Math 20550 Calculus III Tutorial **Name:** ______________________________
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Tutorial Worksheet

Show all your work.

1. Let ℓ be the intersection of the planes given by equations $x - y = 1$ and $x - z = 1$. Find an equation for ℓ in the form $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$.

Solution: To find the equation of a line, we need a point on it and a vector for its direction.

We can find a point on both planes by finding (x, y, z) that satisfy both $x - y = 1$ and $x - z = 1$. For instance, if we set $x = 1$, the first equation tells us that $y = 0$ and gives no restrictions on z, and the second tells us $z = 0$ and gives no restrictions on y. Therefore, $(1, 0, 0)$ lies on both planes and hence on ℓ.

A normal vector for the first plane is $\langle 1, -1, 0\rangle$ and a normal vector for the second plane is $\langle 1, 0, -1\rangle$. If a vector is in both planes, it's perpendicular to both planes' normal vectors. Hence a vector parallel to the line is

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \left\langle \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}, -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} \right\rangle = \langle 1, 1, 1\rangle$$

This gives us a vector parallel to ℓ and a point on ℓ, so an equation for it is $\mathbf{r}(t) = t\,\langle 1, 1, 1\rangle + \langle 1, 0, 0\rangle$.

2. A point moves in space in such a way that at time t its position is given by the vector-valued function $\mathbf{r}(t) = \langle t^2 + 1, 2t^2 - 1, 2 - 3t^2\rangle$. At what time(s) does the point hit the plane $2x + 2y + 3z = 9$?

Solution: The plane can be given by $\langle 2, 2, 3\rangle \cdot \langle x, y, z\rangle = 9$ so we need to solve $\langle 2, 2, 3\rangle \cdot \mathbf{r}(t) = 9$ or $\langle 2, 2, 3\rangle \cdot \langle t^2 + 1, 2t^2 - 1, 2 - 3t^2\rangle = 9$ or $2(t^2 + 1) + 2(2t^2 - 1) + 3(2 - 3t^2) = 9$ or $2t^2 + 2 + 4t^2 - 2 + 6 - 9t^2 = 9$ or $-3t^2 + 6 = 9$ so $t^2 = -1$ and there are no intersections.

3. Determine the *speed* at $t = 1$ of an object whose position function is $\mathbf{r}(t) = \langle 2t^3, 3t, 3t^2 \rangle$.

Solution: $\mathbf{r}'(t) = \langle 6t, 3, 6t \rangle$ so the speed is $|\, \mathbf{r}'(1)\,| = |\, \langle 6, 3, 6 \rangle\,| = \sqrt{36 + 9 + 36} = \sqrt{81} = 9$.

4. Find an equation of the plane perpendicular to the line $x = 1 + 4t$, $y = 1 - t$, $z = -3$ passing through the point $(1, 1, 1)$.

Solution: The line is $t\,\langle 4, -1, 0 \rangle + \langle 1, 1, -3 \rangle$ so we want a plane with normal vector $\langle 4, -1, 0 \rangle$ passing through $\langle 1, 1, 1 \rangle$. Hence $\langle 4, -1, 0 \rangle \cdot \langle x, y, z \rangle = \langle 4, -1, 0 \rangle \cdot \langle 1, 1, 1 \rangle = 3$, which expands out to $4x - y = 3$.

5. Find the distance from the point $(1, -1, 1)$ to the plane $x + 2y - 2z = 6$.

Solution: There are a few ways to solve this. We could find an equation for the line perpendicular to the plane passing through the point, as in 4, find its intersection with the plane, and calculate the distance between these two points.

Alternately, we can calculate the value directly from the formula given by vector components, $d = \dfrac{|(1)(1) + (2)(-1) + (-2)(1) - 6|}{\sqrt{1^2 + 2^2 + (-2)^2}} = \dfrac{|-9|}{3} = 3$.