Multiple Choice

1.(6 pts) Find symmetric equations of the line L passing through the point $(2, -5, 1)$ and perpendicular to the plane $x + 3y - z = 9$.

(a) $\dfrac{x-2}{1} = \dfrac{y+5}{3} = \dfrac{z-1}{-1}$

(b) $2(x-1) = (-5)(y-3) = z+1$

(c) $(x-2) + 3(y-3) - (z-1) = 9$

(d) $\dfrac{x-1}{2} = \dfrac{y-3}{-5} = \dfrac{z+1}{1} = 9$

(e) $\dfrac{x-1}{2} = \dfrac{y-3}{-5} = \dfrac{z+1}{1}$

2.(6 pts) The two curves below intersect at the point $(1, 4, -1) = \mathbf{r}_1(0) = \mathbf{r}_2(1)$. Find the cosine of the angle of intersection

$$\mathbf{r}_1(t) = e^{3t}\mathbf{i} + 4\sin\left(t + \frac{\pi}{2}\right)\mathbf{j} + (t^2 - 1)\mathbf{k}$$
$$\mathbf{r}_2(t) = t\mathbf{i} + 4\mathbf{j} + (t^2 - 2)\mathbf{k}$$

(a) 0 (b) 3 (c) $\dfrac{1}{\sqrt{5}}$ (d) $\dfrac{1}{5}$ (e) $\dfrac{e}{\sqrt{e^2+4}}$

3.(6 pts) Find the projection of the vector $\langle 1, -1, 5\rangle$ onto the vector $\langle 2, 1, 4\rangle$

(a) $\dfrac{1}{5}\langle 2, 1, 5\rangle$ (b) $\langle 6, 3, 12\rangle$ (c) $\langle 1, -1, 5\rangle$ (d) $\langle 3, -3, 15\rangle$ (e) $\langle 2, 1, 4\rangle$

4.(6 pts) Find $\displaystyle\int \mathbf{r}(x)dx$ where

$$\mathbf{r}(x) = (\sec^2 x)\mathbf{i} + e^x\mathbf{k}$$

Recall: $\int \sec^2 x\, dx = \tan x + C$.

(a) $(\tan x + C_1)\mathbf{i} + C_2\mathbf{j} + (e^x + C_3)\mathbf{k}$ (b) $\tan x + e^x + C$

(c) $(\tan x)\mathbf{i} + e^x\mathbf{k}$ (d) $(\tan x + C_1)\mathbf{i} + (e^x + C_2)\mathbf{k}$

(e) $(\tan x + C)\mathbf{i} + C\mathbf{j} + (e^x + C)\mathbf{k}$

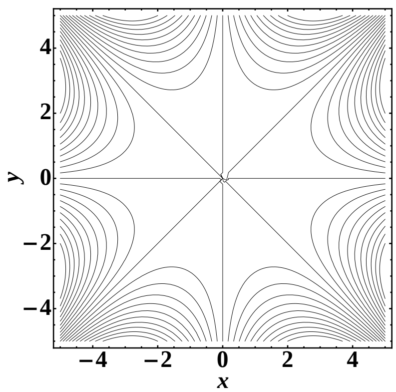
5.(6 pts) Find the volume of the parallelepiped spanned by the three vectors $\langle 1, 2, -1\rangle$, $\langle 0, 1, 2\rangle$ and $\langle 3, 2, 1\rangle$.

(a) $9\sqrt{2}$ (b) $2\sqrt{3}$ (c) 0 (d) 12 (e) $3\sqrt{2}$

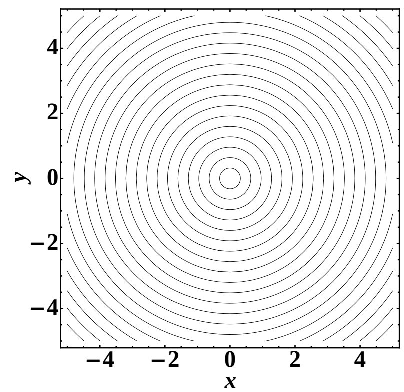
6.(6 pts) Find the area of the triangle formed by the three points $(1, 0, 1)$, $(2, 0, 2)$ and $(3, 3, 3)$.

