft.

(a) Front of terminal showing the telescope, wide field of view camera, LED sources, and track beacon. (b) Back of terminal showing the telescope, narrow field of view camera, and color SHWFS.

Fig. 3  Ground station profiler terminal.

C. Air Station Setup

Figure 4 shows three images: the assembly design, the profiler terminal installed in the aircraft, and an external perspective of the system. The received light from the ground station propagated through the system as follows. Red and blue light from ground station LEDs would propagate through the atmosphere toward the aircraft, through a 30.5cm optical window, reflect off an angled 30cm mirror as part of the AOM360 AeroTech gimbal used for tracking, then
Fig. 4 Aircraft station LED source receiver, laser source emission, and tracking hardware installed in the aircraft.

propagate toward a 20.3cm Celestron telescope. At the base of the telescope aperture some light would be picked off by a mirror and reflected toward a FLIR camera attached to a Nikon Lens which acted as the WFOV camera for tracking. The remaining (majority) of light would be relayed through the telescope and then encounter a 50/50 beam splitter. The splitter sends half of the received light into the color SHWFS and the other half to the Allied Vision Technology GoldEye CL-033 NFOV track camera. The GoldEye NFOV track camera operates in the short-wave infrared range, thus the modified track beacon on the ground station terminal in Fig. 3 enabled beacon tracking for the aircraft station.

The propagation of light out of the aircraft to the ground station utilized laser source injection. As shown in Fig. 4 two lasers, red (637nm) and blue (405nm), were collimated to 5mm diameter out of 19mm lenses to a longpass dichroic mirror. The dichroic reflected the blue laser light and transmitted the red laser light toward a 200mm lens. After passing through the lens, the red and blue laser light (now parallel to each other) enters the back of the 20.3cm Celestron
telescope (the same used for collection of ground station LED light). The combination of the 200mm lens and 2,000mm telescope yielded a 10X magnification. Thus, the diameter of each beam was magnified to 50mm out of the telescope with 10 mrad divergence. Out of the telescope the beams were separated vertically by 12.7cm. After 8km of propagation through the atmosphere, each beam was approximately 160m in diameter. The maximum laser power was 125mW.

IV. Aero-Optical Contamination

The theoretical construct described in Section II and utilized by the profiler in this work, affords an exciting opportunity to extract turbulence parameters along a path with comparable ease relative to previous approaches. For reference, ground tests have been conducted where turbulence was reported along a path between two static profiler terminals. The successes of these ground tests led to the next step of using a profiler in an aircraft to conduct turbulence profiling experiments. However, acquiring atmospheric turbulence measurements from an aircraft has associated challenges. Namely, it is expected that the turbulent boundary layer of the aircraft will introduce high frequency optical turbulence aberrations in front of the profiler’s acquisition terminal, the consequences of which were not known. Since the aero-optical boundary layer does not abide by Kolmogorov statistics, it is reasonable to wonder how the profiler’s measurements may be affected. Therefore, it is beneficial to quantify the expected contamination that the boundary layer will have on these measurements. To do this, data from a previous AAOL-BC campaign was used. For these previous experiments, two aircraft were flying in close formation (∼ 50 m apart) and at high altitude (4,572 m). As such, a beam propagating between these two aircraft is assumed to be subjected to negligible atmospheric distortions. The primary sources of distortions imposed onto the beam were from the naturally occurring boundary layer of the acquisition aircraft as well as the upstream propagating acoustic waves from the aircraft jet engine located downstream of the acquisition window. Therefore, by using the data collected during this close range flight experiment where negligible atmospheric distortions are imposed onto the laser beam, a $C_n^2$ associated with the turbulent boundary layer and acoustic contamination can be estimated. An estimate of aero-optical and aero-acoustical contamination is useful to compare with the turbulence profiling campaign results in proximity of the aircraft. This previous AAOL-BC flight campaign will be addressed as the "boundary layer (BL) campaign," for convenience. For more information pertaining to the BL campaign, see Refs. [29–32]. In order to accurately compare the aero-optical and aero-acoustical environments between the BL campaign and the turbulence profiling campaign described in this work, the beam distortions measured during the BL campaign need to be appropriately scaled for a different altitude of data collection (data collected at an altitude of ∼ 2100 m rather than 4,572 m). Previous work has described in great detail the relevant scaling and properties of an aero-optical turbulent boundary layer $C_n^2$. The root-mean-squared (RMS) of the optical path difference (OPD), $OPD_{RMS}$, is often the metric used to quantify the distortions associated with an aero-optical and aero-acoustical
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extracting a $C_n^2$ in this fashion is useful for quantifying the contamination that may be registered through the profiler due to the aerodynamics of the aircraft. To do this, the tilt removed phase variance of the measured laser beam is used. The tilt removed phase variance or higher order phase variance, $\sigma_{HO}^2$ is related to the aperture diameter, $D$, and atmospheric coherence length, $r_0$, by Eq. [10].

