Math 20550 Calculus III Tutorial **Name:** ______________________________
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Tutorial Worksheet

Show all your work.

1. Find the vector given by the projection of $\mathbf{v} = \langle 3, 5, 4\rangle$ onto $\mathbf{a} = \langle 1, 2, -2\rangle$.

Solution: The projection of $\mathbf{v}$ onto $\mathbf{a}$ is given by $Proj_{\mathbf{a}}\mathbf{v} = \frac{\mathbf{a}\cdot\mathbf{v}}{|\mathbf{a}|^2}\mathbf{a}$. The squared length of $\mathbf{a}$ is given by $|\mathbf{a}|^2 = \mathbf{a}\cdot\mathbf{a} = 1^2 + 2^2 + (-2)^2 = 9$ and the dot product of $\mathbf{a}$ and $\mathbf{v}$ is given by $\mathbf{a}\cdot\mathbf{v} = 3 + 10 - 8 = 5$. Therefore $Proj_{\mathbf{a}}\mathbf{v} = \frac{5}{9}\langle 1, 2, -2\rangle$.

2. Find a vector perpendicular to the plane that passes through the three points $P(1, 4, 5)$, $Q(-2, 5, -2)$ and $R(1, -1, 0)$.

Solution: First find two vectors in the plane: these are given by the displacement vectors from the point P to the point Q and from the point P to the point R. Call these vectors $\mathbf{u}$ and $\mathbf{v}$, respectively. Then $\mathbf{u} = \langle -2-1, 5-4, -2-5\rangle = \langle -3, 1, -7\rangle$ and $\mathbf{v} = \langle 1-1, -1-4, 0-5\rangle = \langle 0, -5, -5\rangle$. A vector that is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$ is the cross-product of $\mathbf{u}$ with $\mathbf{v}$:
$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 1 & -7 \\ 0 & -5 & -5 \end{vmatrix} = \langle 1\cdot(-5)-(-7)\cdot(-5), -\Big((-3)\cdot(-5)-(-7)\cdot 0\Big), (-3)\cdot(-5)-1\cdot 0\rangle =$
$\langle -40, -15, 15\rangle$.

3. Is

$$x^2 - 2x + y^2 + z^2 + 7 = 1 - 5x + 2z$$

an equation of a sphere? If so, find the center of the sphere.

Solution: After completing the square for each of the variables x, y and z one obtains the equation $(x + \frac{3}{2})^2 + y^2 + (z - 1)^2 = -\frac{11}{4}$. This is clearly not the equation of a sphere since we require that the radius squared be a positive number.

4. Let L be a straight line that passes through the points $A(2, 4, -3)$ and $B(3, -1, 1)$. At what point does this line intersect the yz-plane?

Solution: The equation of the line passing through the points A and B is given by $L = \mathbf{r}_0 + t\mathbf{r}$, where $\mathbf{r}_0$ is the position vector of the point A and $\mathbf{r}$ is the displacement vector from A to B. Now $\mathbf{r} = \langle 3-2, -1-4, 1-3\rangle = \langle 1, -5, -2\rangle$ so $L = \langle 2, 4, -3\rangle + t\langle 1, -5, -2\rangle = \langle 2+t, 4-5t, -3-2t\rangle$. Now L intersects the y-z plane when the first coordinate of L is zero. That is, when $2 + t = 0$ or $t = -2$. When $t = -2$, $L = \langle 0, 14, -11\rangle$ so that L intersects the y-z plane at the point $(0, 14, -11)$.

5. A tow truck drags a stalled car along a road. The chain makes an angle of $30°$ with the road and the tension in the chain is 1200 N. How much work is done by the truck in pulling the car 1 km?

Solution: Let $\mathbf{F}$ and $\mathbf{D}$ denote the force and displacement vectors, respectively, with $|\mathbf{F}| = 1200$ and $|\mathbf{D}| = 1$. Then the work done is given by $W = |\mathbf{F}||\mathbf{D}|\cos\theta = 1200 \cdot 1 \cos 30 = 600\sqrt{3}$KJ.

6. Find an equation of the sphere that passes through the origin and has center $(3, -2, 5)$.

Solution: The sphere must touch the origin so the radius is given by the length of the position vector of the point $(3, -2, 5)$. Therefore $r^2 = 3^2 + (-2)^2 + 5^2 = 38$ and the equation of the sphere with the given centre is $(x-3)^2 + (y+2)^2 + (z-5)^2 = 38$.

What is an equation of the intersection of this sphere with the yz-plane?

Solution: Set $x = 0$ in the above equation and we obtain the equation of a circle with radius $\sqrt{38}$ and center $(0, -2, 5)$: $(y+2)^2 + (z-5)^2 = 38$