Math 20550 Calculus III Tutorial **Name:** ______________________________
April 3, 2014

Tutorial Worksheet

Show all your work.

1. Compute $\int_C x^2 ds$, C is the intersection of the surface $x^2+y^2+z^2=4$ and the plane $z=\sqrt{3}$.

Solution: We need to parametrize the curve. First, setting $z=\sqrt{3}$ in the equation of the surface, we get $x^2+y^2+3=4$ or $x^2+y^2=1$, which is a circle of radius 1. Therefore, we can parametrize x and y as a circle, with z as a constant $\sqrt{3}$.

Let $x=\cos t$, $y=\sin t$, $z=\sqrt{3}$, $0\leqslant t\leqslant 2\pi$.

$$ds=\sqrt{x'^2(t)+y'^2(t)+z'^2(t)}dt=dt$$

$$\int_C x^2 ds$$
$$=\int_0^{2\pi}\cos^2 t dt$$
$$=4\int_0^{\pi/2}\cos^2 t dt$$
$$=\pi$$

2. Evaluate $\int_C \sin(\pi y)dy+yx^2dx$ where C is the line segment from $(1,4)$ to $(0,2)$.

Solution: A parametric representation of the line segment is

$$x=1-t, y=4-2t, 0\leqslant t\leqslant 1$$

$$\int_C \sin(\pi y)dy+yx^2dx$$
$$=\int_0^1\sin(\pi(4-2t))(-2)dt+\int_0^1(4-2t)(1-t)^2(-1)dt$$
$$=-\frac{1}{\pi}\cos(4\pi-2\pi t)|_0^1-(-\frac{1}{2}t^4+\frac{8}{3}t^3-5t^2+4t)|_0^1$$
$$=-\frac{7}{6}$$

3. Let R be the region bounded by the ellipse $16x^2 + 4y^2 = 16$. Evaluate the integral $\int\int_R 2y dA$. [**Hint:** Use the transformation $x = u\cos(v)$, $y = 2u\sin(v)$.]

Solution: First, we convert our boundary into the new coordinates, thinking of them as basically polar coordinates. $16x^2 + 4y^2 = 16$ implies $16 = 16u^2\cos^2(v) + 4(4u^2\sin^2(v)) = 16u^2\cos^2(v) + 16u^2\sin^2(v) = 16u^2$, so $u^2 = 1$. Therefore the bounds in u and v of our region should be $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$.

We then convert our argument into u and v. $2y = 2(2u\sin(v)) = 4u\sin(v)$.

We also need the Jacobian:

$$\frac{\delta(x,y)}{\delta(u,v)} = \frac{\delta x}{\delta u}\frac{\delta y}{\delta v} - \frac{\delta x}{\delta v}\frac{\delta y}{\delta u}$$

$$= (\cos(v))(2u\cos(v)) - (-u\sin(v))(2\sin(v))$$

$$= 2u\cos^2(v) + 2u\sin^2(v) = 2u$$

Therefore $\int\int_R 2y dA = \int_0^1 \int_0^{2\pi} (4u\sin(v))(2u) dv du = \int_0^1 \int_0^{2\pi} 8u^2\sin(v) dv du = 0$

4. Let R be the parallelogram between the lines $2x - y = 3$, $2x - y = 5$, $x + y = -1$, and $x + y = 1$. Evaluate the integral $\int\int_R e^{2x-y} dA$.

Solution: This time, the change of variables is easiest to define by writing down T^{-1}, where $2x - y = u$ and $x + y = v$. It's immediately apparent that the bounds of integration must be $3 \leq u \leq 5$, $-1 \leq v \leq 1$.

To find the Jacobian, we solve for x and y in terms of u and v, finding $x = \frac{1}{3}(u + v)$ and $y = \frac{1}{2}(v - u)$. This lets us evaluate:

$$\frac{\delta(x,y)}{\delta(u,v)} = \frac{\delta x}{\delta u}\frac{\delta y}{\delta v} - \frac{\delta x}{\delta v}\frac{\delta y}{\delta u}$$

$$= \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{3}\right)\left(-\frac{1}{2}\right) = \frac{1}{3}$$

Finally we evaluate the integral itself:

$$\int\int_R e^{2x-y} dA = \int_3^5 \int_{-1}^1 (e^u)\left(\frac{1}{3}\right) dv du$$

$$= \int_3^5 \frac{2}{3} e^u du = \frac{2}{3}(e^5 - e^3)$$