(a) 2.2 (b) 0 (c) $\frac{3}{2}\sqrt{2}$ (d) 4 (e) $\frac{\sqrt{3}}{2}$

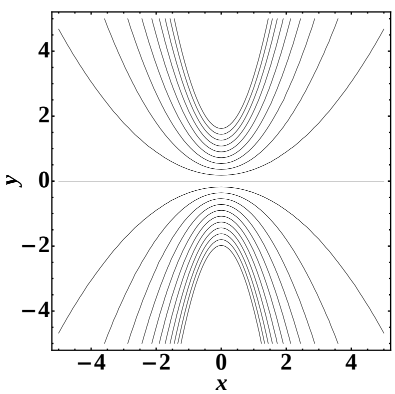
7.(6 pts) Which of the following is a contour map for $f(x, y) = \dfrac{xy}{x^2 + 1}$?



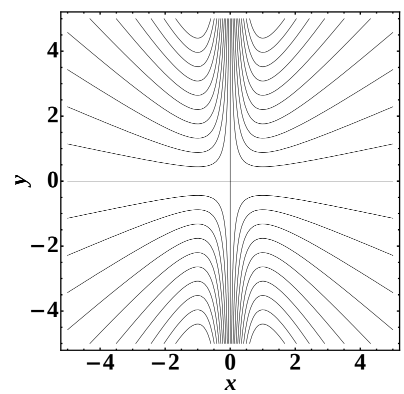
(a)



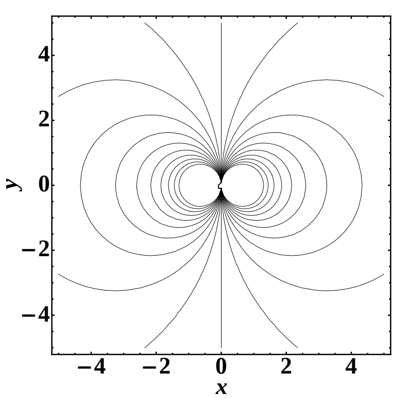
(b)



(c)



(d)



(e)

8.(6 pts) A particle is travelling and has position at time t given by $\mathbf{r}(t) = \left\langle \frac{2}{3}t^3, \frac{\sqrt{12}}{2}t^2, 3t \right\rangle$. How far does it travel from time $t = 0$ to time $t = 3$.

(a) 56 (b) 27 (c) 14 (d) 96 (e) 48

9.(6 pts) Find the radius of the sphere given by the equation

$$x^2 - 8x + y^2 + 2y + z^2 - 10z + 30 = 0.$$

(a) 6 (b) $\sqrt{10}$ (c) 12 (d) $2\sqrt{3}$ (e) $\sqrt{42}$

10.(6 pts) Below are five expressions involving two vectors $\mathbf{a}$ and $\mathbf{b}$. All of them are always equal to either 0 (the scalar) or $\mathbf{0}$ (the vector) except one. Which one can be nonzero?

(a) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{a})$ (b) $(\mathbf{a} \times \mathbf{b}) - (\mathbf{b} \times \mathbf{a})$ (c) $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{b})$

(d) $\mathbf{b} \cdot (\mathbf{a} \times \mathbf{a})$ (e) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{a})$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Consider the curve

$$\mathbf{r}(t) = \sin(2t)\mathbf{i} + t\mathbf{j} - \cos(2t)\mathbf{k}.$$

Give equations for the normal plane and the osculating plane at $t = 0$.

12.(12 pts.) Are the lines $\langle 3, -2, -1\rangle + t\langle 2, 1, 1\rangle$ and $\langle 7, 5, 6\rangle + t\langle 1, 3, 3\rangle$ parallel, intersecting, or skew? If intersecting, find a point of intersection.

