Pressure Fields and Forces Acting on a Spanwise-Oscillating Hemispherical Turret

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Abstract

An experimental Pressure Sensitive Paint (PSP) study of the unsteady pressure fields on a surface of an oscillating hemispherical turret at subsonic Mach numbers between 0.3 and 0.6 is presented. The oscillating turret assembly, consisted of a rigid turret shell mounted on a rectangular aluminum plate, was built to oscillate at a single frequency within the range of the dominant wake frequencies. The resonant-based aero-elastic response of the turret to the unsteady flow resulted in intermittent forced oscillations in the spanwise direction at this fixed frequency. Modal analysis of the pressure fields, utilizing a Joint POD technique, revealed that the oscillating turret primarily suppressed the global spanwise-antisymmetric separation mode by locking its dynamics to the turret motion. The analysis of the unsteady forces acting on the turret revealed that the oscillating turret reduces the spanwise component of the unsteady force, while keeping other force components mostly unchanged. Intermittent nature of the spanwise force, which is responsible for the intermittent motion of the oscillating turret, was observed and discussed. Finally, the turret vibrational response was measured and was shown to be well approximated by a linear second-order under-damped system.

I. Introduction

Hemisphere on-cylinder and hemispherical turrets are often used in airborne optical systems to maximize the potential field-of-regard in a wide variety of adaptive optics and beam control applications. However, when aircraft is instrumented with a turret-based system, a turbulent flow around the turret and especially in the wake induces density gradients and associated variations in index-of-refraction (Gordeyev and Jumper 2010). When a laser light propagates through regions of air containing spatio-temporal variability in index-of-refraction, it will become aberrated (Wang et al. 201; Jumper and Gordeyev 2017). Even at subsonic speeds, these so-called aero-optical distortions around turrets might be significant, potentially reducing laser light focusability on the
target and degrading the point-and-track capabilities of adaptive-optics systems (Gordeyev and Jumper 2010; Jumper and Gordeyev 2017).

A subsonic flow around a turret is fairly complex and unsteady (Gordeyev and Jumper 2010). Briefly, as the flow interacts with the turret, the flow stagnates at the leading portion of the turret and forms a necklace vortex, which legs travel around both sides of the turret base and convect in the streamwise direction through the wake. A portion of the flow accelerates over the turret until it separates near the apex; after the separation it becomes significantly more turbulent and folds downward toward the wall, forming a pair of counter-rotating “horn” vortices on the downstream portion of the turret. The separated region of the flow creates a highly complicated three-dimensional turbulent wake. The turret wake has been found to be dominated by low-frequency, large-scale structures (De Lucca et al. 2018a,b; Gordeyev et al. 2018), which create unsteady forces acting on the turret itself (De Lucca et al. 2013a). These unsteady forces can introduce various mechanical vibrations into optical components via aero-elastic response and, consequently, induce additional aero-mechanical jitter of a laser beam projected from it, further degrading the performance of airborne laser systems.

To address these issues, many experimental (Gordeyev and Jumper 2010; Vukasinovic et al. 2013; De Lucca et al. 2013b; Porter et al. 2013; Morrida et al. 2017) and numerical (Ladd et al. 2009; Morgan and Visbal 2012; Coirier et al. 2014; Matthews et al. 2016; Jelic et al. 2017) studies were conducted to quantify the aero-optical distortions and fluidic performance of turrets. Unsteady spatially-resolved pressure fields on and around the turrets, obtained using pressure sensitive paint (PSP), were used to study the dynamics of the wake at various subsonic and transonic speeds (Gordeyev et al. 2014; De Lucca et al. 2018a,b; Gordeyev et al. 2018). These PSP studies allowed the spatially resolved unsteady global pressure fields to be reconstructed on the surface of and around the turret. Proper orthogonal decomposition (Gordeyev et al. 2014; De Lucca et al. 2018a), and a conditional analysis (Gordeyev et al. 2018) of the pressure fields around and downstream of the hemispherical turret for the range of turret Reynolds numbers on the order of a million revealed that the main dynamic mode in the wake behind the hemispherical turret is a wake shifting mode. The wake shifting mode describes the spanwise-antisymmetric alternating shedding of vortical structures from the opposite spanwise sides of the turret. The corresponding spectral analysis of the temporal evolution of the shifting mode found that the main frequency of the mode lays within the frequency range of $fD/U_\infty = 0.1–0.2$, where $D$ is the hemisphere diameter and $U_\infty$ is the incoming freestream speed. A similar frequency range of the wake shedding vortices over a hemisphere of 0.15–0.2 was observed by McCarthy et al. (2019) at a smaller turret Reynolds number of 6.36 x10^4. An associated wavelet analysis of the fluctuating pressure series on and downstream of the hemisphere revealed an intermittent coherence across the turbulent wake in the aforementioned range of frequencies. A distribution of time intervals of significant intermittency was approximated by an inverse Gaussian probability density function with the mean value of time intervals about $\sim 50 D/U_\infty$.

All aforementioned experimental and numerical studies have assumed that the turret surface was rigid and ignored the mechanical motion of the turret itself. In real applications, however, the turret is not a rigid body, typically constructed as a thin-shell body with various optical and mechanical components inside, mounted on a point-and-steering platform. If this is the case or the mounting platform is not sufficiently rigid, the turbulent flow around the turret will induce unsteady forces applied to the turret skin and/or the mount. These unsteady forces can potentially result in deformation of the turret surface or mechanical motion of the turret as a whole via aero-elastic mechanism, causing changes of the wake dynamics and additional beam jitter.
In the presented studies, a hemispherical turret was allowed to oscillate freely in the spanwise direction by mounting it on a flexible cantilever beam. The oscillating turret system was designed and built such that the dominant vibrational frequency was inside the dominant frequency range of the unsteady wake downstream of the turret, in order to utilize resonance-related oscillation amplitude amplification. For comparison, the turret was also rigidly affixed to the tunnel wall to represent a stationary turret case. The turret and wake region were both painted with fast-response porous pressure sensitive paint (PSP) in order to collect spatially-temporally resolved surface pressure fields and compare the global pressure fields between the oscillating and stationary cases. A companion work (Roeder 2020) analyses the pressure fields in the wake of the turret and discusses the changes in the wake dynamics induced by the oscillating turret. In this paper, pressure fields on the surface of the hemispherical turret and associated forces acting on it are presented and discussed. Section II introduces the concept of the oscillating turret along with an associated mathematical linear model. Section III discusses the details of experimental setup for subsonic wind tunnel testing. Data analysis is described in Section IV, and experimental results are presented and discussed in Section V. Finally, conclusions are summarized in Section VI.

