MATH 20550: Calculus III Name:______________________________

Final *December 12, 2011* Instructor and Section:______________________________

As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Please sign ____

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 20 multiple choice questions worth 7.5 points each for a total of 150 points.

You may use a calculator if you wish.

1.	a	b	c	d	e	11.	a	b	c	d	e
2.	a	b	c	d	e	12.	a	b	c	d	e
3.	a	b	c	d	e	13.	a	b	c	d	e
4.	a	b	c	d	e	14.	a	b	c	d	e
5.	a	b	c	d	e	15.	a	b	c	d	e
6.	a	b	c	d	e	16.	a	b	c	d	e
7.	a	b	c	d	e	17.	a	b	c	d	e
8.	a	b	c	d	e	18.	a	b	c	d	e
9.	a	b	c	d	e	19.	a	b	c	d	e
10.	a	b	c	d	e	20.	a	b	c	d	e

1. Use the change of variables, $x = \frac{1}{3}u + \frac{2}{3}v$ and $y = \frac{2}{3}u + \frac{1}{3}v$, to compute the integral

$$\iint_R 12(2x - y)(x - 2y)dA,$$

where R is the parallelogram region with vertices $(0, 0)$, $(2, 1)$, $(1, 2)$, $(3, 3)$.

(a) -18 (b) -81 (c) 81 (d) 18 (e) -112

2. Find the normal component of acceleration for the curve $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$.

(a) $\sqrt{4 - t^2}$ (b) $\dfrac{\langle 1, 2, 2t\rangle}{\sqrt{5 + 4t^2}}$ (c) $\dfrac{2\sqrt{5}}{\sqrt{5 + 4t^2}}$ (d) $\dfrac{4t}{\sqrt{5 + 4t^2}}$ (e) $\langle 4, -2, 0\rangle$

3. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle x + y, y - x, x - z \rangle$ and S is the surface of the sphere of radius 3 centered at the origin with the outward normal. (Hint: if you use the correct theorem this problem is very simple.)

(a) 48 (b) 0 (c) 36π (d) 12π (e) 118π

4. Evaluate $\int_C x^2 + y^2 - z^2 \, ds$ where C is the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ with initial point $(1, 0, 0)$ and terminal point $(-1, 0, \pi)$

(a) $\frac{\pi^3}{2}$ (b) $\sqrt{2}\left(\pi - \frac{\pi^3}{3}\right)$ (c) $\pi - \frac{\pi^3}{3}$ (d) $\sqrt{2}\pi$ (e) $\frac{1}{6}(3\pi - \pi^3)$

5. Find a vector along which the function

$$f(x, y) = (x^2 + x)y$$

is neither increasing nor decreasing from the point $(1, 1)$.

(a) $\langle 3, -2 \rangle$ (b) $\langle 2, -3 \rangle$ (c) $\langle 2, 3 \rangle$ (d) $\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \rangle$ (e) $\langle \frac{3}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \rangle$

6. Find $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$ where $\mathbf{F} = \langle yz - y, 2xz + x, e^{xyz} \rangle$ and S is the surface of $x^2 + y^2 + \frac{9}{4}z^2 = 4$ for $z \geq 0$ with upward normal.

(a) 0 (b) 8π (c) 2π (d) 4π (e) -4π

7. Suppose a dragon is flying along the helix $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$ while subject to the force field

$$\mathbf{F}(x, y, z) = 2xze^{x^2+y^2}\mathbf{i} + 2yze^{x^2+y^2}\mathbf{j} + e^{x^2+y^2}\mathbf{k}.$$

What is the work done by the force field on the dragon from $t = 0$ to $t = 4\pi$?

(a) 0 (b) $\pi(4e - 1)$ (c) 4π (d) $e - 1$ (e) $4\pi e$

8. Find $\displaystyle\iint_S z\, dS$ where S is the surface $x^2 + y^2 + z^2 = 1$ above the xy-plane.

(a) -1 (b) 0 (c) π (d) $\frac{1}{2}$ (e) 2π

9. Find the tangent plane to the surface $\mathbf{r}(u, v) = \langle uv, u + 2v, 2u - v \rangle$ when $u = v = 1$.

(a) $x + 2y - z = 6$ (b) $\dfrac{x-1}{-5} = \dfrac{y-3}{3} = \dfrac{z-1}{1}$ (c) $-5x + 3y + z = 5$

(d) $2x + 3y = 4$ (e) $-5x + 3y + z = 1$

10. Find the shortest distance of the surface $xyz = 1$ to the origin.

(a) $(\frac{1}{2}, 1, 2)$ (b) $2\sqrt{2}$ (c) $\dfrac{\sqrt{21}}{2}$ (d) $\sqrt{3}$ (e) 3

11. Which of the following is an equation for the osculating plane of the curve

$$\mathbf{r}(t) = \langle \cos t, 1 + t^2, t \rangle$$

at $t = 0$? (That is, at the point $(1, 1, 0)$.)

