MATH 20550: Calculus III Name: Solutions

Final *December 12, 2011* Instructor and Section:

As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Please sign

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 20 multiple choice questions worth 7.5 points each for a total of 150 points.

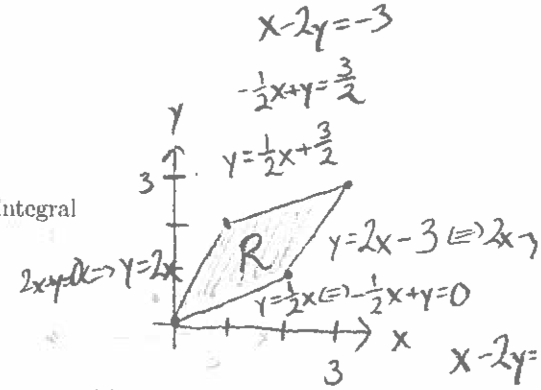
You may use a calculator if you wish.

1. a ~~b~~ c d e
2. a b ~~c~~ d e
3. a b ~~c~~ d e
4. a ~~b~~ c d e
5. a ~~b~~ c d e
6. a ~~b~~ c d e
7. a b c d ~~e~~
8. a b ~~c~~ d e
9. a b ~~c~~ d e
10. a b c ~~d~~ e
11. a ~~b~~ c d e
12. a b c d ~~e~~
13. a b ~~c~~ d e
14. a b ~~c~~ d e
15. a b c ~~d~~ e
16. ~~a~~ b c d e
17. a b ~~c~~ d e
18. a b c d ~~e~~
19. a b c ~~d~~ e
20. ~~a~~ b c d e

1. Use the change of variables, $x = \frac{1}{3}u + \frac{2}{3}v$ and $y = \frac{2}{3}u + \frac{1}{3}v$, to compute the integral

$$\iint_R 12(2x-y)(x-2y)\,dA,$$

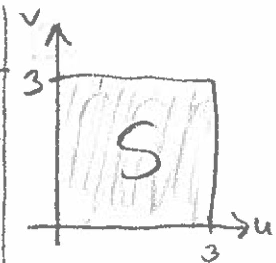
where R is the parallelogram region with vertices $(0,0)$, $(2,1)$, $(1,2)$, $(3,3)$.



(a) -18 (b) -81 (c) 81 (d) 18 (e) -112

$2x-y = \frac{2}{3}u + \frac{4}{3}v - \frac{2}{3}u - \frac{1}{3}v = v$ | $x-2y = \frac{1}{3}u + \cancel{\frac{2}{3}v} - \frac{4}{3}u - \cancel{\frac{2}{3}v} = -u$

R	S
$2x-y=0$	$v=0$
$2x-y=3$	$v=3$
$x-2y=-3$	$u=3$
$x-2y=0$	$u=0$



$12(2x-y)(x-2y) = 12(v)(-u) = -12uv$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{vmatrix} = \frac{1}{9} - \frac{4}{9} = \frac{-3}{9} = -\frac{1}{3}$

$\iint_R 12(2x-y)(x-2y)\,dA = \iint_S -12uv\left|-\frac{1}{3}\right| du\,dv = \int_0^3\int_0^3 -4uv\,du\,dv = \int_0^3 -2u^2v\Big|_0^3 dv$

$= \int_0^3 -18v\,dv = -9v^2\Big|_0^3 = -81$

2. Find the normal component of acceleration for the curve $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$.

(a) $\sqrt{4-t^2}$ (b) $\frac{\langle 1, 2, 2t\rangle}{\sqrt{5+4t^2}}$ (c) $\frac{2\sqrt{5}}{\sqrt{5+4t^2}}$ (d) $\frac{4t}{\sqrt{5+4t^2}}$ (e) $\langle 4, -2, 0\rangle$

$a_N = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|}$

$\vec{r}'(t) = \langle 1, 2, 2t\rangle$

$\vec{r}''(t) = \langle 0, 0, 2\rangle$

$\vec{r}'(t) \times \vec{r}''(t) = \langle 4, -2, 0\rangle$

$= \frac{\sqrt{16+4+0}}{\sqrt{1+4+4t^2}} = \frac{\sqrt{20}}{\sqrt{5+4t^2}}$

3. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle x+y, y-x, x-z \rangle$ and S is the surface of the sphere of radius 3 centered at the origin with the outward normal. (Hint: if you use the correct theorem this problem is very simple.)