II. Oscillating Turret Concept

A concept of an oscillating turret, when a rigid hemispherical turret is mounted on a thin, wide rectangular aluminum plate, is schematically shown in Figure 1. The plate is aligned along the streamwise direction, so the turret can only move in the spanwise direction. In this investigation transverse oscillating cantilever uniform Euler-Bernoulli beam theory for a beam with tip mass attachment, presented in Erturk and Inman (2011) and Repetto et al. (2012), was used to model the oscillating turret system. The vibrational response of this system to the input forcing, \( F_z(t) \), applied at the free end of the beam, are governed by the following differential equation,

\[
m \frac{\partial^2 v(y,t)}{\partial t^2} + \gamma m \frac{\partial v(y,t)}{\partial t} + E \cdot I_p \frac{\partial^4 v(y,t)}{\partial y^4} = 0
\]

The clamped-free boundary conditions for this system are given by,

\[
\left[ \frac{E \cdot I_p}{\partial y^2} + I, \frac{\partial^3 v(y,t)}{\partial t \partial y^2} \right]_{y=L} = 0, \quad \left[ E \cdot I_p \frac{\partial^3 v(y,t)}{\partial y^3} + M_i \frac{\partial^2 v(y,t)}{\partial t^2} \right]_{y=L} = F_z(t)
\]

\[
v(y=0,t) = \left. \frac{\partial v(y,t)}{\partial y} \right|_{y=0} = 0
\]
In this model, \(v(y,t)\) describes the local displacement along the plate in the z-direction due to the unsteady forcing in the spanwise direction, \(E\) is Young’s modulus for aluminum and \(\rho\) is the density of the aluminum plate. \(L\) is the length of the plate, \(w\) is the plate width, \(h\) is the plate thickness, \(m = \rho wh\) is the plate mass per unit length of the beam, \(I_p = wh^3/12\) is the moment of inertia of the cross-sectional area of the plate and \(\gamma\) is the damping coefficient of the aluminum plate. \(M_t = M_{\text{shell}} + M_{\text{disk}}\) is the total mass attached at the end of the beam, which consists of a thin plastic turret shell with the mass \(M_{\text{shell}}\) and an aluminum mounting disk 3 mm thick with the mass \(M_{\text{disk}}\). Finally, \(I_t\) is the total moment of inertia of the turret shell and the mounting disk about the attachment line \(y = L\),

\[
I_t = \frac{1}{4} M_{\text{disk}} R^2 + \frac{1}{3} M_{\text{shell}} R^2
\]

Following the approach, outlined in Erturk and Inman (2011) and Repetto et al. (2012), the solution of the Eqs. 1-3 was found using the separation of variables and assuming a response to a unit single harmonic input, \(F_z(t) = \exp(i \omega t)\) and \(v(y,t) = Y(y; \omega) \exp(i \omega t)\). From the solution, a spectral transfer function, \(G(\omega) = Y(y=L; \omega)\), between the input harmonic forcing and the resulted harmonic motion of the turret, was calculated to be

\[
G(\omega) = G_{\text{steady}} \frac{3 \left[ \sin(\lambda) \cosh(\lambda) - \cos(\lambda) \sinh(\lambda) + \frac{\omega^2 L \cdot I_t}{\lambda E \cdot I_p} (\cos(\lambda) \cosh(\lambda) - 1) \right]}{\lambda^3 \cdot \Delta}, \tag{4}
\]

where

\[
\Delta = 1 + \cos(\lambda) \cosh(\lambda) + \frac{\omega^2 L^3 \cdot M_t}{\lambda^3 E \cdot I_p} \left[ \cos(\lambda) \sinh(\lambda) - \sin(\lambda) \cosh(\lambda) \right]
\]

\[
- \frac{\omega^2 L \cdot I_t}{\lambda E \cdot I_p} \left[ \cosh(\lambda) \sin(\lambda) + \sinh(\lambda) \cos(\lambda) \right] + \frac{\omega^4 L^4 \cdot M_t \cdot I_t}{\lambda^5 (E \cdot I_p)^2} \left[ 1 - \cos(\lambda) \cosh(\lambda) \right] = 0
\]

and
\[
\lambda = L \left[ \frac{(\omega^2 - i\gamma \omega)\rho}{E \cdot I_p} \right]^{1/4}
\]

In Eq. 4, \( G_{\text{steady}} = \frac{L^3}{3E \cdot I_p} \) is a steady turret position under steady unit forcing.

For the presented studies, the turret diameter was chosen to be \( D = 0.254 \text{ m} \), and the dimensions of the aluminum plate was chosen to be \( h = 12.7 \text{ mm} \), \( L = 235 \text{ mm} \) and \( w = 254 \text{ mm} \). The plate material was 7075-T651 aluminum alloy, which was selected for its fatigue-resistant properties. All relevant parameters of the turret and the plate used to compute the transfer function are presented in Table 1.