Using the $OPD_{RMS_{Scaled}}$ comprised of scaled boundary layer and acoustic contamination, a tilt removed phase variance of $0.4532 \text{rad}^2$ was calculated. Consequently, an $r_0$ of $9.63 cm$ was extracted using Eq. [10]

$$\sigma_{HO}^2 = 0.134 \left( \frac{D}{r_0} \right)^{5/3}$$

With $r_0$, Eq. [11] can be used to calculate $C_n^2$. In this equation, $\kappa$ is the laser wavenumber and $L$ is the propagation range. The propagation range is taken to be the size of the bin for the profiler ($L/30$), which is approximately $267 m$.

$$r_0 = 1.68 (C_n^2 L \kappa^2)^{-3/5}$$

As such, a $C_n^2$ value of $3.16 \times 10^{-15} \text{ m}^{-2/3}$ was calculated. Later on, this value will be compared with the measured profiles to show whether the profiler measurements reveal a similar degree of contamination from the boundary layer and acoustic environment in $C_n^2$ values closest to the aircraft.

V. Turbulence Measurements and Atmospheric Model Comparison

The profiler measurements provided the $C_n^2$ values along the optical propagation path. For every SHWFS data sequence that passes the filtering process, a 30-bin solution is outputted where each bin corresponds to approximately $267 m$ of propagation path. Each terminal naturally generates its own single-sided profile solution. In order to get a double-sided profile, the profile solution must be computed from the simultaneously collected sets of DDTV measurements from air and ground terminals. The double-sided solution utilizes the path weighting functions from both ends resulting in full path sensitivity. The measured double-sided solutions are compared to the traditional Hufnagel-Valley 5/7 (HV57) model [6,34], as well as the state-of-the-art Laser Environmental Effects Definition and Reference (LEEDR) weather cube atmospheric model [35–37]. The following subsections describe how double-sided solutions are calculated, how the aero-contamination of the atmospheric profiler is removed, and how the measurements compared to atmospheric models.

A. Double Ended Profile Generation and Results

Due to the relative source separation distance ($\approx 15 cm$) versus furthest subaperture separation distance ($\approx 19 cm$), single-sided profiles only provide DDTV weighting functions on the side of the propagation path closest to receiver. Single-sided profiles are independently generated for both the air and ground station. Data collected by the air station resolves the portion of the path closest to the aircraft whereas the ground station resolves the lower half of the propagation
path. Double-sided profiles are generated by using time synced statistics from both stations allowing the weighting functions to more accurately resolve the entirety of the path. The number of weighting functions available comes from the SHWFS data and the number of subaperture separations available. The profiling solution algorithm iterates until convergence.

