Multiple Choice

1.(6 pts) Find dz/dt when $t = 0$, where $z = x^2 + y^2 + 2xy$, $x = \ln(t+1)$ and $y = e^{3t}$.

(a) 8 (b) 2 (c) 1

(d) 6 (e) 5

2.(6 pts) Calculate the directional derivative of $f(x, y, z) = x^2 + y^2 + z^2$ at the point $(2, 4, 2)$ in the direction of the vector $\langle 1, 2, 1 \rangle$.

(a) $\dfrac{24}{\sqrt{6}}$ (b) $\sqrt{6}$

(c) $-\dfrac{1}{12}\langle 1, 4, 1 \rangle$ (d) -9.79

(e) $\left\langle \dfrac{1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right\rangle$

3.(6 pts) What is the normal line to $z^2 = 9x^2 - 4y^2$ at $(2, 3, 0)$?

(a) $\langle 2, 3, 0\rangle + t\,\langle 36, -24, 0\rangle$

(b) $\langle 2, 3, 0\rangle + t\,\langle 18x, -8y, -2z\rangle$

(c) $\langle 2, 3\rangle + t\langle 36, -24\rangle$

(d) $36x - 24y = 0$

(e) $\langle 2, 3, 0\rangle + t\,\langle 24, 36, -12\rangle$

4.(6 pts) Find and classify the critical points of $f(x, y) = x^2 + 6xy - 4y^2$.

(a) $(0, 0)$, saddle point.

(b) $(0, 0)$, local maximum.

(c) $(0, 0)$, local minimum.

(d) $(1, 1)$, saddle point.

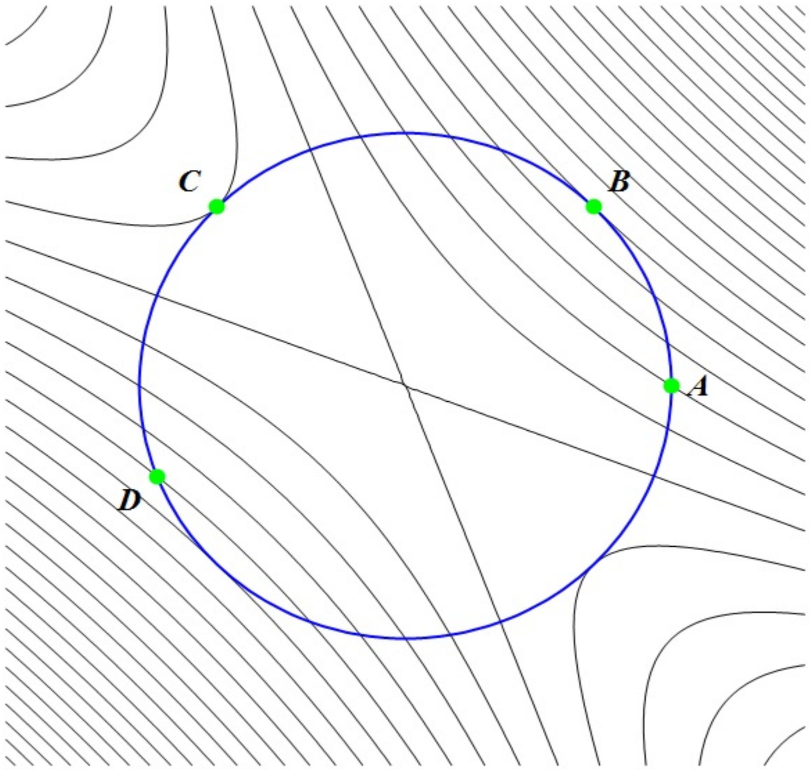
(e) $(1, 1)$, local maximum.

5.(6 pts) Suppose $f(x, y) = x^2 y$ with domain $D = \{(x, y) | x \geq 0, y \geq 0, x^2 + y^2 \leq 3\}$. What is the absolute maximum value of $f(x, y)$?