(a) 48 (b) 0 (c) 36π (d) 12π (e) 118π

$\operatorname{div} \vec{F} = 1+1-1 = 1$

$\iint_S \vec{F} \cdot d\vec{S} = \iiint_B (\operatorname{div} \vec{F})\, dV = \iiint_B dV = \mathrm{vol}(B)$

$= \frac{4}{3}\pi(3)^3 = 36\pi$

4. Evaluate $\int_C (x^2 + y^2 - z^2)\, ds$ where C is the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ with initial point $(1, 0, 0)$ and terminal point $(-1, 0, \pi)$

$t=0$

$t=\pi$

(a) $\frac{\pi^3}{2}$ (b) $\sqrt{2}\left(\pi - \frac{\pi^3}{3}\right)$ (c) $\pi - \frac{\pi^3}{3}$ (d) $\sqrt{2}\pi$ (e) $\frac{1}{6}(3\pi - \pi^3)$

$\vec{r}'(t) = \langle -\sin t, \cos t, 1 \rangle$, $|\vec{r}'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$

$x^2 + y^2 - z^2 = \cos^2 t + \sin^2 t - t^2 = 1 - t^2$

$\int_C (x^2+y^2-z^2)\, ds = \int_0^{\pi} (1-t^2)\sqrt{2}\, dt = \sqrt{2}\left(t - \frac{1}{3}t^3\right)\Big|_0^{\pi} = \sqrt{2}\left(\pi - \frac{1}{3}\pi^3\right)$

5. Find a vector along which the function

$$f(x, y) = (x^2 + x)y$$

is neither increasing nor decreasing from the point $(1, 1)$.

(a) $\langle 3, -2\rangle$ (b) $\langle 2, -3\rangle$ (c) $\langle 2, 3\rangle$ (d) $\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\rangle$ (e) $\langle \frac{3}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\rangle$

Want something $\perp \nabla f(1,1)$

$\nabla f = \langle (2x+1)y, x^2+x\rangle$, $\nabla f(1,1) = \langle 3, 2\rangle$

6. Find $\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$ where $\mathbf{F} = \langle yz - y, 2xz + x, e^{xyz}\rangle$ and S is the surface of $x^2 + y^2 + \frac{9}{4}z^2 = 4$ for $z \geq 0$ with upward normal. $\Rightarrow \partial S$ counterclockise Stokes!

(a) 0 (b) 8π (c) 2π (d) 4π (e) -4π

∂S: $z=0$, $x^2+y^2=4$: $\vec{r}(t) = \langle 2\cos t, 2\sin t, 0\rangle$, $0 \leq t \leq 2\pi$

$\vec{F}(\vec{r}(t)) = \langle 0 - 2\sin t, 0 + 2\cos t, e^0\rangle = \langle -2\sin t, 2\cos t, 1\rangle$

$\vec{r}'(t) = \langle -2\sin t, 2\cos t, 0\rangle$

$$\iint_S (\operatorname{curl} \vec{F}) \cdot d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\, dt$$

$$= \int_0^{2\pi} (4\sin^2 t + 4\cos^2 t + 0)\, dt = \int_0^{2\pi} 4\, dt = 8\pi$$

Also could use $\vec{F} = \nabla f$, $f = ze^{x^2+y^2}$

& FToLI

7. Suppose a dragon is flying along the helix $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$ while subject to the force field

$$\mathbf{F}(x, y, z) = 2xze^{x^2+y^2}\mathbf{i} + 2yze^{x^2+y^2}\mathbf{j} + e^{x^2+y^2}\mathbf{k}.$$

What is the work done by the force field on the dragon from $t = 0$ to $t = 4\pi$?

(a) 0 (b) $\pi(4e - 1)$ (c) 4π (d) $e - 1$ (e) $4\pi e$

$$\vec{F}(\vec{r}(t)) = \langle 2(\cos t)te^{\cos^2 t + \sin^2 t}, 2(\sin t)te^{\cos^2 t + \sin^2 t}, e^{\cos^2 t + \sin^2 t} \rangle$$

$$= \langle 2et\cos t, 2et\sin t, e \rangle$$

$\vec{r}\,'(t) = \langle -\sin t, \cos t, 1 \rangle$, $\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = -2et\sin t\cos t + 2et\sin t\cos t + e = e$

$$\text{Work} = \int_C \vec{F} \cdot d\vec{r} = \int_0^{4\pi} e\,dt = 4e\pi$$

8. Find $\displaystyle\iint_S z\,dS$ where S is the surface $x^2 + y^2 + z^2 = 1$ above the xy-plane.

(a) -1 (b) 0 (c) π (d) $\frac{1}{2}$ (e) 2π

Parametrize : $\vec{r}(\theta, \varphi) = \langle \cos\theta\sin\varphi, \sin\theta\sin\varphi, \cos\varphi \rangle$, $0 \le \theta \le 2\pi$, $0 \le \varphi \le \frac{\pi}{2}$.