<table>
<thead>
<tr>
<th>Plate Thickness, ( h )</th>
<th>Plate Width, ( w )</th>
<th>Plate Length, ( L )</th>
<th>Plate Moment of Inertia, ( I_p )</th>
<th>Damping coefficient, ( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.7 mm</td>
<td>254 mm</td>
<td>235 mm</td>
<td>0.812 kg( \cdot )m(^2)</td>
<td>65.2 1/s</td>
</tr>
<tr>
<td>Total Turret Mass, ( M_t )</td>
<td>Total Turret Moment of Inertia, ( I_t )</td>
<td>Young’s Modulus for 7075-T651 Aluminum alloy, ( E )</td>
<td>Density for 7075-T651 Aluminum alloy, ( \rho )</td>
<td></td>
</tr>
<tr>
<td>1.511 kg</td>
<td>6.4( \cdot )10(^{-3} ) kg/m(^3)</td>
<td>71.7 GPa</td>
<td>2,810 kg/m(^3)</td>
<td></td>
</tr>
</tbody>
</table>

To measure the damping coefficient of the aluminum plate, \( \gamma \), additional measurements were conducted on a separate rectangular aluminum bar of the same thickness. One end of the bar was clamped, and the free end of the beam was deflected and suddenly released. The resulted damping motion of the bar was recorded by a single-axis accelerometer, mounted on the free end of the bar. Analysis of the bar response was conducted and the damping coefficient was measured to be \( \gamma = 65.2 \text{ 1/s} \), see Roeder (2020) for additional details of the experiment and the analysis.

Using the parameters in Table 1, the theoretical transfer function, \( G(\omega) \), normalized by the steady position, \( G_{\text{steady}} \), was numerically calculated and its amplitude and phase is shown in Figure 2. The first resonant frequency was found to be 89.4 Hz, where the amplitude reaches 32 dB and the phase drops from zero to -180 degrees. Above this frequency, the amplitude decays at a rate of -40 dB/decade and the phase stays at -180 degrees. The only exception is near the second resonant frequency of 423.3 Hz, where the amplitude has a drop, followed by a secondary peak, while the phase approaches zero before dropping back to -180 degrees. With the exception of this narrow frequency range, the oscillating turret can be modelled as a linear second-order underdamped system with the main resonant frequency of 89.4 Hz.
In these studies, the freestream Mach numbers were chosen to be between $M = 0.3$ and $0.6$, and for these speeds and the selected turret diameter, the peak in the wake response at $St_D = 0.15$ corresponds to a range of frequencies between 80 and 100 Hz. Thus, the main resonant frequency of the turret assembly lays inside this range, and the unsteady wake should induce resonant-based turret oscillations in the spanwise direction.

**III. Experimental Studies**

Based on the theoretical studies, outlined above, the turret assembly was designed and built, as shown in Figure 3. To allow for the turret motion, the bottom of the mounting plate with the hemisphere affixed to it was aligned with the tunnel wall with a small, in the order of a millimeter, gap in between. The oscillating turret assembly was converted to the stationary turret configuration by inserting metal spacers between the mounting disk and the vibrating plate, so the mounting disk was pressed against the tunnel wall. These configurations enabled two different sets of experiments in order for the flow physics and dynamic effects of the oscillating turret to be compared to the same, but stationary turret configuration.

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**Fig. 2** Amplitude and phase of the theoretical spectral response to the input forcing, Eq. 4

![Graph showing amplitude and phase of the theoretical spectral response]
Both the oscillating and stationary turret configurations were tested in the University of Notre Dame White Field 3’ x 3’ wind tunnel facilities for freestream Mach numbers of \( M = 0.3, 0.4, 0.5 \) and 0.6, with the corresponding range of Reynold numbers \( 1.7 \times 10^6 \leq Re_D \leq 2.6 \times 10^6 \). The complete description of the experimental set-up and the procedure is presented in Roeder (2020), so only essential details will be presented here. A schematic of the experimental setup is presented in Figure 4. A fast-response Pressure-Sensitive Paint (PSP) was used to collect unsteady pressure fields on the surface of and around the turret. A very thin uniform layer, < 1mm, of PSP was sprayed on the plastic dome and the surrounding tunnel wall before testing was conducted. The binder material was polymer-ceramic PSP (PC-PSP), which consisted of Silicone RTV as the polymer and bathophen ruthenium as the luminophore. Four 400 nm ultraviolet lights (ISSA LM2-400 type) were used to illuminate the PSP. Two high-speed cameras, Phantom v1611 and Phantom v2512, were used to simultaneously record the fluorescence intensities emitted by the PSP for several seconds. Both cameras were run at the same sampling frequency of \( f_{sam} = 3 \) kHz at the maximum pixel resolution of 1280 x 800. Several subminiature differential pressure transducers, Type XT-140(M), were used to perform in-situ calibration of the PSP; four pressure transducers were installed on the turret surface and three pressure transducers were instrumented on the tunnel wall in the wake region. The transducers were simultaneously sampled with the cameras at 30 kHz for 30 seconds. The origin of the frame of reference was chosen to be at the center of the hemispherical turret, with x-axis denoting the streamwise direction, y-axis denoting the wall normal direction and z-axis denoting the spanwise direction.
IV. Data Analysis

The instantaneous intensity variations, $I$, emitted by the PSP are related to the instantaneous local static pressure values, $P$, via the Stern-Volmer equation,

$$\frac{I_{\text{ref}}}{I} = A + B \frac{P}{P_{\text{ref}}} \ ,$$

where $I_{\text{ref}}$ and $P_{\text{ref}}$ are the time-averaged reference intensity and pressure values, and $A$ and $B$ are experimentally determined coefficients. Reference data were collected before each test under no flow conditions with the ultraviolet lights turned on. The temporal-spatial averaged intensities from PSP inside a small annular region immediately surrounding each of the seven pressure sensors were extracted at all tested Mach numbers, and the static pressures from the pressure transducers were used in order to determine $A$ and $B$ for each test case. An example of calibration
curve from the in-situ calibration is presented in Figure 5. Using these results, $A$ and $B$ coefficients in Eq. 5 were found to be $A = 0.64$ and $B = 0.33$. The dynamic response of PSP was studied in Roeder (2020) and the PSP was found to correctly resolve the unsteady pressure spectrum up to approximately 700 Hz, which was deemed sufficient for the present experiments. Also, while technically the calibration coefficients in Eq. 5 also depend on the flow temperature (Hayashi and Sakaue 2020), the temperature changes during the tunnel runs were measured to be less than $2^\circ$C, and the temperature variation of $A$ and $B$ coefficients was neglected.