Turbulence profiling results obtained from the profiling campaign are shown in Fig. 5. Here, $C_n^2$ is plotted versus altitude above ground level for different orbits and the data lines on each plot correspond to measurements that took place at different times within each orbit (data recorded between 9:28 a.m. and 10:12 a.m. on August 29, 2019). For comparison, the widely used HV57 model is also plotted in green. It can be seen that at the ground, the empirical data and HV57 model are in agreement. However, the measured values show that $C_n^2$ decreases from $1 \times 10^{-14}$ to $1 \times 10^{-16}$ m$^{-2/3}$ about 100 m off the ground whereas HV57 predicts a more gradual decrease in turbulence strength with altitude. In general, HV57 tends to over predict the turbulence strength at low altitudes. Interestingly, at the highest altitude corresponding to the data collected next to the aircraft (altitude above ground level $\approx$ 2,000 m), there exists a seemingly spurious spike in turbulence strength. Current atmospheric models do not predict a drastic rise in turbulence intensity at these altitudes like what is seen in the measured data. The aerodynamic turbulence associated with the aircraft boundary layer imposes localized distortions onto the incoming light in front of the receiving aircraft profiler terminal. Therefore, the spikes in optical turbulence strength seen above approximately 1,750 m are not believed to be attributed to atmospheric turbulence, rather, they are due to aero-optical and aero-acoustical effects in proximity to the aircraft acquisition window. These measured bins closest to the aircraft very closely match the predicted level of boundary layer and acoustic contamination described in Section IV. In Fig. 5 the predicted aero-optical contamination $C_n^2$ value from
Section IV of $3.16 \times 10^{-15} \text{ m}^{-2/3}$ is depicted at 2,000 m altitude above ground level as a red triangle.

For comparison, a measured double-sided turbulence profile for a ground-to-ground traverse is shown in the left plot of Fig. 6. There was no significant change in terrain or altitude, which correspondingly, produces a turbulence profile that remains fairly constant across the path. It is clear that the phenomena seen in the air to ground experiments is not present here. The right plot of Fig. 6 highlights this aero-optical induced contamination. The contamination is isolated in the first 2-3 bins nearest to the aircraft. A key advantage of using a DDTV based algorithm is that it allows the non-atmospheric optical disturbances, which in this case includes the aero-optical and aero-acoustical contamination, to be removed. This will be discussed in greater detail in the next section. Also seen in the air-to-ground turbulence profiles of Fig. 5 is a significant increase in $C_n^2$ between 1,000 to 1,500 m altitude above ground level, where HV57 seems to under predict turbulence strength. It is difficult to make conclusive statements about the reasons for the turbulence enhancement between 1,000 to 1,500 m altitude above ground level, but one reasonable hypothesis is that the measurements are capturing the turbulence effects of the Earth boundary layer. This spike in $C_n^2$ is followed by a decay in turbulence strength which converges to a value in agreement with HV57 between altitudes of 1,500 and 1,700 m.

Fig. 6 Comparison between a typical ground-to-ground (G2G) profile versus a ground-to-air (G2A) profile.

**B. Removing Aero-Optical Contamination**

Aero-optical contamination is removed by omitting the affected bins. An illustrative example of this process is shown in Fig. 7. The two plots each show the measured $C_n^2$ profiles as a function of altitude from a single orbit. The x-axes show a logarithmic-scale of $C_n^2$ and the y-axes are linear-scale altitude above ground level. Figure 7(a) contains the raw profiles with the contamination and the HV57 profile (green) modeled from the path geometry. Each raw profile is labeled with a timestamp representing the time when the data was collected. Since HV57 is only altitude dependent and the aircraft altitude did not significantly fluctuate during the orbit, there is a single model profile curve. Similar to the observations in Fig. 5 Fig. 7(a) shows the apparent consistency in profiles near ground-level and bumps in $C_n^2$ from 900m to 1400m altitude above ground level. Turbulence dips in magnitude around 1500m for nearly all the
Fig. 7 Eliminating the profile bins closest to the aircraft reduced the aero-optical contamination of the atmospheric profile measurements and enables a direct comparison to $C_n^2$ modeling.

