Andrew J. Sommese, March 19, 2012

Let's compute Legendre polynomial series for functions.

```
> #Digits := 37;
```

```
> a := -1;b:=1;
```

$$a := -1$$
$$b := 1 \qquad (1)$$

```
> h := t -> 1;
```

$$h := t \rightarrow 1 \qquad (2)$$

Here is the new inner product

```
> IP := proc(f,g,x) evalf(Int(f*g*h(x),x=a..b)) end;
```

$$IP := \textbf{proc}(f, g, x)\ \ evalf(Int(f * g * h(x), x = a..b))\ \ \textbf{end proc} \qquad (3)$$

```
> with(orthopoly);
```

$$[G, H, L, P, T, U] \qquad (4)$$

```
> int(P(4,x)*P(4,x),x=-1..1);
  IP(P(4,x),P(4,x),x);
```

$$\frac{2}{9}$$
$$0.2222222222 \qquad (5)$$

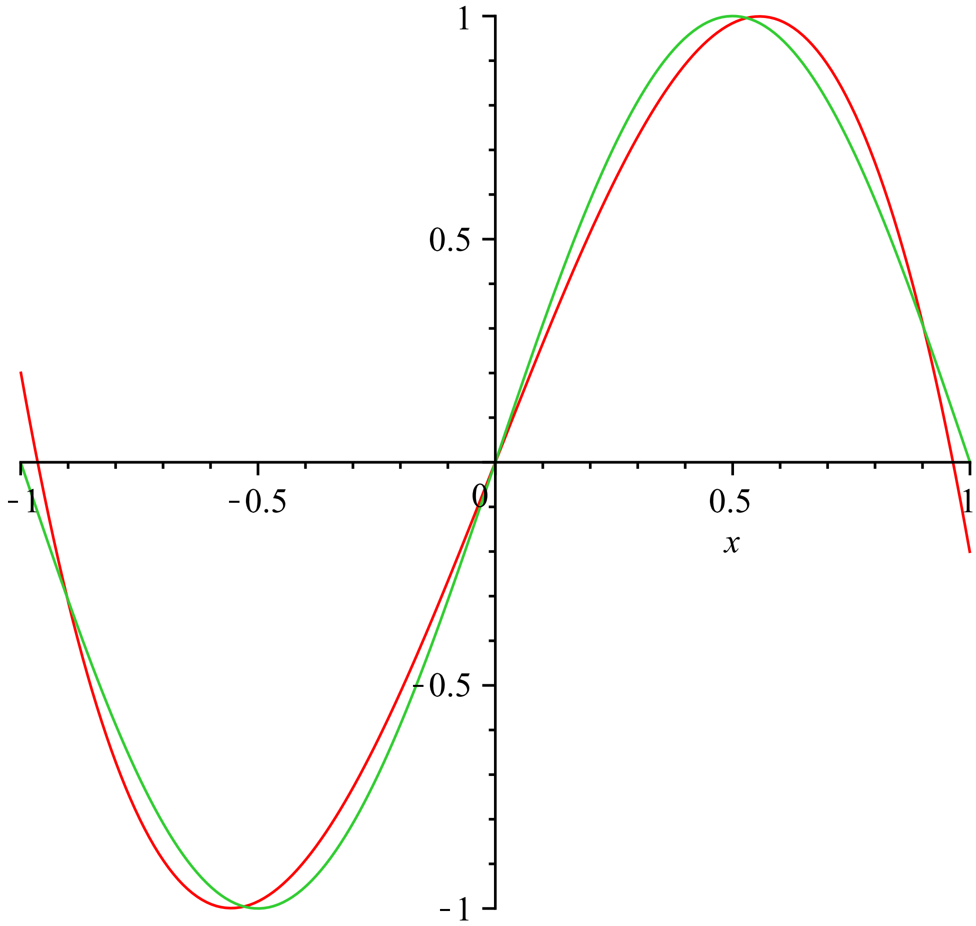
```
> IP(P(4,x),P(5,x),x);
```

$$0. \qquad (6)$$

Let's compute the series for sin(Pi*x)

```
> N:=4;
  for j from 0 to N do
  c[j]:= IP(P(j,x),sin(Pi*x),x)/(2/(2*j+1));
  od;
> u:=0:x:='x';
  for j from 0 to N do
  u:= u+c[j]*P(j,x);
  od:
  f := unapply(u,x);
> plot({sin(Pi*x),f(x)},x=-1..1);
```

$$N := 4$$
$$c_0 := 0.$$
$$c_1 := 0.9549296585$$
$$c_2 := 0.$$
$$c_3 := -1.158241912$$
$$c_4 := 0.$$
$$x := x$$
$$f := x \to 2.692292526\,x - 2.895604780\,x^3$$



```
> N:=6;
  for j from 0 to N do
  c[j]:= IP(P(j,x),sin(Pi*x),x)/(2/(2*j+1));
  od:
> u:=0:x:='x';
  for j from 0 to N do
  u:= u+c[j]*P(j,x);
  od:
  f := unapply(u,x);
> plot({sin(Pi*x),f(x)},x=-1..1);
```

$$N := 6$$

$$x := x$$

$$f := x \rightarrow 3.103460427\, x - 4.814388318\, x^3 + 1.726905184\, x^5$$