Fig. 5 Calibration curve using in-situ PSP data for freestream Mach numbers $M = 0.3 – 0.6$

Knowing exact location of each camera, relative to the turret, it is possible to map the pressure variation from 2-D images onto the 3-D surface of the turret, using Perspective Transformation Matrix (PTM) technique (Tan et al. 1993). Similar to the procedure, used in Gordeyev et al. (2014), the turret surface was split into two regions to provide a unique mapping between the turret surface and the corresponding cameras’ images. Using PTM, pressure values on the surface of the turret in each region were computed and, after combining the regions, the full surface pressure field on the turret was reconstructed.

After extracting the surface pressure fields, the time-averaged fields were computed and removed from the instantaneous pressure fields to produce the fluctuating pressure fields, $p(x,y,z,t)$, for different test cases and Mach numbers. Using these data, the normalized mean-removed coefficients of pressure distributions, $C_p(x,y,z,t) = \frac{p(x,y,z,t) - p_\infty}{0.5 \rho_\infty U_\infty^2}$, were computed for each Mach number and turret configuration, where $p_\infty$ and $\rho_\infty$ denotes the freestream pressure and density, respectively. The spatial distribution of the temporal root-mean-squared coefficients of the fluctuating pressure, $C_{p,\text{rms}}(x,y,z) = \sqrt{C_p^2(x,y,z,t)}$, were calculated in order to study the unsteady pressure distribution on the turret. The overbar here and later in this paper denotes time averaging.

Proper Orthogonal Decomposition (POD) is commonly used as a means for creating a low-order reconstruction of the spatio-temporal data, where the data can be adequately represented using a reduced number of spatial orthonormal modes with the corresponding temporal coefficients (Taira et al. 2017). However, POD decomposition depends on a particular dataset and for different datasets POD modes will be different, so comparing POD modes and their coefficients
between different sets becomes complicated. An alternative approach, called Joint Proper Orthogonal Decomposition (JPOD), introduced by Gordeyev et al. (2014), was used here as a data analysis technique to compute common or joint modes of the pressure fields on the turret surface. Briefly, JPOD is a specialized version of POD, in which the spatio-temporal pressure fields for different cases are combined into a single joint pressure field,

\[ p_{\text{joint}}(s,t) = \{ p(s,t;\Lambda_1), p(s,t;\Lambda_2), \ldots \} \]

where \{\Lambda_1, \Lambda_2, \ldots\} are some unique parameters which characterize each case and \( s = (x, y, z) \) is a point on the turret surface. These joint pressure field data are used to extract the spatial modes, which are common to all cases. In the present studies, the pressure fields are characterized by only two parameters, the oscillating and the stationary turret configurations. It is easy to see that the corresponding joint correlation matrix, \( R_{\text{joint}} \), is given by the average of all of the individual correlation matrices (Gordeyev et al. 2014),

\[
R_{\text{joint}}(s,s') = \frac{1}{K} \sum_{k=1}^{K} R(s,s';\Lambda_k) .
\]  

Using \( R_{\text{joint}}(s,s') \), the JPOD modes can be found by solving the eigenvalue problem,

\[
\int R_{\text{joint}}(s,s')\varphi_n(s')ds' = \lambda_n \varphi_n(s),
\]

By the construction, the new set of JPOD modes does not depend on the parameters \{\Lambda_1, \Lambda_2, \ldots\}, so they are the same of all included cases. However, when the pressure field for each case is projected onto the spatial JPOD modes, the temporal coefficients, \( a_n \), and the corresponding eigenvalues or energies of the modes, \( \lambda_n \), will be the functions of the \( \Lambda \)-parameters and therefore will differ for each case,

\[
a_n(t;\Lambda_k) = \int p(s,t;\Lambda_k)\varphi_n(s)ds, \quad \lambda_n(\Lambda_k) = \frac{\bar{a}_n^2(t;\Lambda_k)}{\lambda_n}
\]

Thus, the differences in the pressure fields are reflected only in the temporal coefficients and the related eigenvalues. Using the eigenvalues, individual energies of JPOD modes, normalized by the total amount of energy, \( \lambda_n / \sum_k \lambda_k \), were computed. Similarly, the cumulative, up to the \( n \)-th mode, energies, defined as \( \sum_k \lambda_k / \sum_k \lambda_k \), were also computed. Instantaneous pressure fields can be reconstructed from JPOD modes and the coefficients as,

\[
p(s,t;\Lambda_k) = \sum_n a_n(t;\Lambda_k)\varphi_n(s)
\]

Using the pressure fields, the unsteady fluctuating forces, \( \bar{F}(t) = \{ F_x(t), F_y(t), F_z(t) \} \), acting on the turret were computed by integrating the unsteady fluctuating pressure field over the hemispherical turret surface,

\[
\bar{F}(t) = \int p(s,t)\bar{n}(s)ds ,
\]

where \( \bar{n} = (n_x, n_y, n_z) \) are the normal outward vector and the integration is performed over the hemisphere. All three components of the force were normalized by the dynamic pressure and the
cross-sectional area of the turret, \( S_{cross} = 0.5\pi R^2 \), yielding a non-dimensional time-dependent force coefficients, \( \{C_x(t), C_y(t), C_z(t)\} \). Using these force coefficients, temporal root-mean-squared values and the power spectra were calculated for each force component.