13.(12 pts.) Suppose the curve C has parametric equations:

$$x(t) = t^3 - t, \quad y(t) = 1 - 2\sqrt{t}, \quad z(t) = t^2 + t$$

Find the parametric equation for the tangent line to the above curve C at the point $P = (0, -1, 2)$.

Name: ____________________

Instructor: ANSWERS

Math 20550, Practice Exam 1
September 23, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 0 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(•)	(e)
6.	(a)	(b)	(•)	(d)	(e)
7.	(a)	(b)	(c)	(•)	(e)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(•)	(e)
10.	(a)	(•)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	
11.	
12.	
13.	
Extra Points.	4
Total	

1. The symmetric equations of line are given by $(x-x_0)/a = (y-y_0)/b = (z-z_0)/c$, where (x_0, y_0, z_0) is a point on the line and $\langle a, b, c\rangle$ is a direction vector. Since L is perpendicular to the plane $x + 3y - z = 9$, then we can take the normal to the plane as the direction vector, this is, $\langle 1, 3, -1\rangle$ is a direction vector of L. Therefore, the symmetric equations are $\dfrac{x-2}{1} = \dfrac{y+5}{3} = \dfrac{z-1}{-1}$.

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2. Note

$$\mathbf{r}_1'(t) = \left\langle 3e^{3t}, 4\cos\left(t + \frac{\pi}{2}\right), 2t\right\rangle$$
$$\mathbf{r}_2'(t) = \langle 1, 0, 2t\rangle$$

To compute the angle of intersection we find $\mathbf{r_1'}(0) = \langle 3, 0, 0\rangle$ $\mathbf{r_2'}(1) = \langle 1, 0, 2\rangle$ so that $\cos\theta = \dfrac{\mathbf{r}_1'(0)\cdot\mathbf{r}_2'(1)}{|\mathbf{r}_1'(0)||\mathbf{r}_2'(1)|} = \dfrac{3}{3\sqrt{5}} = \dfrac{1}{\sqrt{5}}$.

3.

$$\operatorname{proj}_{\langle 2,1,4\rangle}\big(\langle 1, -1, 5\rangle\big) = \frac{\langle 2, 1, 4\rangle\cdot\langle 1, -1, 5\rangle}{\langle 2, 1, 4\rangle\cdot\langle 2, 1, 4\rangle}\langle 2, 1, 4\rangle = \frac{21}{21}\langle 2, 1, 4\rangle = \langle 2, 1, 4\rangle$$

4.

$$\begin{aligned}\int \mathbf{r}(x)dx &= \int \left((\sec^2 x)\mathbf{i} + e^x\mathbf{k}\right)dx \\ &= \left(\int \sec^2 x dx\right)\mathbf{i} + \left(\int 0 dx\right)\mathbf{j} + \left(\int e^x dx\right)\mathbf{k} \\ &= (\tan x + C_1)\,\mathbf{i} + C_2\mathbf{j} + (e^x + C_3)\,\mathbf{k}\end{aligned}$$

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5. Answer is the absolute value of the triple product

$$\begin{vmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 1\cdot\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - (2)\cdot\begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} + -1\cdot\begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix} = -3 + 12 + 3 = 12$$

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6. Two vectors which form two sides of the triangle are $\langle 1, 0, 1\rangle = \langle 2, 0, 2\rangle - \langle 1, 0, 1\rangle$ and $\langle 2, 3, 2\rangle = \langle 3, 3, 3\rangle - \langle 1, 0, 1\rangle$. Hence

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 2 & 3 & 2 \end{vmatrix} = \langle 3 - 0, -(2-2), 3 - 0\rangle = \langle 3, 0, 3\rangle$$

The area of the parallelogram is $|\langle 3, 0, 3\rangle| = \sqrt{9+0+9} = 3\sqrt{2}$ and the area of the triangle is half this.