$\vec{r}_\theta = \langle -\sin\theta\sin\varphi, \cos\theta\sin\varphi, 0 \rangle$

$\vec{r}_\varphi = \langle \cos\theta\cos\varphi, \sin\theta\cos\varphi, -\sin\varphi \rangle$

$\vec{r}_\theta \times \vec{r}_\varphi = \langle -\cos\theta\sin^2\varphi, -\sin\theta\sin^2\varphi, -\sin\varphi\cos\varphi\sin^2\theta - \sin\varphi\cos\varphi\cos^2\theta \rangle$

$= \langle -\cos\theta\sin^2\varphi, -\sin\theta\sin^2\varphi, -\sin\varphi\cos\varphi \rangle$

$$|\vec{r}_\theta \times \vec{r}_\varphi|^2 = \cos^2\theta\sin^4\varphi + \sin^2\theta\sin^4\varphi + \sin^2\varphi\cos^2\varphi = \sin^4\varphi + \sin^2\varphi\cos^2\varphi = \sin^2\varphi(\sin^2\varphi + \cos^2\varphi) = \sin^2\varphi$$

$$\iint_S z\,dS = \int_0^{\pi/2}\int_0^{2\pi} \cos\varphi\,|\vec{r}_\theta \times \vec{r}_\varphi|\,d\theta\,d\varphi = \int_0^{\pi/2}\int_0^{2\pi} \sin\varphi\cos\varphi\,d\theta\,d\varphi = 2\pi\int_0^{\pi/2} \sin\varphi\cos\varphi\,d\varphi$$

$$= \pi\sin^2\varphi\Big|_0^{\pi/2} = \pi$$

9. Find the tangent plane to the surface $\mathbf{r}(u, v) = \langle uv, u + 2v, 2u - v\rangle$ when $u = v = 1$.

(a) $x + 2y - z = 6$ (b) $\frac{x-1}{-5} = \frac{y-3}{3} = \frac{z-1}{1}$ (c) $-5x + 3y + z = 5$

(d) $2x + 3y = 4$ (e) $-5x + 3y + z = 1$

$\vec{r}_u = \langle v, 1, 2\rangle$ @$(1,1)$ $= \langle 1, 1, 2\rangle$

$\vec{r}_v = \langle u, 2, -1\rangle$ @$(1,1)$ $= \langle 1, 2, -1\rangle$

$\vec{n} = \vec{r}_u(1,1) \times \vec{r}_v(1,1) = \langle -5, 3, 1\rangle$

$\vec{r}(1,1) = \langle 1, 3, 1\rangle$

$0 = \langle -5, 3, 1\rangle \cdot \langle x-1, y-3, z-1\rangle = -5x + 5 + 3y - 9 + z - 1$

$-5x + 3y + z = 5$

10. Find the shortest distance of the surface $xyz = 1$ to the origin.

(a) $(\frac{1}{2}, 1, 2)$ (b) $2\sqrt{2}$ (c) $\frac{\sqrt{21}}{2}$ (d) $\sqrt{3}$ (e) 3

Minimize $f(x,y,z) = (d[(x,y,z),(0,0,0)])^2 = x^2 + y^2 + z^2$ subject to $g(x,y,z) = xyz = 1$.

$\nabla f = \langle 2x, 2y, 2z\rangle$, $\nabla g = \langle yz, xz, xy\rangle$

$\begin{cases} \nabla f = \lambda \nabla g \\ g = 1 \end{cases} \Rightarrow \begin{cases} 2x = \lambda yz & ① \\ 2y = \lambda xz & ② \\ 2z = \lambda xy & ③ \\ xyz = 1 & ④ \end{cases}$

Notice $x, y, z \neq 0$ since they're not on the surface $xyz=$

So, $\lambda \overset{①}{=} \frac{2x}{yz} \overset{②}{=} \frac{2y}{xz} \overset{③}{=} \frac{2z}{xy} \Rightarrow x^2 = y^2 = z^2$