JPOD analysis is also useful to analyze the contribution of JPOD modes to the unsteady force components. By substituting Eq. 9 into Eq. 10 and recalling that \( \lambda_n(\Lambda_k) = a_n^2(t; \Lambda_k) \), so \( a_n(t; \Lambda_k) \sim \sqrt{\lambda_n(\Lambda_k)} \), it is possible to obtain,

\[
F_i(t; \Lambda_k) = \sum_n \frac{a_n(t; \Lambda_k)}{\sqrt{\lambda_n(\Lambda_k)}} \int S \sqrt{\lambda_n(\Lambda_k)} \phi_n(s) n_i(s) ds = \sum_n \frac{a_n(t; \Lambda_k)}{\sqrt{\lambda_n(\Lambda_k)}} b^n_i,
\]

where

\[
b^n_i = \int S \sqrt{\lambda_n(\Lambda_k)} \phi_n(s) n_i(s) ds.
\]

B-coefficients are the influence coefficients, reflecting a contribution of \( n \)-th JPOD mode to \( i \)-th component of force with \( i = (x, y, z) \). It here it follows that the force is linearly proportional to the temporal coefficients of the JPOD modes, weighed by the influence coefficients.

The temporal spanwise displacement of the turret in the oscillating case was measured directly by optically tracing the motion of the turret edge. This was accomplished by extracting a single 1 x 30 column of pixels (0.5 mm x 1.6 mm) spanning the gap region and tracking the relative position of the intensity threshold corresponding to the turret edge. The exact position of the turret edge was resolved for each frame by interpolating the intensity values of adjacent pixels in order to achieve displacement resolution within the subpixel range. The edge position of the oscillating turret, \( z_{turret} \), was computed for the entire time series at all three tested Mach numbers.

Finally, in order to study the intermittency of the applied forcing and the turret motion during the stationary and the oscillating cases, a conditional analysis using a Morlet wavelet analysis (Gordeyev and Thomas 1999) was applied. The wavelet transformation of a continuous signal, \( g(t) \), is generally defined as,

\[
G_{\psi}(\kappa, \tau) = \kappa \int_{-\infty}^{\infty} g(t) \Psi^*(\kappa (t-\tau)) dt
\]

where \( \Psi(\eta) \) is the wavelet mother function, \( \kappa \) is a dilatation parameter, \( \tau \) is a translation or shift parameter and an asterisk denotes a complex conjugate. Morlet mother function is defined as

\[
\Psi(\eta) = \exp\left( id\eta - \frac{\eta^2}{2}\right) - \exp\left( \frac{d^2}{2} - \frac{\eta^2}{2}\right), \text{ where } d = 6 \text{ was chosen for this study.}
\]

The Morlet wavelet transform is useful to study temporal amplitude variation of the signal at a given frequency. The frequency is related to the dilatation parameter as \( f(\kappa) = (d\kappa) / (2\pi) \approx 0.96\kappa \). The local amplitude of the Morlet wavelet transform was computed as \( A_{\psi}(\tau, f) = |G_{\psi}(f(\kappa), \tau)| \).

V. Experimental Results

A. Oscillating Turret Motion

Time series of the turret position in the spanwise direction for the oscillating case were extracted, as discussed before, and auto-correlation spectral densities at \( M = 0.4, 0.5 \) and 0.6 are presented in Figure 6. The spectra for the oscillating turret reveals the presence of a single strong peak at a frequency of 87 Hz. This value is close to the theoretical main resonant frequency of 89.4
Hz, predicted by the oscillating turret model. In addition to the dominant peak, the first and the second harmonics at 174 Hz and 260 Hz, respectively, with much smaller levels of spectral energy are also present in the spectra, indicating a small degree of non-linear effects in the turret response. A range of low frequencies, centered around 15 Hz, are present in the spectra, and, as it will be shown later in this paper, they are related to the intermittent nature of the turret motion.

![Auto-correlation spectral densities of turret displacement for the oscillating case at M = 0.4, 0.5 and 0.6](image)

**Fig. 6** Auto-correlation spectral densities of turret displacement for the oscillating case at $M = 0.4$, 0.5 and 0.6

The time series of the extracted turret position for the oscillating case at $M = 0.5$ are presented in Figure 7 as a thin blue line. The time series show intermittent bursts up to 0.5 mm in amplitude with durations of few hundred of $D/U_{∞}$. The amplitude of Morlet wavelet transform of the edge motion was extracted at the 87 Hz main resonant frequency and is presented in Figure 7 as a thick red line. The amplitude increases closely match the bursts of the signal. Similar intermittency of the turret spanwise motion were observed at other test Mach numbers. As the source of the turret oscillations is the spanwise unsteady forcing, this behavior confirms the intermittent nature of the unsteady spanwise forcing, acting on the turret. We will provide additional analysis of the related intermittent forcing later in this paper.
Fig. 7 Time series of the turret spanwise motion, \( z_{\text{turret}} \), (thin red line) and amplitude of Morlet wavelet transform at the main resonant frequency, \( A_{\Psi}(\tau, f = 87 \text{ Hz}) \) (thick red line) for the oscillating case at \( M = 0.5 \)

**B. Unsteady Pressure Fields**

The spatial distribution of the temporal root-mean-squared pressure coefficients, \( C_{p,\text{rms}}(x, y, z) \), on and around the turret for the oscillating and stationary turret cases at \( M = 0.5 \) are presented in Figure 8. The unsteady pressure fields for both cases are approximately symmetric in the spanwise direction, indicating a symmetric flow. The unsteady necklace vortex, formed in front of the turret, is responsible for the increase in \( C_{p,\text{rms}} \) in front of the turret. The unsteady separation over the turret is responsible for the increase in the pressure fluctuations, visible as a narrow region aligned in the spanwise direction on top of the turret. Finally, an inherently unsteady re-attachment line downstream of the turret creates a crescent-shaped region of large pressure fluctuations. Overall, the spatial map of the fluctuating pressure is very similar to the one, observed in other studies of the pressure fields on and around the hemisphere at subsonic speeds (De Lucca et al. 2018a,b; Gordeyev et al. 2018).
For the oscillating turret, the source of the turret motion is the unsteady force acting on the turret. As it requires energy to move the turret, the overall turbulent energy in the wake should decrease. Indeed, as it can be seen in Figure 8, the oscillating turret does weaken the turbulent wake, as well as increases its streamwise extent. Detailed analysis of the pressure fields in the wake for both the stationary and the oscillating cases at subsonic Mach numbers can be found in Roeder (2020).