7.The level curves are given by $f(x,y) = \dfrac{xy}{x^2+1} = c$ where c is a constant. Solving for y we get

$$y = \frac{c}{x} + cx.$$

So each level curve for $c \neq 0$ has a vertical asymptote at $x = 0$. Of the five given curves, that only leaves (a) and (d) as the possible correct answers. Also notice that for each x, there is only one corresponding value of y i.e. y is a well defined function of x. So each level curve must pass the vertical test. But clearly there are curves in (a) which fail this. Hence the correct answer is (d).

8.

$\mathbf{r}'(t) = \langle 2t^2, \sqrt{12}t, 3\rangle$. So the distance travelled is given by

$$L(0,3) = \int_0^3 |\mathbf{r}'(t)|\, dt = \int_0^3 \sqrt{4t^4 + 12t^2 + 9}\, dt = \int_0^3 (2t^2+3)\, dt = 27.$$

For the penultimate equality, notice that $4t^4 + 12t^2 + 9 = (2t^2+3)^2$.

9.Completing squares $x^2+8x = (x+4)^2-16$, $y^2+2y = (y+1)^2-1$, $z^2-10z = (z-5)^2-25$. So

$$\begin{aligned} x^2 - 8x + y^2 + 2y + z^2 - 10z + 30 &= 0 \\ \Leftrightarrow (x+4)^2 + (y+1)^2 + (z-5)^2 - 16 - 1 - 25 + 30 &= 0 \\ \Leftrightarrow (x+4)^2 + (y+1)^2 + (z-5)^2 &= 12 \end{aligned}$$

So radius is $\sqrt{12} = 2\sqrt{3}$.

10.(a) is always zero since $\mathbf{a}\times\mathbf{b} = -(\mathbf{b}\times\mathbf{a})$ and cross product of parallel vectors is zero.
(b) is zero if and only if $\mathbf{a}\times\mathbf{b} = 0$. So (b) can be non zero.
(c) is always zero since $\mathbf{a}\times\mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ and hence perpendicular to $\mathbf{a}+\mathbf{b}$.
(d) is always zero since $\mathbf{a}\times\mathbf{a} = 0$ for any vector $\mathbf{a}$.
(e) is always zero since $\mathbf{a}$ is perpendicular to $\mathbf{b}\times\mathbf{a}$.

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11. At $t = 0$, $\mathbf{r}(0) = \langle 0,0,-1\rangle$, $\mathbf{r}'(0) = \langle 2,1,0\rangle$, $\mathbf{r}''(0) = \langle 0,0,4\rangle$. The normal vector to the normal plane is given by the velocity vector. So equation of the normal plane is

$$2x + y = 0.$$

The osculating plane at 't' is a plane containing $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$. So a normal to the osculating plane at $t = 0$ is given by

$$\mathbf{r}'(0)\times\mathbf{r}''(0) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ 0 & 0 & 4 \end{vmatrix} = \langle 4, -8, 0\rangle$$

The equation to the osculating plane is thus

$$4x - 8y = 0.$$

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12. They are not parallel, since $\langle 2,1,1\rangle$ is not a multiple of $\langle 1,3,3\rangle$. If they intersect, then one can find a t and s such that

$$3+2t=7+s$$
$$-2+t=5+3s$$
$$-1+t=6+3s$$

Solving the first two equations we get $s=-2, t=1$. This clearly satisfies the third equation also. So the lines do intersect. To get the point of intersection we plug in $t=1$ in the equation of the first line (or equivalently $s=-2$ in the equation of the second line). Doing so we obtain the point of intersection as $(5,-1,0)$.

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13. Let $\mathbf{v}(t)=\langle 2,-1,3\rangle$. Then the vector equation for the tangent line to C at P is given by $t\mathbf{v}(1)+\langle 0,-1,2\rangle=\langle 2t,-t,3t\rangle+\langle 0,-1,2\rangle=\langle 2t,-t-1,3t+2\rangle$.

Then the parametric equation for the tangent line to C at P is given by
$x(t)=2t, \quad y(t)=-t-1, \quad z(t)=3t+2.$