measured profiles. After the dip is another bump in turbulence from 1800m to 2000m altitude above ground level. These high-altitude bumps are the result of aero-optical contamination from the aircraft, as discussed above. Figure 7(b) is a statistical summary illustration of the results for the same orbit, however the contaminated bins are now removed, as shown by the lack of measured data above 1800m. Figure 7(b) illustrates the same measured profiles from Fig. 7(a), but plotted as a mean in solid black and min/max as dotted blue lines. By omitting the contaminated bins, the measurements and modeled atmospheric profiles are more directly comparable. Also illustrated in Fig. 7(b) are LEEDR model profiles computed for times surrounding the orbit time, 09:00 in green and 10:00 in cyan.

The LEEDR model takes in numerical weather prediction data to generate a 3D atmospheric model. Specifically, the LEEDR models shown in Fig. 7(b) are generated using a particular kind of numerical weather prediction data, referred to as High-Resolution Window (HIRESW). The HIRESW data comes from regional weather research and forecasting models. This data is provided on a 5km uniform grid and spans 0 to 10km altitude above ground level. Since the LEEDR profiles are path and terrain dependent, there are multiple profiles per orbit. Each modeled profile is dependent on the geometries and terrain underneath the propagation path along the orbit. Figure 7(b) is illustrating the mean curve for the 09:00 and 10:00 HIRESW sets. At surface level, Fig. 7(b) shows that the LEEDR model and measured $C_n^2$ profiles have similar magnitudes. Additionally, the bumps in turbulence from 900m to 1400m captured by the profiler are also apparent in the weather cubes. Generally, the measured profiles have similar shape to the LEEDR model.

In Fig. 7(b), the magenta and red lines in the plot are the HV57 model profiles calculated with and without an $\alpha$ multiplier, respectively. The $\alpha$ multiplier is derived from the average of the measured coherence diameter, $r_0^\text{PROPS}$, for each profile and calculated HV57 coherence diameter, $r_0^{HV57}$, for the path geometry. It scales the HV57 model profile
so its $r_0$ matches the measured $r_0$. The equations for $\alpha$ and $r_0^{PROP5}$ are shown in Eqs. 12 and 13, respectively.

$$\alpha = \left( \frac{r_0^{PROP5}}{r_0^{HV57}} \right)^{-5/3} \quad (12)$$

$$r_0^{PROP5} = \frac{r_0^A + r_0^B}{2} \quad (13)$$

Figure 7(b) shows that the average contamination-removed profiles in orbit 14 roughly agree in shape with the LEEDR weather profiles from surface level to 1000m altitude above ground level. Beyond 1,000m the average measured profiles match very well with the LEEDR weather cubes.

The measured profiles are typically analyzed on a normalized path position plot. Figure 8 presents an alternative visual representation of the measured $C_n^2$ profiles with and without the aero-optical contamination. Here, logarithmic $C_n^2$ is plotted as a function of normalized path position. On the x-axis, the zero-position is the air terminal and the one-position is the ground terminal. Plots (a) and (b) show a single measured profile with and without aero-optical contaminants removed. Plots (c) and (d) show all measurements with and without aero-optical contaminants removed.

Fig. 8 The aero-optical contamination can appear as high $C_n^2$ values in the profile bins closest to the aircraft.
contamination, respectively. The red line illustrates the normalized path position each bin represents, whereas the
black line simply connects the centers of each bin. Also shown in green is the reference HV57 model. Turbulence
strength at the ground terminal (x-axis equals 1) is similar to HV57, but measurements indicate turbulence falls off
with slant-path range significantly faster than modeled. Additionally, the measured bump in turbulence about 40%
into the path from the air-perspective is not captured by the model. With aero-optical contamination removed, plot
(b), the turbulence characteristics near the air station are more consistent with expectation and model, indicating the
importance of isolating those bins. Plots (c) and (d) in Fig. 8 contain all measurements for a single orbit with and without
aero-optical contamination, as well as HV57 plotted for reference in green. The predicted aero-optical contamination
\( C_n^2 \) described in Section IV is shown as a red triangle. These results show the same trend as in Fig. 8 (a) and (b). The
ground terminal model and measurements are in agreement, but measurements indicate turbulence falls off faster than
modeled. The characteristic bump 40% into the path from air perspective is still evident. The spike in turbulence close
to the air terminal is consistent in all profiles.