(a) 2 (b) 1 (c) 3

(d) 4 (e) 5

6.(6 pts) Consider the following contour plot for a function $f(x, y)$:



The circle is a level curve $g(x, y) = k$. Which of the following must ALWAYS be true?

(a) Subject to $g(x, y) = k$, $f(x, y)$ has a possible extremum at C.

(b) Subject to $g(x, y) = k$, $f(x, y)$ has a possible maximum at A.

(c) Subject to $g(x, y) = k$, $f(x, y)$ has a possible minimum at D.

(d) Subject to $g(x, y) = k$, $f(x, y)$ has an absolute maximum at B.

(e) $f(x, y)$ has a possible absolute maximum or absolute minimum at C.

7.(6 pts) Evaluate the following double integral

$$\iint_R (5 - x)$$

for $R = \{(x, y) \mid 0 \leq x \leq 4, 0 \leq y \leq 3\}$.

(a) 36

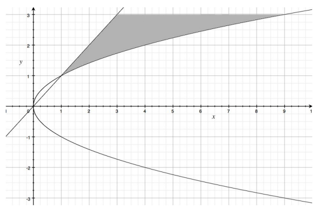
(b) 24

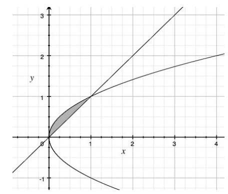
(c) 60

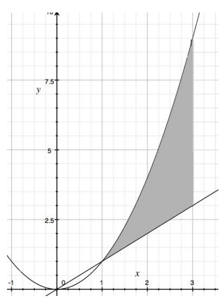
(d) 12

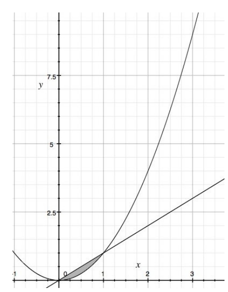
(e) 52

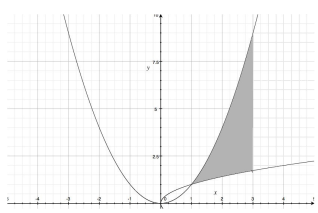
8.(6 pts) Consider the double integral of a function f over a region R, $\iint_R f\,dA$. Suppose $\iint_R f = \int_1^3 \int_y^{y^2} f(x,y)\,dx\,dy$. Which gray region below is R?

(a) 

(b) 

(c) 

(d) 

(e) 

Partial Credit

You must show your work on the partial credit problems to receive credit!

9.(12 pts.) (a) Find an equation for the tangent line (in vector or parametric form) at the point $(2, 2, 1)$ to the curve of intersection of the two surfaces $g(x, y, z) = 2x^2+2y^2+z^2 = 17$ and $h(x, y, z) = x^2 + y^2 - 3z^2 = 5$. (8 pts)

(b) Suppose $f(x, y, z)$ is a function with $\nabla f = \langle 0, 1, 0\rangle$ at the point $(2, 2, 1)$. Starting at $(2, 2, 1)$, which direction should one travel along the curve of intersection from part (a) in order to increase f? (2 pts)

Note: You can specify a direction along the curve by saying whether the variable in your equation from (a) would increase or decrease, or by choosing a vector tangent to the curve.

10.(12 pts.) Find the absolute maximum and minimum of $f(x, y, z) = 2x+y$ with respect to the constraints $g(x, y, z) = 2x^2 + z^2 = 4$ and $h(x, y, z) = 2x + y + 3z = 6$.

11.(12 pts.) Find and classify all critical points of $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

12.(12 pts.) A cylinder containing an incompressible fluid is being squeezed from both ends. If the length of the cylinder is changing at a rate of -3m/s, calculate the rate at which the radius is changing when the radius is 2m and the length is 1m. (Note: An incompressible fluid is a fluid whose volume does not change.)

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13.(12 pts.) Evaluate $\iint_R 4xy\, dA$ where R is the region bounded above by $y = \sqrt{x}$ and below by $y = x^3$.