EXAM 1. Solutions

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11. Clearly the origin $(0,0,0)$ lies on both the planes and hence the intersection. To get equation of line, we also need the direction. The line is perpendicular to the normals to both the planes. The normals are $(3,-1,1)$ and $(2,-3,1)$ respectively. So the direction of the line is given by

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 1 \\ 2 & -3 & 1 \end{vmatrix} = \langle 2,-1,-7\rangle$$

So the equation of the line is

$$\frac{x}{2} = \frac{y}{-1} = \frac{z}{-7}.$$

12.

(a) $\mathbf{r}'(t) = 2t\mathbf{i} + (1/t)\mathbf{k}$ and $\mathbf{r}''(t) = 2\mathbf{i} - (1/t^2)\mathbf{k}$. Pluggin in $t = 1$, $\mathbf{r}'(1) = 2\mathbf{i} + \mathbf{k}$ and $\mathbf{r}''(1) = 2\mathbf{i} - \mathbf{k}$. So

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{2\mathbf{i} + \mathbf{k}}{\sqrt{5}}.$$

Next, recall that $\mathbf{B} = \mathbf{r}' \times \mathbf{r}''/|\mathbf{r}' \times \mathbf{r}''|$. But,

$$\mathbf{r}'(1) \times \mathbf{r}''(1) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 1 \\ 2 & 0 & -1 \end{vmatrix} = 4\mathbf{j}$$

So, $\mathbf{B}(1) = \mathbf{j}$. But then,

$$\mathbf{N}(1) = \mathbf{B}(1) \times \mathbf{T}(1) = \mathbf{j} \times \left(\frac{2\mathbf{i} + \mathbf{k}}{\sqrt{5}}\right) = \frac{2\mathbf{j} \times \mathbf{i} + \mathbf{j} \times \mathbf{k}}{\sqrt{5}} = \frac{\mathbf{i} - 2\mathbf{k}}{\sqrt{5}}.$$

(b) $\mathbf{r}(1) = (1,-1,0)$. Normal plane consists of $\mathbf{B}$ and $\mathbf{N}$, so the normal vector to the normal plane is given by $\mathbf{T}$. So the equation (after canceling out $\sqrt{5}$) is

$$2(x-1) + z = 0$$

or $2x + z = 2$.

(c) The normal to the osculating plane is given by the bi-normal $\mathbf{B}$. And so the equation is

$$y = -1.$$

13.Step-1. Find equation of the plane. It contains vectors $\mathbf{PQ} = (1,1,0)$ and $\mathbf{PR} = (1,0,-4)$. So the normal to this plane is given by

$$\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & 0 & -4 \end{vmatrix} = (-4,4,-1),$$

and the equation of the plane is $-4x + 4y - (z - 3) = 0$ or

$$-4x + 4y - z + 3 = 0.$$

Then the distance of the point $(-4, 1, 4)$ from this plane is obtained by putting $a = -4, b = 4, c = -1, d = 3$ in the distance formula to get

$$D = \frac{|-4(-4) + 4(1) - (4) + 3|}{\sqrt{4^2 + 4^2 + 1^2}} = \frac{19}{\sqrt{33}}.$$