C. Spatial Comparison of Measurements vs Modeling

A unique aspect of this data collection is the varying terrain and geometries at ground level below the orbital path of
the aircraft. The measurements versus modeling analysis is summarized by comparing the spatial representation of an
ensemble of measurements to the turbulence models generated for that time. Figures 9a, 9b, and 9c show the HV57
profile, a LEEDR weather cube profile, and measured profiles, respectively. The \( C_n^2 \) strength is represented by the color
of each bin. Red indicates strongest turbulence, \( C_n^2 \) of \( 1 \times 10^{-13} [m^{-2/3}] \), and blue indicates weakest turbulence, \( C_n^2 \)
of \( 1 \times 10^{-17} [m^{-2/3}] \). Since HV57 is only dependent on altitude, the profiles around the orbit are identical as seen in
Fig. 9a. The 10:00 LEEDR weather cube generated from HIRESW numerical weather prediction data in Fig. 9b is
dependent on terrain so there are slight variations in each spoke, but over relatively short distances the differences are
minimal. There is a strong contrast between Figs. 9a and 9b. HV57 models turbulence that is much stronger at the
ground station (center of the spokes) and falls off quickly with altitude. However, LEEDR models turbulence at medium
strength (approximately \( 1 \times 10^{-15} [m^{-2/3}] \)) over the entire propagation path with an enhancement about 30% into the
path from ground terminal to air terminal. Figure 9c contains all double-ended profiler measurements over the course
of the one hour of flight. The aero-optical contamination is apparent as the orange-red bins at the edge of the spokes.
Like HV57, but unlike LEEDR, measurements indicate strong turbulence at the ground terminal (center of spokes).
Measurements show that turbulence near the surface decreases rapidly with increasing altitude. Then, turbulence begins
to increase again several hundred meters above the ground, which is possibly due to the Earth boundary layer. The
LEEDR model in Fig. 9b captures a turbulence enhancement in the path as well, indicated by the orange ring. Overall,
the spatial comparison of profiler measurements reveals interesting similarities and differences in the characteristics
between measurements and modeling. These measurements will be used to further enhance the modeling capabilities.
VI. Modeling and Measurements Extended to System Performance

Up to this point, the analysis focuses on comparing double-ended profiler measurements with HV57 and LEEDR atmospheric models. This section extends the analysis by examining directed energy system performance analysis for an aircraft system looking down and for a ground system looking upward. The comparison of air-to-ground engagements versus ground-to-air engagements will use realistic directed energy system parameters with a 30cm aperture operating a 1MW, and 1μm laser.

A. System Performance Modeling With and Without the Aero-Optics

Both atmospheric conditions as well as an aero-optical environment impact the performance capability of a directed energy system. Atmospheric optical turbulence is caused by total temperature variations in the atmosphere. Aero-optical effects on the other hand, are optical distortions imposed on a laser beam due to a varying density field around an aircraft, caused by either compressibility effects at flight Mach numbers above 0.3 or by pressure variations \[1,38,40\]. The physical cause of aero-optical effects is fundamentally different from atmospheric optical effects.

