MATH 20550: Calculus III Name:____________________________

Exam III *November 15, 2011* Instructor and Section:____________________________

As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Please sign ____

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 10 multiple choice questions worth 6 points each and 3 partial credits problems worth 14 points each. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Score

DO NOT WRITE IN THIS	COLUMN
Multichoice	
10	
11	
12	
Total	

1.	a	b	c	d	e	6.	a	b	c	d	e
2.	a	b	c	d	e	7.	a	b	c	d	e
3.	a	b	c	d	e	8.	a	b	c	d	e
4.	a	b	c	d	e	9.	a	b	c	d	e
5.	a	b	c	d	e	10.	a	b	c	d	e

1. Which of the following gives

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} x^2 + y^2 \; dz \, dx \, dy$$

in cylindrical coordinates?

(a) $\displaystyle\int_0^1 \int_0^{\frac{\pi}{2}} \int_{r^2}^{r} r^3 \; dz \, d\theta \, dr$ (b) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{r^2}^{r} \int_0^1 r^3 \; dr \, dz \, d\theta$ (c) $\displaystyle\int_0^1 \int_0^{2\pi} \int_0^{r} r^3 \; dz \, d\theta \, dr$

(d) $\displaystyle\int_0^1 \int_0^{\frac{\pi}{2}} \int_{r^2}^{r} r \; dz \, d\theta \, dr$ (e) $\displaystyle\int_0^1 \int_0^{2\pi} \int_{r^2}^{r} r^4 \; dz \, d\theta \, dr$

2. Let $\mathbf{F}(x, y, z) = (e^{xy} + xye^{xy} + z^2)\,\mathbf{i} + (e^y + x^2e^{xy})\,\mathbf{j} + 2xz\,\mathbf{k}$. Which of the following is a potential for the vector field $\mathbf{F}$.

(a) $f(x, y, z) = e^y + ye^{xy} + yz^2$ (b) $f(x, y, z) = e^y + xe^{xy} + xz^2$ (c) $f(x, y, z) = e^{xy} + yz^2$

(d) $f(x, y, z) = e^{xy} + xye^{xy} + z^2$ (e) $f(x, y, z) = e^y + xe^{xy}$

3. Find the center of mass of a lamina occupying the region $D = \{(x, y) : -y \leq x \leq y, 0 \leq y \leq 1\}$ with density function $\rho(x, y) = 1$. (Hint: the mass of the lamina is 1.)

(a) $(0, \frac{3}{4})$ (b) $\left(0, \frac{2}{3}\right)$ (c) $\left(\frac{2}{3}, 0\right)$ (d) $(0, 1)$ (e) $\left(0, \frac{1}{2}\right)$

4. Evaluate $\int_C yz\, ds$ where C is the line segment from $(1, 0, 0)$ to $(1, 3, 4)$.

(a) $4\sqrt{26}$ (b) 20 (c) 5 (d) 4 (e) 6

5. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \sin y\ \mathbf{i} + x \cos y\ \mathbf{j} - \sin z\ \mathbf{k}$ and C is any smooth curve from $(0, 0, 0)$ to $\left(3, \frac{\pi}{2}, \pi\right)$.

(a) $\frac{3}{\sqrt{2}}$

(b) 0

(c) 2

(d) Cannot be calculated with the information given.

(e) 1

6. Which of the following is equal to

$$\int_C \sqrt{1+x^3}\,dx + 2xy\,dy$$

where C is the triangle with vertices $(0,0)$, $(1,0)$ and $(1,3)$?

(a) $\displaystyle\int_0^1 \int_0^{3x} 2y\,dy\,dx$

(b) $\displaystyle\int_0^1 \int_{3x}^{3} 2y\,dy\,dx$

(c) $\displaystyle\int_0^1 \int_0^{3} 2x\,dy\,dx$

(d) $\displaystyle\int_0^3 \int_0^{\frac{y}{3}} 2x\,dx\,dy$

(e) $\displaystyle\int_0^1 \int_0^{3x} (2xy - \sqrt{1+x^3})\,dy\,dx$

7. Which of the following integrals is equal to the work done by the force field $\mathbf{F}(x, y, z) = z\,\mathbf{i} + y\,\mathbf{j} - x\,\mathbf{k}$ in moving a particle from $(3, 0, 0)$ to $\left(0, \frac{\pi}{2}, 3\right)$ along the helix $x = 3\cos t,\ y = t,\ z = 3\sin t$.

(a) $\displaystyle\int_0^{\frac{\pi}{2}} t - 9\; dt$

(b) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{10}(t - 9)\; dt$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} 3\sin t + t - 3\cos t\; dt$

(d) $\displaystyle\int_0^{3} t^2\; dt$

(e) $\displaystyle\int_0^{\pi} \sqrt{10}t^2\; dt$

8. Which of the following is equal to the volume of the solid enclosed by $z = 0$, $z = x + y$, $y = x^2$ and $x = y^2$ in the first octant? (The first octant is where $x \geq 0, y \geq 0$, and $z \geq 0$.)

(a) $\displaystyle\int_0^1 \int_{\sqrt{x}}^{x^2} \int_0^{x+y} dy\, dx$

(b) $\displaystyle\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x+y} dz\, dy\, dx$

(c) $\displaystyle\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x+y} x + y\; dz\, dy\, dx$

(d) $\displaystyle\int_1^2 \int_{\sqrt{x}}^{x^2} \int_0^{x+y} z\; dz\, dy\, dx$

(e) $\displaystyle\int_0^1 \int_0^{x^2} z\; dy\, dx$

9. Evaluate $\int_C (x-y)\,dx + (x+y)\,dy$ where C is the curve with initial point $(2,0)$ and terminal point $(-2,0)$ given by the upper-half circle of radius 2.

(a) -4π (b) 2π (c) 4π (d) 0 (e) 4

10. Evaluate $\int\int_D e^{x^2+y^2}\,dy\,dx$ where D is the region between the circles of radius 1 and 2 in the 1st quadrant ($x \geq 0$ and $y \geq 0$).

(a) $\pi(e^4-1)$ (b) $\frac{\pi}{4}\left(e^4-e\right)$ (c) 4π (d) $2\pi e$ (e) $\frac{3}{4}\pi$

11. Find the z component of the center of mass of the solid above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 2$ with density given by the constant function $\rho(x, y, z) = k$. The mass of this solid is equal to $k\frac{4\pi}{3}(\sqrt{2} - 1)$. You should have to calculate one integral to solve this problem. You must show each step in the integration process to receive full credit.

12. Use the transformation $u = x - y$, $v = x + y$ to evaluate

$$\iint_R \frac{x - y}{x + y} \, dA$$

where R is the square with vertices $(0, 2)$, $(1, 1)$, $(2, 2)$ and $(1, 3)$.

13. Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = x^2y\,\mathbf{i} - xy^2\,\mathbf{j}$ where C is the lower-half of the circle of radius 1 from $(1, 0)$ to $(-1, 0)$ and the line segment from $(-1, 0)$ to $(1, 0)$. Indicate clearly where Green's Theorem is applied in the calculation. Be sure to check the orientation of C.