$x^2 = y^2$ $y^2 = z^2$

$x^2 = z^2$

$\Rightarrow x = \pm y = \pm z$

④ $\Rightarrow x^3 = \pm 1 \Rightarrow x = \pm 1, y = \pm 1, z = \pm 1$

$f(\pm 1, \pm 1, \pm 1) = 1 + 1 + 1 = 3$ dist $= \sqrt{3}$

$\vec{B}(t) = \frac{\vec{r}'(t) \times \vec{r}''(t)}{| \quad '' \quad |}$

11. Which of the following is an equation for the osculating plane of the curve

$$\mathbf{r}(t) = \langle \cos t, 1 + t^2, t \rangle$$

at $t = 0$? (That is, at the point $(1, 1, 0)$.)

(a) $x - y + 1 = 0$

(b) $2x + y - 3 = 0$

(c) $x - 2y + 1 = 0$

(d) $2x + y + z - 3 = 0$

(e) $x + y = 0$

Can use $\vec{r}'(0) \times \vec{r}''(0)$ as normal vector:

$\vec{r}'(t) = \langle -\sin t, 2t, 1 \rangle$ $\quad$ $\vec{r}''(t) = \langle -\cos t, 2, 0 \rangle$

$\vec{r}'(0) = \langle 0, 0, 1 \rangle$ $\quad$ $\vec{r}''(0) = \langle -1, 2, 0 \rangle$

$\vec{r}'(0) \times \vec{r}''(0) = \langle -2, -1, 0 \rangle = \vec{n}$

$\langle -2, -1, 0 \rangle \cdot \langle x-1, y-1, z-0 \rangle = -2(x-1) - (y-1) + 0 = 0$

$= -2x + 2 - y + 1 = 0$

$2x + y = 3$

12. Which of the following triple integrals represents the volume of the region inside both the cylinder $x^2 + y^2 = 4$ and the sphere $x^2 + y^2 + z^2 = 9$?

$r=2$

↑ use cylindrical

(a) $\cancel{\int_{-2}^{2}\int_{-2}^{2}\int_{-3}^{3} 2\sqrt{9-x^2-y^2}\, dz\, dy\, dx}$ (b) $\cancel{\int_0^{2\pi}\int_0^{\pi}\int_0^{3} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta}$

(c) $\cancel{\int_0^{2}\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{9-x^2-y^2}} dz\, dy\, dx}$ (d) $\cancel{\int_0^{2\pi}\int_0^{3}\int_{r^2-4}^{4-r^2} r\, dz\, dr\, d\theta}$

(e) $\int_0^{2\pi}\int_0^{2}\int_{-\sqrt{9-r^2}}^{\sqrt{9-r^2}} r\, dz\, dr\, d\theta$

$x^2+y^2+z^2=9 \Leftrightarrow r^2+z^2=9 \Leftrightarrow$ top: $z=\sqrt{9-r^2}$ bot: $z=-\sqrt{9-r^2}$

13. Find $\iint_S \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = \langle x, 3y^2, z\rangle$ and S is the surface of the cylinder $4x^2 + z^2 = 4, 0 \le y \le 1$, with outward normal.

(a) 0 (b) -2π (c) 4π (d) 2π (e) 8π

Parametrize: $\vec{r}(\theta, y) = \langle \cos\theta, y, 2\sin\theta\rangle$, $0 \le \theta \le 2\pi$, $0 \le y \le 1$

$\vec{r}_\theta = \langle -\sin\theta, 0, 2\cos\theta\rangle$, $\vec{r}_\theta \times \vec{r}_y = \langle -2\cos\theta, 0, -\sin\theta\rangle$

$\vec{r}_y = \langle 0, 1, 0\rangle$ These point inward, so need to use $\vec{r}_y \times \vec{r}_\theta = \langle 2\cos\theta, 0, \sin\theta\rangle$

$\vec{F}(\vec{r}(\theta,y)) = \langle \cos\theta, 3y^2, 2\sin\theta\rangle$, $\vec{F}(\vec{r}(\theta,y))\cdot(\vec{r}_y \times \vec{r}_\theta) = 2\cos^2\theta + 0 + 2\sin^2\theta = 2$

$\iint_S \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^{1} 2\, dy\, d\theta = \boxed{4\pi}$

14. Find $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle z, x, 2y \rangle$ and C is the curve given by the intersection of the plane $x + 2y + z = 2$ with the coordinate planes in a counterclockwise direction when viewed from above.