C. JPOD Analysis of Unsteady Pressure Fields

Before presenting the modal analysis of the pressure fields, it is useful to discuss the relation between the unsteady separation location over the turret and the related changes in the unsteady pressure field, as schematically illustrated in Figure 9. Here, the time-averaged pressure distribution over a hemisphere is shown as a dashed line. The flow accelerates at the front portion of the hemisphere, so the pressure decreases. After passing the apex, the flow starts slowing down and the pressure begins to raise, resulted in adverse pressure gradient environment. At some point, the flow separates and the pressure becomes approximately constant inside the separation region. Let us say at some moment, a change in the wake topology results in decreasing the pressure gradient downstream of the apex. Because of the smaller pressure gradient, the flow stays attached longer and the separation point moves downstream, relative to the time-averaged separation location. Also, the pressure after the separation might be higher than the time-averaged pressure. Recall that the mean pressure is removed from the pressure field, so only the unsteady pressure is analyzed. Looking at the difference between the instantaneous and the time-averaged pressure distributions, the downstream shift in the separation point would result in a negative pressure difference upstream of the instantaneous separation location, and possibly in a small positive pressure difference near and after the separation location. Similarly, if the separation point moves upstream of the time-averaged location, it would result in the positive pressure difference, followed by the negative pressure change. Another word, the fluctuating pressure field near the separation location should be negatively correlated with the fluctuating pressure inside the separated region.
Using the pressure data on the turret surface, the first six JPOD spatial modes were computed for $M = 0.5$ and are presented in Figure 10. The first dominant mode, as well as the third and the sixth modes are anti-symmetric in the spanwise direction, while modes #2, #4 and #5 are symmetric in the spanwise direction. The symmetric mode #2 reveals a region of negative pressure change near the separation line, followed by a global positive pressure change in the separated region. Remember, that JPOD modes are multiplied by the temporal coefficients, and the coefficients can be either positive or negative. Thus, using the argument outlined in the previous section, the positive value of the temporal coefficient would correspond to the separation line located downstream of the time-averaged location, and the negative coefficients would indicate times where the separation line is upstream of the time-averaged location. Thus, mode #2 and similarly, mode #4 describe the symmetric motion of the separation line near the hemisphere apex in the streamwise direction, with the related changes in the separated region, as discussed before. This wake dynamics corresponds to the wake global breathing mode, discussed in Gordeyev et al. (2018), De Lucca et al. (2018a).

The dominant mode #1, however, is anti-symmetric in the spanwise direction and primarily non-zero near the separation line. It represents the situation, when the separation line on one side of the hemisphere moves upstream, while the separation line on the opposite side shifts downstream. This anti-symmetric separation motion was shown to be related to the global shifting motion of the wake (Gordeyev et al. 2018). Mode #3 is also anti-symmetric and, as it will be shown later, is related only to the turret motion. Higher-order modes #4 and #6 can be viewed as perturbations of the separation line, as well as pressure signature of vortical structures, present in the separation region.

Fig. 10 First six dominant JPOD spatial modes for $M = 0.5$
The relative individual and cumulative energies of the first 100 JPOD modes, computed using Eq. 8, for the oscillating and the stationary cases at $M = 0.5$ are presented in Figure 11. The first JPOD mode contains about 40% of the total “energy” for both the stationary and the oscillating cases, and the first ten modes contribute more than 80% of the unsteady pressure “energy” on the turret surface. The energy of the main anti-symmetric mode # 1 is reduced by almost 25% in the oscillating case, indicating the oscillating turret primarily suppresses the global shifting wake mode. The energies of other modes, with the exception of the mode # 3, are not significantly affected by the oscillating turret. The mode # 3 is virtually absent in the stationary case and present only for the oscillating case, further relating this mode to the turret motion.

Fig. 11 JPOD cumulative and individual energies, for the oscillating and stationary cases at $M = 0.5$, normalized by the total energy for the stationary case

To study a connection between the turret motion and the dynamics of JPOD modes for the oscillating case, reflected in the temporal coefficients, the standard normalized correlations between the turret spanwise location, $z_{turret}(t)$, and the modes’ coefficients, $a_k(t)$ were calculated. For the mode # 1, the correlation was found to approximately 0.5, further indicating a dynamic connection between the turret motion and the resulted wake shifting mode. For mode # 2, the correlation coefficient is much less, about 0.18, so the dynamics of the global breathing mode is not significantly affected by the oscillating turret. For all other modes, except mode # 3, the correlations are close to zero.

The correlation for the spanwise anti-symmetric mode # 3 was found to be very high, about 0.9. Also, as mentioned before, this mode is not present in the stationary case, as shown in Figure 11. Thus, this mode is related to the turret motion only. There are several possible mechanisms which are associated with the pressure variation on the surface of the moving object. When the turret moves in the spanwise direction, the flow will be compressed on one side of it and expanded on the opposite side, so it is possible to create the spanwise anti-symmetric pressure distribution observed in mode # 3. Another mechanism is actually an artifact of the moving turret. When the turret moves, the distances between points on the surface and the cameras vary. The closer the
point to the camera, the brighter is the fluorescence intensity of PSP and visa versa. As the pressure is reconstructed from the intensity, this apparent pressure variation is only a function of the turret motion and will also have spanwise anti-symmetric distribution.