This section presents system performance modeling as a function of the measured $C_n^2$ profile. The analysis focuses on capturing differences in performance with and without the inclusion of aero-optical contamination. Removal of
the two bins closest to the air terminal allows for a better understanding of how atmospheric optical turbulence alone affects system performance. Leaving the aero-optical contamination in the data set allows the error associated with these disturbances to be assessed. In the following figures, measured and modeled system quantities are presented with and without the aero-optical contamination. Then, using these measurements and modeled system quantities (with HV57 & LEEDR), the system performance is computed. Figure 10 compares four system parameters – spherical Rytov number, Isoplanatic Angle ($\theta_0$), Greenwood Frequency, and Tyler Frequency – between results with and without aero-optical contamination. Each plot contains profiler measurements, LEEDR weather cube (in blue), and HV57 model (in green). The hourly temporal resolution of LEEDR results in lulls in this model’s results. First, in Fig. 10(a) and (b), spherical Rytov number is analyzed. Generally, the removal of aero-optical contamination does not yield significant differences in Rytov number. The clustering of measurements between each plot are similar, and magnitude differences are minimal. Rytov is most sensitive to the turbulence in the middle of the propagation path and the mid-path conditions do not change between Fig. 10(a) and (b). The only differences in the turbulence profile are in the bins closest to the aircraft where Rytov has minimal sensitivity. The Rytov number computed from the profiler measurements have more agreement with the HV57 model than the LEEDR model. In Fig. 10(c) and (d), isoplanatic angle, $\theta_0$ is not affected by the removal of aero-optical contamination. $\theta_0$ is sensitive to turbulence deep into the path. Thus, for an air-to-ground (A2G) system, the removal of the first two bins nearest to the aircraft does not have much impact on the $\theta_0$ measurements. Generally, the HV57 and LEEDR models underestimate the measured isoplanatic angle. The measured and modeled Greenwood Frequencies are presented in Fig. 10(e) and (f). The removal of the aero-optical contamination significantly influences the results. With contamination, many of the measurements are around 200 Hz, but without contamination those same measurements fall to around 100 Hz. Several measurements are originally over 700 Hz, but without contamination, the magnitudes are significantly lower and more closely align with the HV57 model. This is expected since the boundary layer of the aircraft introduces high frequency, higher order aberrating structures. The Tyler Frequency, presented in Fig. 10(g) and (h), is influenced similarly. The Tyler Frequency measurements are mostly halved due to the removal of the aero-optical contamination. Similarly, the Tyler Frequency measurements are more consistent with the HV57 model and LEEDR model. The Greenwood Frequency and Tyler Frequency are computed from the $C_n^2$ profile and wind profile. Therefore, by removing the aero-optical contamination, the magnitude of the $C_n^2$ and wind profiles are reduced. Consequently, the Greenwood and Tyler Frequencies also significantly decrease when those bins are removed.

These optical parameters can be extended to system performance parameters. Figure 11 compares system performance parameters – spherical coherence diameter $r_0$, open loop jitter, and open loop Strehl – for the same air-to-ground system between results with and without the aero-optical contamination. The top row contains results with aero-optical contamination and the bottom row contains results without contamination. Each of these performance measurements change significantly with the removal of aero-optical contamination. Beginning with coherence diameter, $r_0$, in Fig. 11(a) and (d), the removal of aero-optical contamination results in significantly larger values for many of the
Fig. 10  System performance parameters with and without aero-optical contamination.
measurements. This result is expected since the bins closest to the aircraft impose a turbulence enhancement, which naturally will result in a smaller $r_0$ value. With contamination, $r_0$ is more consistent with the sparse LEEDR modeling than the HV57 model. The $r_0$ measurements with the aero-optical contamination removed are less clustered but are in closer agreement with the HV57 model results. Figure 11(b) and (e) show atmospheric open loop jitter which mostly decreases with the removal of aero-optical contamination. Originally, the measurements are clustered more around the LEEDR model, but with contamination removal the open loop jitter values become clustered closer to the HV57 model. Figure 11(c) and (f) present open loop Strehl with and without tilt removal. The measured open loop Strehl significantly increases when aero-optical contamination is removed. Many of the tilt-removed Strehl measurements (red points) are clustered around HV57 tilt-removed Strehl.

