
11.(a) Here we need to compute $\mathbf{r}_u \times \mathbf{r}_v = \langle -\sin u, \cos u, 0\rangle \times \langle 0, 0, 1\rangle = \langle \cos u, \sin u, 0\rangle$.

So at $(u, v) = (0, 2)$ we get $\mathbf{r}(0, 2) = \langle 1, 0, 2\rangle$, so we evaluate $\mathbf{r}_u \times \mathbf{r}_v$ at this point, to get $\langle 1, 0, 0\rangle$. Thus the tangent plane is given by

$$\langle 1, 0, 0\rangle \cdot \langle x, y, z\rangle = \langle 1, 0, 0\rangle \cdot \langle 1, 0, 2\rangle$$

which gives the equation $x = 1$.

(b) For the second part, we need to compute $|\mathbf{r}_u \times \mathbf{r}_v|$ in order to integrate it. As we have done part of this in part (a) we can see that $|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{\cos^2 u + \sin^2 u + 0} = 1$, so we need to compute the following integral

$$A(S) = \int_0^{2\pi} \int_0^{u} 1\, dv du = \int_0^{2\pi} v|_0^u \, du = \int_0^{2\pi} u\, du = 2\pi^2$$

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12. First we need to compute the Jacobian, $\dfrac{\partial(x, y)}{\partial(u, v)} = 4uv$. Then we note that the equation $\sqrt{x} + \sqrt{y} = 1$ becomes $u + v = 1$ and $x = 0$ and $y = 0$ imply respectively that $u = 0$ and $v = 0$. So we now want to integrate over the region bounded by $u = 0$, $v = 0$,and $u + v = 1$.

Let's use the convention of calling the orignal region in the xy-plane R and our region in the uv-plane S. Then we want to compute

$$\iint_R 1\, dA = \iint_S 1|4uv|\, dudv = \int_0^1 \int_0^{1-v} 4uv\, dudv$$

. So computing this integral we get

$$\int_0^1 2u^2 v|_0^{1-v}\, dv = \int_0^1 2v(1 - 2v + v^2)\, dv = v^2 - 4/3v^3 + 1/2v^4|_0^1 = 1 - 4/3 + 1/2 = 1/6$$

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13. First let's compute $\mathbf{r}'(t) = \langle -\sin t, \cos t, 8\rangle$. Now

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^{\pi/3} (\cos^2 t)(-\sin t) + (-\cos t \sin t)(\cos t) + 0*8\, dt = \int_0^{\pi/3} -2\cos^2 t \sin t\, dt = 2/3 \cos^3 t|_0^{\pi/3} =$$