The spectra of the temporal coefficients, normalized by the square of the dynamic pressure, for the stationary turret case at $M = 0.4$, $0.5$ and $0.6$ are presented in Figure 12. The frequency was normalized by the turret diameter and the freestream velocity, $St_D = fD / U_\infty$. All modal spectra collapse over the range of the measured Mach numbers, demonstrating a self-similar behavior of the pressure field on the turret surface. The dominant global wake shifting mode, represented by JPOD mode #1 has a distinct peak in the spectra at approximately of $St_D = 0.17$, independent of the Mach number. Another JPOD mode # 6, also related to the global shifting mode, has a similar peak of this frequency in its spectra, although less energetic. Spectral analysis of the pressure field in the wake of the stationary turret, presented in Roeder (2020), revealed the presence of the same spectral peak in wake. Also, a similar spectral peak was observed in other studies by Gordeyev et al. (2014) and De Lucca et al. (2018a) and the peak was also found to be approximately constant for a range of Mach numbers between 0.35 and 0.66. The spanwise-symmetric modes # 2, # 4, and #5, related to the global breathing mode, not exhibit any significant peaks in their spectra, and have most of their energy contained in low, $St_D < 0.1$, frequencies.

Fig. 12 JPOD temporal coefficient normalized spectra corresponding to the first six dominant JPOD modes for the stationary turret case at $M = 0.4$, $0.5$ and $0.6$

A comparison between normalized spectra of the temporal coefficients for the stationary and oscillating cases at $M = 0.5$ is presented in Figure 13. The spectra of the spanwise symmetric modes # 2, # 4 and #5 are largely unchanged between the stationary and the oscillating cases, with only a modest suppression at low frequencies, $St_D < 0.1$. In contrast, the peaks in the spectra of the global wake shifting modes, represented by JPOD modes # 1 and # 6, are significantly suppressed for the
oscillating case, compared to the stationary case. Also, the spectra of these anti-symmetric modes for the oscillating case have an additional peak exactly at the location of the main oscillating turret frequency of 87 Hz, indicated in Figure 13 as a vertical dashed line. Very similar trends were observed for $M = 0.4$ and 0.6 and therefore are not shown. This behavior provides a strong evidence that the resulting mechanical motion of the turret driven by the unsteady turbulent flow suppresses the natural motion of the global shifting wake mode and dynamically locks the mode to the turret oscillation instead, while keeping the dynamics of the global breathing motion mostly unchanged. Consistent with the earlier discussion, mode #3 for the oscillating case has a strong single peak at the main oscillating turret frequency, as this mode is related only to the turret motion.

![Fig. 13 JPOD temporal coefficient normalized spectra corresponding to the first six dominant JPOD modes, for the stationary and the oscillating turret cases at $M = 0.5$. The main oscillating frequency of 87 Hz is indicated as a vertical dashed line](image)

**F. Analysis of Unsteady Forces Acting on Turret**

The computed normalized coefficients of unsteady force components are presented in Figure 14 for the oscillating and stationary cases at $M = 0.4$, 0.5, and 0.6. The highest magnitude of the fluctuating force on the turret occurs in the spanwise $z$-direction for both cases. The coefficient of force is over 50% higher in the $z$-direction than in the wall-normal $y$-direction and between three and four times higher than the streamwise $x$-direction for both turret configurations at all three Mach numbers. The computed force coefficients for the stationary turret agree reasonably well with the values, obtained on a hemisphere-on-cylinder turret at $M = 0.5$ in De Lucca et al. (2013a),
De Lucca (2015) and presented in Figure 14 as filled stars. The differences in actual force coefficients are attributed to different turret geometries.

![Fig 14. The coefficients of different force components for various Mach numbers and both the stationary (open symbols) and the oscillating (filled symbols) turrets. Results for the hemisphere-on-cylinder turret for $M = 0.5$ from De Lucca et al. (2013a) are presented as star symbols for comparison.](image)

Generally, the coefficients either weekly increase or stay approximately constant with the increasing Mach number. The force coefficients for the oscillating case are smaller than the coefficients for the stationary case for z-coefficients and, in lesser degree, for y-coefficients. The drag-related x-coefficients are virtually unchanged by the oscillating turret. One way to explain these trends is to recall that the oscillating turret responds to the unsteady pressure acting on it. Let us say the pressure is higher on one side compared to the opposite site. The oscillating turret will start moving in the spanwise direction and the resulted work produced by the flow will reduce the pressure and, consequently, the z-force, acting on the oscillating turret. Since the turret cannot move in the streamwise direction, the turret motion does not affect the streamwise force.

As shown in Eq. 11, the force spectra are weighted sums of individual JPOD modes’ spectra, where weights or influence coefficients are computed using Eq. 12. The influence coefficients for all three force components for the first ten JPOD modes are presented in Figure 15. As the force is a surface integral of the pressure field, local pressure features, present in JPOD modes might cancel each other during the integration. Thus, while a pressure mode could be important to describe unsteady pressure field, its influence coefficient could be small for a given force component. For instance, it takes only a single mode # 1 to describe the temporal evolution of the z-force, and only mode # 2 to properly represent the y-force. The unsteady x-force is mostly defined by the dynamics of modes # 4 and # 7. While mode # 3 contributes about 20% to the total “energy” of the pressure field for the oscillating case, it has zero influence on the forces acting on the turret.
Fig. 15 Influence coefficients, computed using 10 JPOD modes for different force components. Open symbols correspond to the stationary case and filled symbols represent the oscillating case.

Spectra of the $x$, $y$, and $z$-components of the unsteady forces on the oscillating and stationary cases are presented as a function of the normalized frequency at $M = 0.5$ in Figure 16. For the stationary case, see Figure 16(a), the spectrum of $z$-component has a sharp peak at the same normalized frequency of $St_D = 0.17$, as observed in the spectra of the main mode # 1 from JPOD analysis. Similarly, the spectrum of $y$-component resembles the spectrum of the pressure mode # 2. The spectra of $x$-component do not have any distinct peaks. In contrast, the dominant peak, present in the $z$-component of the oscillating turret, see Figure 16(b), coincides with 87 Hz resonant frequency. There is a second peak in $z$-spectra located at the normalized frequency of $St_D = 0.17$, and it appears to be significantly dampened with respect to the stationary case. The spectra for other force components are not affected by the oscillating turret.
A comparison of the spectra of z-component of the force for the stationary and the oscillating cases at different Mach numbers are presented in Figure 17. For the stationary case, Figure 17(a), the location of the main spectral peak was found to be independent of the Mach number. For the oscillating turret case, Figure 17(b), this peak is suppressed at all Mach numbers, while the resonant-related peak at 87 Hz appears.