B. System Performance Modeling Comparison Between Air-to-Ground and Ground-to-Air

This section presents a comparison of system measurements and modeling between air-to-ground and ground-to-air engagements. All results in this section are free from aero-optical contamination, so some of the air-to-ground plots are identical to air-to-ground “Aero Removed” results presented in the previous section. For slant path engagements, the system performance and system capabilities will drastically differ between up-looking systems versus down-looking systems. Figure 12 compares air-to-ground (left column) and ground-to-air (right column) measurement parameters: spherical wave Rytov number, Isoplanatic Angle, Greenwood Frequency, and Tyler Frequency. Like before, each plot contains three sets of data: profiler measurements, LEEDR weather cube models from High-Resolution Window data,
and HV57 model. Plots (a) and (b) confirm that Rytov weighting function is symmetric between each terminal. For a particular turbulence profile, Rytov number does not change between an air-to-ground and ground-to-air system. Plots (c) and (d) compare Isoplanatic Angle, $\theta_0$. It can be seen that there is a pronounced difference between air-to-ground and ground-to-air system. The measured isoplanatic angles from ground-to-air are generally higher than air-to-ground, as expected. Since $\theta_0$ is sensitive to turbulence out in the path, an air-to-ground system observes smaller isoplanatic angles than an up-looking ground-to-air system. Plots (e) and (f) show the same Greenwood Frequency results from each terminal. This symmetry is expected since Greenwood Frequency is derived from $C_n^2$ and wind profiles. If the measurement is made from one terminal, the results are the same as from the other terminal because the $C_n^2$ and wind profiles reverse orientation, yielding the same data for the calculation of Greenwood Frequency. For the same reason, plots (g) and (h) show the same Tyler Frequency results from each terminal.

Furthermore, the optical parameters can be extended to other system performance metrics. The results are obtained using the same example system with a 30cm aperture operating at wavelength of 1 $\mu$m. Figure 13 shows the system performance modeling parameters – spherical wave $r_0$, open loop jitter, and open loop Strehl – for a ground engagement. It is apparent that the models are pessimistic, showing $r_0$ values that are generally lower than measurements. While the HV57 and LEEDR models forecast $r_0$ to be approximately 6cm, the profiler measurements show $r_0$ values ranging from 5cm to 15cm. The open loop jitter plot in the center of Fig. 13 shows a similar performance trend. Open loop jitter is a function of $r_0$ and inversely proportional $[41,42]$. The modeling shows a standard deviation of beam jitter to be 5-6$\mu$rad. However, the measurement mostly reported open loop atmospheric jitter under 5$\mu$rad. For context, the tracking terminal at the ground station described in Section III had two optical tracking systems (acquisition track system and fine track system). The instantaneous field of view of the acquisition track system, which provides the wide field of view, was set to 62.7$\mu$rad and the instantaneous field of view of the fine track system, which provides the narrow field of view, was set to 1.65$\mu$rad.
Fig. 12  Air-to-Ground vs Ground-to-Air system performance parameters without aero-optical contamination.
For a ground-to-air system with similar instantaneous field of views, the jitter created by uncompensated atmosphere shown in Fig. 13 is a small fraction of a pixel in the aquisition track system and 2-4 pixels in the fine track system. In those terms, the HV57 and LEEDR model’s forecast anticipates around 4 pixels of atmospheric jitter however the measurements indicated an observed 2-4 pixels throughout the data collection. Similarly, Fig. 13 also shows the open loop Strehl ratio (right). The Strehl ratio and tilt-removed Strehl are also a function of $r_0$. This plot contains several key trends. First, for a ground-to-air system, the strongest turbulence along the propagation path is near the aperture, and therefore the Strehl ratio is relatively small. Second, the HV57 model and LEEDR model are in agreement and indicate a Strehl of approximately 3% to 5%. The profiler measurements show the Strehl varying from 2% to 16%. Third, by removing the tilt, the Strehl ratio significantly increases. The tilt-removed values are representative of having an ideal fast steering mirror in the system that corrects for the tilt disturbances. The tilt-removed values, indicated in Fig. 13 by a ‘TR’, are 10% to 15% for the models and 10% to 60% for the profiler measurements. Lastly, as observed in previous plots, the LEEDR model does not have the temporal resolution to capture trends occurring at the minute-by-minute scale. Overall, the models tend to be underrating the ground-to-air system performance measurements.