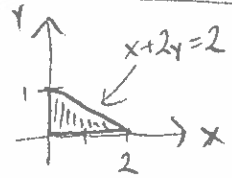
upward

(a) -4 (b) 6 (c) 5 (d) 4 (e) 0

$$\text{curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ z & x & 2y \end{vmatrix} = \langle 2, 1, 1 \rangle$$

S = piece of $x+2y+z=2$ inside first octant

Parametrize surface: $\vec{r}(x,y) = \langle x, y, 2-x-2y \rangle$, D =



$x+2y=2 \iff x = 2-2y$

$\vec{r}_x = \langle 1, 0, -1 \rangle$

$\vec{r}_y = \langle 0, 1, -2 \rangle$

$\vec{r}_x \times \vec{r}_y = \langle 1, 2, 1 \rangle$ ✓

Stokes': $\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\text{curl } \vec{F}) \cdot d\vec{S} = \iint_D \langle 2,1,1 \rangle \cdot \langle 1,2,1 \rangle \, dA$

$= \iint_D 5 \, dA = 5 \text{ area}(D) = 5\left(\frac{1}{2}(1)(2)\right) = 5$

15. Find the absolute maximum and minimum of the function

$$f(x, y) = (x-1)^2 + y^2$$

on the region

$$D = \left\{ (x, y) : \frac{x^2}{4} + y^2 \leq 1 \right\}$$

(a) Absolute minimum 0; no absolute maximum.

(b) ~~Absolute minimum~~ $\frac{2}{3}$; absolute maximum 9.

(c) Absolute minimum 0; absolute maximum $\frac{2}{3}$.

(d) Absolute minimum 0; absolute maximum 9.

(e) Absolute ~~minimum~~ $\frac{2}{3}$; absolute maximum 1.

Inside: $\nabla f = \langle 2(x-1), 2y \rangle = \vec{0}$: $x=1, y=0$ ✓ $(1,0)$

Boundary: $\frac{x^2}{4} + y^2 = 1 \iff x^2 + 4y^2 = 4 = g(x,y)$

$\nabla g = \langle 2x, 8y \rangle$ $\begin{cases} \nabla f = \lambda \nabla g \\ g = 4 \end{cases} \iff \begin{cases} 2x - 2 = 2\lambda x & (1) \\ 2y = 8\lambda y & (2) \\ x^2 + 4y^2 = 4 & (3) \end{cases}$

(2) $\Rightarrow y = 0$ or $\lambda = \frac{1}{4}$

$\boxed{y=0}$: $x = \pm 2$: $\boxed{x=2}$ (1) $2 = 4\lambda \Rightarrow \lambda = \frac{1}{2}$ $(2,0)$

$\boxed{x=-2}$ (1) $-6 = -4\lambda \Rightarrow \lambda = \frac{3}{2}$ $(-2,0)$

$\boxed{\lambda = \frac{1}{4}}$: (1) : $2x - 2 = \frac{1}{2}x \Rightarrow \frac{3}{2}x = 2 \Rightarrow x = \frac{4}{3} \overset{(3)}{\Rightarrow} \frac{16}{9} + 4y^2 = 4 \Rightarrow y^2 = 1 - \frac{4}{9} = \frac{5}{9} \Rightarrow y = \pm \frac{\sqrt{5}}{3}$

$\left(\frac{4}{3}, \frac{\sqrt{5}}{3}\right), \left(\frac{4}{3}, -\frac{\sqrt{5}}{3}\right)$.

pt	$(1,0)$	$(2,0)$	$(-2,0)$	$\left(\frac{4}{3}, \frac{\sqrt{5}}{3}\right)$	$\left(\frac{4}{3}, -\frac{\sqrt{5}}{3}\right)$
val	0	1	9	$\frac{1}{9} + \frac{5}{9} = \frac{6}{9}$	$\frac{6}{9}$
	min		max		

16. Find the rate of change of the function
$$f(x,y,z) = xy^2e^{x-z}$$
at the point $(2,1,2)$ in the direction of the vector $\langle 0,1,1\rangle$.

