Homework set # 3

Due on 2/6

0. The following problems from Artin “Algebra” edition 2: 15.3.9, 15.3.10

1. (a) Let $K$ be an extension of $F$ of degree $n$. For any $\alpha \in K$ prove that $\alpha$ acting by left multiplication on $K$ is an $F$-linear transformation of $K$.

(b) Show that if the matrix of the linear transformation “multiplication by $\alpha$” is the matrix $A$ then $\alpha$ is a root of the characteristic polynomial for $A$.

(c) Use the procedure in part (b) to find the monic polynomials of degree 3 satisfied by $\sqrt[3]{2}$ and by $1 + \sqrt[3]{2} + \sqrt[3]{4}$.

(d) The polynomials found in part (c) are in fact the irreducible polynomials for these elements (Why?). Give an example (of a $K$, $F$, and an $\alpha$) where the procedure in part (b) does not produce the irreducible polynomial of $\alpha$ over $F$.

2. Use the fact that $\alpha = 2\cos(2\pi/7)$ satisfies the equation $x^3 + x^2 - 2x - 1 = 0$ to prove that the regular 7-gon is not constructible by straightedge and compass.

3. Use the fact that $\alpha = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1 = 0$ to conclude that the regular 5-gon is constructible by straightedge and compass.