In a similar way, the system performance can be modeled using the same system parameters and the same turbulence profiles, but with a reversed operation geometry. Figure 14 shows air-to-ground engagement system performance modeling results. Here, the LEEDR models are highly pessimistic, mostly resulting in underrated performance relative to measurements. The $r_0$ of the LEEDR model range from 6cm to 10cm, while the HV57 model indicates an $r_0$ around 34cm. The profiler $r_0$ measurements fluctuate significantly spanning the range of 2cm to 40cm. These fluctuations in the measurements translate to large fluctuations in the measured open loop jitter and open loop Strehl. The open loop jitter indicated by the LEEDR is 4-5 $\mu$rad, while the HV57 model shows 1.5 $\mu$rad. The profiler measurements fall in between the two models. For context, similar to the ground station, the tracking profiler terminal at the air station also had two optical systems (acquisition track system and fine track system). The instantaneous field of view of the aquisition track system, which provides the wide field of view, was set to 43.1 $\mu$rad and the instantaneous field of view of the fine track system, which provides the narrow field of view, was set to 7.5 $\mu$rad. For an air-to-ground system with

![Fig. 13 Ground-to-Air system performance.](image-url)
similar instantaneous field of views, the jitter created by the uncompensated atmosphere is a negligible fraction of a pixel in the acquisition track system and about $\frac{1}{5}$ pixels in the fine track system. The Strehl and tilt-removed Strehl ratios are significantly higher for air to ground engagements as compared to the ground to air system. Since the strong turbulence is away from the aperture, and the aero-contamination has been removed, the air-to-ground system provides more favorable atmospheric conditions for imaging and optical propagation. In Fig. 14, the open loop Strehl plot (right) shows the HV57 model at 50% and LEEDR spanning 15% to 40%. The profiler measurements vary from 2% to 60%. The tilt removed Strehl significantly improves the system performance. The tilt-removed HV57 model and many tilt-removed profiler measurements show a 90% Strehl ratio. Overall, the HV57 model has reasonable agreement with the measurements while the LEEDR model under rates the air-to-ground system performance.

VII. Conclusion

The work presented here demonstrates a new approach for making air-to-ground path-resolved turbulence measurements. The DDTV path resolved turbulence profiling technique, along with the described experimental hardware/software tools, enable these types of measurements. A unique spatial filtering technique allowed atmospheric and aero-optical induced effects to be decoupled. The predicted strength of aero-optical and aero-acoustical contamination showed that the turbulence measurements reasonably isolated the contamination to within the first two bins of the profile. The turbulence profile measurements were compared to the standard HV57 model and state-of-the-art LEEDR atmospheric model. The analysis showed occasional strong agreement in trends between modeling and measurements at higher altitudes. However, the analysis also revealed important deviations between modeling and measurements. The results showed that models consistently predict higher turbulence close to the ground than indicated by the measurements. However, at altitudes between 1 to 1.5km, the measurements confirmed a trend seen in the LEEDR model but absent in the HV57 model that shows an increase in $C_n^2$. The increased turbulence strength at this altitude could be attributed to the Earth boundary layer. The modeling-versus-measurement analysis examined how differences in turbulence profiles impact directed energy system performance parameters. Analyzing system performance metrics such as open-loop jitter and open-loop Strehl shows that conditions are significantly different for air-to-ground and ground-to-air systems.
operating along the same turbulence path. For the ground-to-air engagements, both models underrated the system performance. For the air-to-ground scenarios, the HV57 model tends to be optimistic and LEEDR model tends to be pessimistic. The similarities and differences between measurements and models affords an exciting opportunity for future campaigns to build on the findings presented in this manuscript.

References