(a) $x - y + 1 = 0$

(b) $2x + y - 3 = 0$

(c) $x - 2y + 1 = 0$

(d) $2x + y + z - 3 = 0$

(e) $x + y = 0$

12. Which of the following triple integrals represents the volume of the region inside both the cylinder $x^2 + y^2 = 4$ and the sphere $x^2 + y^2 + z^2 = 9$?

(a) $\int_{-2}^{2} \int_{-2}^{2} \int_{-3}^{3} 2\sqrt{9 - x^2 - y^2}\, dz\, dy\, dx$

(b) $\int_{0}^{2\pi} \int_{0}^{\pi} \int_{0}^{3} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

(c) $\int_{0}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} dz\, dy\, dx$

(d) $\int_{0}^{2\pi} \int_{0}^{3} \int_{r^2-4}^{4-r^2} r\, dz\, dr\, d\theta$

(e) $\int_{0}^{2\pi} \int_{0}^{2} \int_{-\sqrt{9-r^2}}^{\sqrt{9-r^2}} r\, dz\, dr\, d\theta$

13. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle x, 3y^2, z \rangle$ and S is the surface of the cylinder $4x^2 + z^2 = 4, 0 \leq y \leq 1$, with outward normal.

(a) 0 (b) -2π (c) 4π (d) 2π (e) 8π

14. Find $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle z, x, 2y \rangle$ and C is the curve given by the intersection of the plane $x + 2y + z = 2$ with the coordinate planes in a counterclockwise direction when viewed from above.

(a) -4 (b) 6 (c) 5 (d) 4 (e) 0

15. Find the absolute maximum and minimum of the function

$$f(x, y) = (x - 1)^2 + y^2$$

on the region

$$D = \left\{(x, y) : \frac{x^2}{4} + y^2 \leq 1\right\}$$

(a) Absolute minimum 0; no absolute maximum.

(b) Absolute minimum $\frac{2}{3}$; absolute maximum 9.

(c) Absolute minimum 0; absolute maximum $\frac{2}{3}$.

(d) Absolute minimum 0; absolute maximum 9.

(e) Absolute minimum $\frac{2}{3}$; absolute maximum 1.

16. Find the rate of change of the function

$$f(x,y,z) = xy^2e^{x-z}$$

at the point $(2,1,2)$ in the direction of the vector $\langle 0,1,1\rangle$.

(a) $\sqrt{2}$ (b) 4 (c) $3\sqrt{2}$ (d) 2 (e) 5

17. Find the projection $\text{proj}_{\mathbf{a}}\,\mathbf{b}$ of $\mathbf{b} = \langle 4,2,5\rangle$ onto $\mathbf{a} = \langle 1,-1,2\rangle$.

(a) $\langle \frac{16}{15}, \frac{8}{15}, \frac{4}{3}\rangle$ (b) $\langle \frac{1}{2}, -\frac{1}{2}, 1\rangle$ (c) $\langle 2,-2,4\rangle$ (d) $\langle \frac{4}{15}, -\frac{4}{15}, \frac{8}{15}\rangle$ (e) 2

18. The cylinder $x^2 + y^2 = 4$ intersects the plane $x + y + z = 1$ in an ellipse. Find an equation of the tangent line to the ellipse at $(2, 0, -1)$.

(a) $\mathbf{r}(t) = \langle 0, -t, t\rangle$ (b) $\mathbf{r}(t) = \langle 2 + 4t, 0, -1 + t\rangle$ (c) $x = 2,\ \dfrac{y}{4} = z$

(d) $\dfrac{x - 2}{4} = z + 1,\ y = 0$ (e) $\mathbf{r}(t) = \langle 2, -4t, -1 + 4t\rangle$

19. Find the surface area of the part of the plane $3x + 2y + z = 6$ in the first octant (so $x \geq 0, y \geq 0, z \geq 0$).

(a) $3\sqrt{13}$ (b) 18 (c) 12 (d) $3\sqrt{14}$ (e) $3\sqrt{15}$

20. Consider the function $f(x, y) = x\sin(y^2)$. Which of the following is true of the point $(0, 0)$?

(a) $(0, 0)$ is a critical point of f, but the Second Derivative Test is inconclusive.

(b) $(0, 0)$ is not a critical point of f.

(c) $(0, 0)$ is a local maximum of f.

(d) $(0, 0)$ is a saddle point of f.

(e) $(0, 0)$ is a local minimum of f.