
11. We want to minimize $f(x, y, z) = d^2 = x^2 + y^2 + z^2$ subject to the constraint $2x - y + 3z + 14 = 0$. Using Lagrange multipliers we get the following system of equations:

$$\begin{aligned} 2x &= 2\lambda \\ 2y &= -\lambda \\ 2z &= 3\lambda \\ 2x - y + 3z + 14 &= 0 \end{aligned}$$

To solve this we see that $x = \lambda$, $y = -\lambda/2$, and $z = 3\lambda/2$. Plugging into the last equation we get:

$$2\lambda + \lambda/2 + (9/2)\lambda = -14$$

Solving for lambda we get $\lambda = -2$ as the only solution. Clearly this is a minimum since the $f(-2, 1, -3) = 14$ and $f(-7, 0, 0) = 49$ (and same will be true for any other point on the plane). Answer: $(-2, 1, -3)$

12. We want to compute the iterated integral:

$$\int_0^2 \int_0^{2-x} 9 - x^2 - 2y \, dy \, dx$$

. So we get

$$\int_0^2 9y - x^2y - y^2 \Big|_0^{2-x} dx$$

which simplifies to

$$\int_0^2 14 - 5x - 3x^2 + x^3 \, dx$$

which is

$$14x - 5/2x^2 - x^3 + 1/4x^4 \Big|_0^2 = 14$$

Answer: 14

13. To find the critical points in the region look at the following equations:

$$f_x(x, y) = (4 - y^2)4xe^{2x^2} = 0$$

and

$$f_y(x, y) = -2ye^{2x^2} = 0.$$

From here we see that the only critical point occurs at $(0, 0)$ which is inside our region. The value of the function here is $f(0, 0) = 4$.

To check the boundary we can either use Lagrange multipliers or we can use substitution. Here we will opt to use substitution by substituting $y^2 = 4 - 4x^2$ in for y^2 in $f(x, y)$.

We get a function in x , call it $g(x) = 4x^2e^{2x^2}$ where we allow x to be in the interval $[-2, 2]$. The critical values of $g(x)$ in this interval are the solutions to $g'(x) = 8xe^{2x^2} + 16x^3e^{2x^2} = 0$ which is $x = 0$. This is in the interval $[-2, 2]$ and so including the end points this means we need to check the points $(0, -2)$, $(0, 2)$, and $(-2, 0)$, and $(2, 0)$.

Checking values of f we get $f(\pm 2, 0) = 4e^8$. And $f(0, \pm 2) = 0$. The global maximum of the function is $4e^8$ and the global minimum is 0.