Math 20550 Calculus III Tutorial
April 23, 2015

Name: _____

Tutorial Worksheet

1. Calculate $\iint_S yx^2 dS$ where S is the surface $z = x + y^2$, $0 \leq x \leq 1$ and $0 \leq y \leq 2$.

2. Let S be the part of the cylinder $y^2 + z^2 = 1$, with $z \geqslant 0$, and $0 \leqslant x \leqslant 1$, and let S have the upward orientation. Determine which of the following equals $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle 0, 0, z \rangle$.

(a) $\displaystyle\int_0^1 \int_{-1}^1 \sqrt{1 - y^2}\, dy\, dx$ (b) $\displaystyle\int_0^1 \int_{-1}^1 \sqrt{1 - x^2}\, dy\, dx$ (c) $\displaystyle\int_0^1 \int_{-1}^1 (1 - y^2)\, dy\, dx$

(d) $\displaystyle\int_0^1 \int_{-1}^1 (1 - x^2)\, dy\, dx$ (e) $\displaystyle\int_0^1 \int_{-1}^1 \left[\sqrt{1 - y^2}\right]^{-1} dy\, dx$

3. Let S be portion of the plane $z = 2y + 3$ inside the cylinder $x^2 + y^2 = 1$ oriented with the upward normal. Calculate $\iint_S (\operatorname{curl} \mathbf{F}) \cdot d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = \langle 2e^y - z, \cos(yz), xe^y \rangle$$

(a) 0 (b) 2π (c) $\dfrac{\pi}{\sqrt{5}}$ (d) $\sqrt{5}\pi$ (e) $\dfrac{2\pi}{\sqrt{5}}$

4. Use Stokes' theorem to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x^2 y\mathbf{i} + \frac{1}{3}x^3\mathbf{j} + xy\mathbf{k}$ and C is the curve of intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$ oriented counterclockwise as viewed from above.