Math 20550 Calculus III Tutorial **Name:** ______________________________
February 12, 2015

Tutorial Worksheet

Show all your work.

1. Do the following limits exist? If it exists, compute the limit, if not, explain why it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4}, \quad \lim_{(x,y)\to(0,0)} \frac{x^4y^4}{x^4+y^4}$$

Solution: Along x-axis, $y=0$, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4} = \lim_{x\to 0} \frac{x^2 0^2}{x^4+3\cdot 0^4} = 0.$

Along the path $x=y$, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4} = \lim_{x\to 0} \frac{x^4}{4x^4} = \frac{1}{4}.$

Hence $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4}$ does not exist.

2. Find $f_{yx}(1,2)$, $f_{xx}(1,2)$, $f_{yy}(1,2)$ and $f_{xy}(1,2)$ for the function $f(x,y) = x^3 + 2x^2y^2 + y^3$.

Solution: We compute

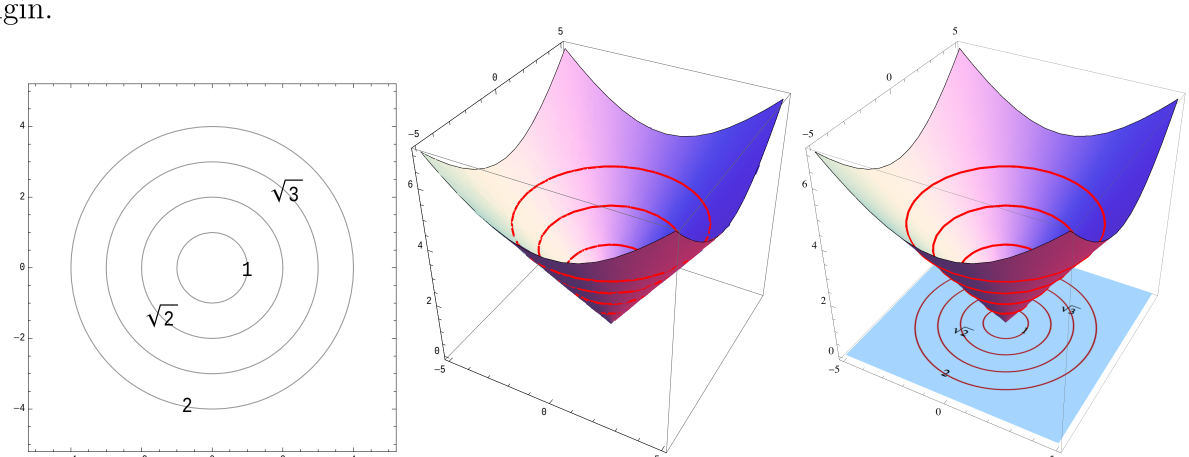
$$f_x = 3x^2 + 4xy^2, \; f_y = 3y^2 + 4x^2y$$
$$f_{xx} = 6x + 4y^2, \; f_{yy} = 4x^2 + 6y, \; f_{xy} = f_{yx} = 8xy.$$

We get

$$f_{xx}(1,2) = 22$$
$$f_{yy}(1,2) = 16$$
$$f_{xy}(1,2) = f_{yx}(1,2) = 16.$$

3. Identify the level curves of $f(x,y) = \sqrt{x^2+y^2}$. Using the level curves, plot the surface.

Solution: We first notice that $z^2 = x^2 + y^2$ is the formula for a positive cone. Setting $z^2 = k^2$, we see that the curves $x^2 + y^2 = k^2$ are circles with radius k and centers at the origin.



4. If the position of an object is given by $r(t) = (\frac{1}{2}t^2 + 1)i + (t^2 + t - 2)j + (t^3 - t + 3)k$, then determine the tangential and normal components of acceleration.

Solution: This is a formulaic problem. Computing

$$r'(t) = ti + (2t + 1)j + (3t^2 - 1)k$$
$$r''(t) = i + 2j + 6tk,$$

we have

$$r'(t) \cdot r''(t) = t + 4t + 2 + 18t^4 - 6t = 18t^3 - t + 2,$$

and

$$r'(t) \times r''(t) = (6t^2 + 6t + 2)i - 3t^2 + 1)j - k.$$

Calculating the magnitudes of r' and the above cross product we have

$$|r'(t)| = \sqrt{t^2 + (2t+1)^2 + (3t^2 - 1)^2} = \sqrt{9t^4 - t^2 + 4t + 2}$$
$$|r'(t) \times r''(t)| = \sqrt{(6t^2 + 6t + 2)^2 + (3t^2 + 1)^2 + 1} = \sqrt{45t^4 + 72t^3 + 66t^2 + 24t + 6}.$$

Therefore, the tangential component of acceleration is

$$a_T = \frac{18t^3 - t + 2}{\sqrt{9t^4 - t^2 + 4t + 2}},$$

and the normal component of acceleration is

$$a_N = \frac{\sqrt{45t^4 + 72t^3 + 66t^2 + 24t + 6}}{\sqrt{9t^4 - t^2 + 4t + 2}}.$$

5. A cannon fires a ball with mass of 2 kg with an initial speed of 100 m/s at an angle of 45 degrees to the ground in the easterly direction. A southwesterly wind applies a steady force of $2\sqrt{8}$ N to the ball in a northeasterly direction. At what time does the ball land?

Solution: Let i be the north direction, j the east direction and k the upwards direction. Since force = mass $\times$ acceleration, we see that the force creates an acceleration of $2\sqrt{8}/2$ $=\sqrt{8}$ m/s^2 in the northeasterly direction. The vector of length $\sqrt{8}$ in the northeasterly direction can be described by $2i + 2j$. Combined with the acceleration due to gravity, the acceleration acting on the ball is $a(t) = 2i + 2j - 9.8k$. Therefore,

$$v(t) = \int a(t)dt = 2ti + 2tj - 9.8tk + \mathbf{C}.$$

Initially, $v(0) = 100\cos\pi/4j + 100\sin\pi/4k = 50\sqrt{2}j + 50\sqrt{2}k$. Therefore

$$v(t) = \int a(t)dt = 2ti + 2tj - 9.8tk + 50\sqrt{2}j + 50\sqrt{2}k.$$

Now, we integrate to solve for $r(t)$

$$r(t) = \int v(t)dt = \int 2ti + (2t + 50\sqrt{2})j - (9.8t - 50\sqrt{2})kdt = t^2i + (t^2 + 50\sqrt{2}t)j - (4.9t^2 - 50\sqrt{2}t)k + \mathbf{D}.$$

Initially, $r(t) = 0 \implies \mathbf{D} = 0$. So $r(t) = t^2i + (t^2 + 50\sqrt{2}t)j - (4.9t^2 - 50\sqrt{2})k$. The ball lands when $4.9t^2 - 50\sqrt{2}t = 0 \implies t = \frac{50\sqrt{2}}{4.9}$.