(a) $\sqrt{2}$ (b) 4 (c) $3\sqrt{2}$ (d) 2 (e) 5

$$\nabla f = \langle y^2 e^{x-z} + xy^2 e^{x-z},\ 2xye^{x-z},\ -xy^2 e^{x-z}\rangle$$

$$\nabla f(2,1,2) = \langle 1+2, 4, -2\rangle = \langle 3,4,-2\rangle$$

$$\nabla f \cdot \left(\frac{1}{\sqrt{2}}\langle 0,1,1\rangle\right) = \frac{1}{\sqrt{2}}(0+4-2) = \frac{2}{\sqrt{2}}$$

17. Find the projection $\text{proj}_{\mathbf{a}}\,\mathbf{b}$ of $\mathbf{b} = \langle 4,2,5\rangle$ onto $\mathbf{a} = \langle 1,-1,2\rangle$.

(a) $\langle \frac{16}{15}, \frac{8}{15}, \frac{4}{3}\rangle$ (b) $\langle \frac{1}{2}, -\frac{1}{2}, 1\rangle$ (c) $\langle 2,-2,4\rangle$ (d) $\langle \frac{4}{15}, -\frac{4}{15}, \frac{8}{15}\rangle$ (e) 2

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{b}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{4-2+10}{1+1+4}\langle 1,-1,2\rangle = \frac{12}{6}\langle 1,-1,2\rangle = \langle 2,-2,4\rangle$$

$\overset{g}{''}$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $\overset{h}{''}$

18. The cylinder $x^2 + y^2 = 4$ intersects the plane $x + y + z = 1$ in an ellipse. Find an equation of the tangent line to the ellipse at $(2, 0, -1)$.

(a) $\mathbf{r}(t) = \langle 0, -t, t\rangle$ (b) $\mathbf{r}(t) = \langle 2 + 4t, 0, -1 + t\rangle$ (c) $x = 2,\ \dfrac{y}{4} = z$

(d) $\dfrac{x-2}{4} = z + 1,\ y = 0$ (e) $\mathbf{r}(t) = \langle 2, -4t, -1 + 4t\rangle$

$\nabla g = \langle 2x, 2y, 0\rangle \quad \nabla h = \langle 1,1,1\rangle$

$\nabla g(2,0,-1) = \langle 4,0,0\rangle$

$\vec{v} = \nabla g(2,0,-1) \times \nabla h(2,0,-1)$

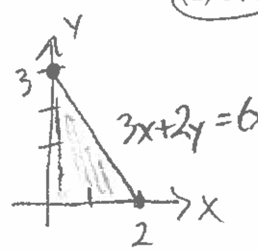
$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \langle 0,-4,4\rangle$

$\vec{r}(t) = P + t\vec{v} = \langle 2,0,-1\rangle + t\langle 0,-4,4\rangle = \langle 2, -4t, -1+4t\rangle$

19. Find the surface area of the part of the plane $3x + 2y + z = 6$ in the first octant (so $x \geq 0, y \geq 0, z \geq 0$).

(a) $3\sqrt{13}$ (b) 18 (c) 12 (d) $3\sqrt{14}$ (e) $3\sqrt{15}$

$\vec{r}(x,y) = \langle x, y, 6-3x-2y\rangle$, D:



$\vec{r}_x = \langle 1,0,-3\rangle$ $\quad \vec{r}_x \times \vec{r}_y = \langle 3,2,1\rangle \quad |\vec{r}_x \times \vec{r}_y| = \sqrt{9+4+1} = \sqrt{14}$

$\vec{r}_y = \langle 0,1,-2\rangle$

$S.A. = \iint_S dS = \iint_D \sqrt{14}\, dA = \sqrt{14}\ \text{area}(D) = \sqrt{14}\left(\tfrac{1}{2}(2)(3)\right) = 3\sqrt{14}$

20. Consider the function $f(x, y) = x\sin(y^2)$. Which of the following is true of the point $(0, 0)$?

(a) $(0, 0)$ is a critical point of f, but the Second Derivative Test is inconclusive.

(b) $(0, 0)$ is not a critical point of f.

(c) $(0, 0)$ is a local maximum of f.

(d) $(0, 0)$ is a saddle point of f.

(e) $(0, 0)$ is a local minimum of f.

$\nabla f = \langle \sin(y^2), 2xy\cos(y^2) \rangle$

$\nabla f(0,0) = \langle 0, 0 \rangle$ ✓ $(0,0)$ is a critical point.

$$Hf = \begin{pmatrix} 0 & 2y\cos(y^2) \\ 2y\cos(y^2) & 2x\cos(y^2) + 4xy^2\sin(y^2) \end{pmatrix}$$

$$Hf(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\det Hf(0,0) = 0 \Rightarrow$ indeterminont