To investigate the intermittent nature of the spanwise z-force, a Morlet wavelet analysis was applied to the time series of the spanwise force coefficient for the stationary and the oscillating cases at $M = 0.5$. A map of wavelet amplitude for the stationary case as a function of the normalized frequency and normalized time is shown in Figure 18(a). The force amplitude is clearly intermittent, with a typical burst durations of 20-40 $D/U_\infty$; similar time durations of the wake coherence around the hemisphere were observed in McCarthy et al. (2019). Most of the force bursts occur at the range of $St_D = 0.1 - 0.25$, so some of the bursts would coincide with the natural resonant frequency of 87 Hz, indicated in Figure 18(a) as a horizontal dashed white line. When it happens, the oscillating turret will responds to these force bursts by the increased turret motion. When the force burst either shifts away from the resonant 87 Hz or decreases in amplitude, the turret motion will decay. For this reason, the observed bursts in the force are very similar to the bursts in the turret motion observed in Figure 7.
As it was demonstrated in Figure 17, due to aero-elastic interaction, the turret motion will modify the spectrum of the spanwise force by suppressing the natural peak at $St_0 \sim 0.17$ and enhancing the spectral energy at the resonant frequency of 87 Hz. It can be seen in the wavelet map for the oscillating case in Figure 18(b), where most of the force bursts now occur near 87 Hz. The durations of the bursts appear to be unchanged, while the frequency and the amplitudes of the burst occurrences are reduced, compared to the stationary case. These less frequent and less energetic bursts result in the overall reduced values of the spanwise force coefficients, reported in Figure 14.

Using the computed unsteady force in the $z$-direction, $F_z$, and the resulting turret position, $z_{\text{turret}}$, it is possible to extract the spectral transfer function between these quantities as a ratio of Fourier transforms, $G(f) = FT\{F_z\} / FT\{z_{\text{turret}}\}$, and to compare it with the theoretical transfer function, Eq. 4. The amplitude and the phase of the experimentally calculated transfer function, normalized by the steady value of the transfer function, $G_{\text{steady}}$, are presented in Figure 19, along with the theoretical model from Eq. 4. These results agree quite well with the linear second-order model up to 150 Hz, including the location and the amplitude of the resonant peak, as well as the phase jump, verifying that the turret motion can be approximately modelled as a linear second-order underdamped system. Above 150 Hz, the experimental transfer function deviates from the linear
model. One reason for it is possible non-linear effects of these high frequencies; similar non-linear effects were observed in the spectra of the turret position in Figure 6. Another reason is that the spectrum of the turret position, presented in Figure 6 is essentially flat for frequency above 100 Hz due to the noise related to accuracy of the optical technique to extract the turret position.

![Fig 19](image)

**Fig 19** a Amplitude and b phase of experimentally-measured harmonic normalized transfer function, $G(f)/G_{\text{steady}}$, between the unsteady z-force and the resulted turret z-displacement at $M = 0.5$. The theoretical linear model from Eq. (4) is also shown for comparison

**VI. Conclusions**

In the presented studies, the oscillating turret assembly has been built and experimentally tested in a subsonic tunnel over a range of Mach numbers between 0.3 and 0.6. The hemispherical turret was mounted on a thin aluminum beam, allowing the turret to freely move in the spanwise direction only. Only the hemispherical turret was inserted into the flow. The oscillating turret was designed using Euler-Bernoulli beam theory to oscillate at a predominantly single resonant frequency. The parameters of the mounting beam were chosen such that the main oscillating frequency coincides with the wake-dominant frequency of $St_D = 0.15$, so the resonant aero-elastic interaction would result in the forced turret oscillations.

An extensive study using fast-response pressure-sensitive paint was conducted in order to investigate the globally fluctuating pressure fields on the turret surface for both oscillating and stationary turrets. The turret motion in the oscillating case was observed to be at a single frequency, with the value close to the one predicted by the linear theory. The turret oscillations exhibited intermittent behavior, with bursts lasting approximately few hundred characteristic times. JPOD modal analysis technique was applied in order to decompose the pressure fields into the spatial modes and the corresponding temporal coefficients for the oscillating and the stationary cases. It was found that the spanwise anti-symmetric modes are related to the global shifting wake mode and their spectra have a sharp peak at $St_D \sim 0.17$. These modes were found to be substantially suppressed for the oscillating case at all tested Mach numbers, while the spanwise-symmetric modes, corresponding to the global breathing mode, were mostly unaffected by the turret motion. Due to aero-elastic interaction, the wake global shifting mode was dynamically locked with the mechanical motion of the oscillating turret. An analysis of unsteady forces acting of the turret itself revealed that most of the unsteady forcing for both turret configurations occurs in the spanwise
direction, and unsteady force spectra showed that the dominant frequency corresponded to the resonant oscillating frequency for the oscillating turret and the dominant wake response frequency for the stationary turret. The spanwise unsteady force was also demonstrated to be intermittent in time, explaining the bursts of the periodic turret motion for the oscillating case. The spectral transfer function, extracted from the simultaneous force and the resultant turret motion, agree reasonably well with the prediction from a linear second-order underdamped model.

The presented work can be useful to interpret the changes in the unsteady forces acting on the turret due to aero-elastic effects and can be used to validate numerical codes, capable of solving coupled aero-elastic response and aero-dynamic performance of the hemispherical turrets.

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